Abstract—A permanent-magnet synchronous motor drive that employs an adaptive control to find the maximum efficiency operating point at any speed and load is demonstrated. Active damping is provided by modulating the inverter frequency in proportion to the perturbations in the average inverter dc link current. All principal control functions, including efficiency optimization and frequency modulation, are performed by a real-time digital control algorithm, using only the filtered inverter dc link current as a feedback signal from the inverter. Laboratory tests and computer simulations demonstrate the performance of the efficiency-optimizing control and the frequency-modulation feedback loop.

INTRODUCTION

PERMANENT-MAGNET (PM) synchronous motors can offer significant efficiency advantages over induction machines when employed in adjustable-speed drives. The PM motor operates at synchronous speed and therefore does not have the slip losses inherent in induction motor operation. In addition, since much of the excitation in the PM motor is provided by the magnets, the PM motor will have smaller losses associated with the magnetizing component of stator current. These factors make the PM motor an attractive alternative to the induction machine in drives where overall efficiency is critical, notably in pump, compressor, or fan drives.

The PM motor can be operated at its optimal efficiency for a given speed and torque by adjusting the amplitude of the voltage to minimize the total losses. This condition departs somewhat from the condition of maximum torque per ampere in machines where there is appreciable core loss [1], [2], with the departure becoming increasingly significant at high speed. PM synchronous motors have traditionally been employed in self-controlled drives, in which the inverter frequency is dictated by the rotor speed. These so-called "brushless dc motor" drives typically operate at a fixed torque angle between the stator MMF and rotor position, with the result that the voltage is proportional to the speed. This feature, while advantageous for torque control, makes the brushless dc drive ill-suited to an efficiency-optimizing control where the voltage must be adjusted to minimize the losses. An efficiency-optimizing control is more easily achieved with a frequency-programmed drive, where there is independent control of the voltage and frequency.

This paper describes a frequency-controlled permanent-magnet synchronous motor drive that employs an adaptive control to find the maximum-efficiency operating point at any speed and load. A similar control was described in [3] for optimal efficiency control of an induction motor drive. A functional block diagram of the PM motor drive is given in Fig. 1. The efficiency control operates by measuring the current on the dc link and adjusting the voltage output of the inverter to minimize this quantity. In this way the system optimizes the combined motor-inverter efficiency, not just the motor efficiency. The speed is maintained constant by an independent open-loop control of the inverter frequency.

The stability problems associated with frequency-controlled synchronous motor drives have been extensively reported in the literature [4]-[10]. Studies of wound field machines [4], [5] and reluctance synchronous machines [6] have demonstrated the existence of low-frequency instability in this class of machine.

The surface-magnet PM motor differs from these examples in that there is no damper winding on the rotor. It may be expected to exhibit a mid-frequency instability similar to that observed in hybrid PM stepping motors [9], [10]. A linearized small-signal model shows that this instability is characterized by weak coupling between the electrical and mechanical modes, where the eigenvalues that represent the mechanical modes make a small bounded excursion into the right-half plane for operating frequencies above a certain value [10]. Several authors have shown that synchronous motor drives can be stabilized by appropriate modulation of the inverter frequency [7], [12]-[14]. Krause [13] used tachometer feedback to modulate the phase of the voltage applied to a reluctance synchronous machine in proportion to the perturbations in the rotor velocity. It is desirable to avoid using a tachometer or other direct feedback device, but instead to develop the feedback signals from electrical measurements made at the motor terminals or inside the inverter. A quasi-steady-state analysis of the PM motor drive is developed that shows how the perturbations in the dc link current can be used as a feedback signal. The analysis also yields a simple approximation useful in selecting the feedback gain for the stabilizer loop.

It is important to note that the two principal control
functions of this drive, the efficiency optimization and stabilization, are implemented with just one easily measured signal, the dc link current. No ac side current sensors or speed sensors are required.

**Efficient Operation of PM Motors**

There are basically two approaches to optimal efficiency control in electric machines [15]. One method employs a loss model of the machine and regulates the controlled quantities (voltages and currents) to minimize the estimated loss. This approach can be used to good advantage where the losses in the machine can easily be modeled in terms of the controlled quantities. For example, in a PM motor with no core loss a drive that operates at maximum torque per ampere will be optimally efficient.

It has been shown [1] that a PM motor operating at a fixed speed and torque has a unique optimum voltage that will minimize the total electrical loss. The existence of core loss will cause this optimum to deviate from the maximum torque per ampere condition, especially for high-speed operation.

A second approach to optimal efficiency control is to measure the power delivered to the drive and use a search algorithm to adjust a control variable until it detects a minimum in the power. The optimizing controller has the advantage that it does not depend on a loss model of the machine and therefore is insensitive to variations in the motor parameters, such as the temperature change in resistance, and does not require accurate modeling of complicated phenomena such as core loss.

For the optimizing control to work it is necessary that the losses be related to the control variable by a convex function, i.e., one with a single minimum. The natural choice of control variable for the PM synchronous motor is the voltage amplitude. However, the minimum in the loss-versus-voltage curve for the PM motor tends to be broad and flat, indicating that a search algorithm will have some difficulty converging to the true minimum. At the same time, the flatness of the minimum mitigates the consequences of error in the voltage. Any voltage near the optimum will yield nearly the same loss.

**Stabilization of PM Synchronous Motor Drives**

The damping of the local oscillations in an adjustable-speed drive can be improved by appropriate modulation of the supply frequency. An analysis of the small-signal dynamics in the quasi-steady-state approximation shows how this approach can be used to add damping. In the quasi-steady-state approximation a change in torque angle $\Delta \delta$ away from equilibrium gives rise to a restoring torque $\Delta T_e$ that moves the system back toward equilibrium. One may write

$$\Delta T_e = K_e \Delta \delta$$

where the electromagnetic spring constant $K_e$ is equal to the slope of the torque-angle curve at the equilibrium torque angle $\delta_0$, i.e.,

$$K_e = \frac{\partial T_e}{\partial \delta} |_{\delta_0}$$

The torque angle $\delta$ is defined as

$$\delta = \theta_e - \theta_r$$

where $\theta_e$ is the angle of the applied voltage vector with respect to a fixed reference, and $\theta_r$ is the angle of the internal voltage of the synchronous machine with respect to the same reference. These angles in turn are defined as

$$\theta_e = \int_0^t \omega_e(t) \, dt + \theta_e(0)$$
$$\theta_r = \int_0^t \omega_r(t) \, dt + \theta_r(0)$$

where $\omega_e$ and $\omega_r$ are the instantaneous electrical frequency and rotor velocity, in electrical rad/s. The angle $\delta$ is the usual torque angle of synchronous machine theory, and as used here is positive for motor action.

The rotor velocity, electrical frequency, and torque angle are expressed as the sum of steady-state and perturbation components, i.e.,

$$\omega_r = \omega_r + \Delta \omega_r$$

$$\omega_e = \omega_e + \Delta \omega_e$$

$$\delta = \delta_0 + \Delta \delta$$

Using these definitions in (3)-(5) gives

$$\Delta \delta = \int (\Delta \omega_e - \Delta \omega_r) \, dt.$$
frequency modulation proportional to the fluctuation in the average dc link current,

$$\Delta \omega_e = -K_c \Delta i_{dc}$$

$$= \frac{K_c}{V_{BUS}} \left[ \left( \frac{2}{n} \right)^2 J \omega_0 \frac{d}{dt} (-\Delta \omega_e) + \frac{2}{n} \frac{T_i}{T} \right]$$  \hspace{1cm} (14)$$

should provide the necessary damping.

This form of the feedback is incorporated in the block diagram in Fig. 3. Reduction of the block diagram gives the characteristic equation of this system as

$$s^2 + \frac{2}{n} \frac{K_c}{V_{BUS}} K_c \omega_0 s + \frac{n}{2 J} K_c \left( 1 + \frac{K_c}{n V_{BUS}} T_i \right) = 0.$$  \hspace{1cm} (15)$$

The damping factor $\alpha$, i.e., the real part of the complex roots, introduced by the dc link current feedback is equal to

$$\alpha = - \frac{1}{n V_{BUS}} \frac{K_c}{K_c \omega_0}.$$  \hspace{1cm} (16)$$

A constant damping factor can be achieved by maintaining the product $K_c \omega_0$ constant. A decrease in damping with frequency was noted in [7], in a study of an inverter-driven synchronous motor. The quasi-steady-state analysis presented here shows the reason for this.

The effect of the dc link current feedback on the small-signal dynamics can be more accurately evaluated by incorporating the dc link current and frequency modulation into the linearized small-signal model of the PM motor. The basic system equations from which the small-signal model is derived are presented in the Appendix, and a full derivation of the model is given in [16]. The perturbation component of dc link current is extracted by a first-order high-pass filter, with gain $K_c$ and time constant $\tau_c$.

The effect of a constant gain-frequency product on the system poles is seen in Fig. 4, which shows the locus of the principal natural frequencies for a permanent-magnet machine at full-load maximum torque per ampere as a function of the applied frequency. The motor is a four-pole inset-magnet machine of the type described in [17], and the parameters and nameplate data are given in the Appendix. The gain-frequency product in the simulation is $K_c \omega_0 = 2 \pi$ (300), and the filter time constant is 0.256 s. The locus of the natural frequencies without feedback is also shown for comparison.

The figure clearly shows the expected mid-frequency instability in the absence of feedback. It can also be seen on the figure that the feedback has made the damping of the poles nearly independent of the applied frequency.

A computer simulation of the PM synchronous motor drive demonstrates the need for and the effectiveness of the feedback stabilizer. The fundamental component only of the inverter voltage was modeled. Fig. 5 shows the response of the drive to a $-0.1 \text{ N} \cdot \text{m}$ step change in torque at 60-Hz full load, with and without the stabilizer. The damping due to the feedback is clearly manifest.
AN EXPERIMENTAL PM MOTOR DRIVE

System Description

A prototype PM synchronous motor drive was assembled in the laboratory to demonstrate the performance of the optimal efficiency control and the dc link current feedback stabilizer loop. The PWM voltage source inverter has sine-triangle modulation with independent control inputs for the frequency and amplitude of the sine-wave reference signals. A Hall-effect amplifier and four-pole Butterworth low-pass filter provide a signal proportional to the average dc link current.

The principal control functions, including the efficiency optimization routine and generation of the frequency and voltage commands, are performed by a real-time interrupt-driven digital control algorithm, implemented on a PC-DOS computer. The algorithm operates on a 2-ms interrupt cycle. Interface between the computer and inverter is via a 12-bit A/D-D/A board. The functions of the control algorithm are indicated in the flowchart of Fig. 6 and are described as follows.

DC Link Current Perturbation: The perturbation component of the dc link current, \( \Delta I \), is computed by subtracting the average current, \( I_{av} \), from the latest sample of the low-pass filtered dc link current, \( I_{DF} \).

Average Current: The average dc link current is computed as the running average of \( N_t \) samples of the filtered dc link current \( I_{DF} \). A value of \( N_t = 128 \) is used to give an average over 0.256 s. This signal is also used as an indication of the average power.

Acceleration Ramp: The nominal frequency \( F_0 \) is compared with the commanded value \( F^* \) and incremented or decremented as appropriate. This is done every second interrupt to give an acceleration time of 0-60 Hz in 5.5 s.

Voltage Command: The voltage command is obtained by adding a dc offset to the product of the nominal frequency and a \( V/f \) ratio.

Frequency Modulation: The dc link current perturbation \( \Delta I \) is multiplied by the feedback gain to obtain \( \Delta F \). The gain is inversely proportional to frequency above 9 Hz, and is stored in a look-up table indexed to the nominal frequency. \( \Delta F \) is subtracted from \( F_0 \) to obtain the actual frequency command.

Efficiency Optimization: The algorithm adjusts the voltage every \( N_e \) interrupts to search for the minimum-loss operating point. A value of \( N_e = 256 \) is used to cause a voltage adjustment every 0.512 s. The voltage adjustment is made by changing the voltage/frequency \( (V/f) \) ratio by a fixed amount. A test is performed at each iteration to check whether the power has increased or decreased. An increase in the power causes the search direction to change, and the voltage step size is reduced to permit closer convergence to the optimum. The search direction is unchanged if the power decreases. If four consecutive steps are taken in the same direction the step size is restored to its nominal value to permit faster convergence to the optimum.

System Performance

Efficiency Optimizing Control: The performance of the efficiency-optimization algorithm can be seen in Fig. 7, which shows the input power to the drive and the voltage command for an acceleration from 0 to 1800 rpm at no load. The nominal voltage/frequency ratio is set to give 220 V at 60 Hz, which is approximately the voltage for optimal efficiency operation at full load. During the acceleration phase, which
A measure of the possible energy savings can be made by running the drive at fixed speed, variable load, and by plotting the inverter input power against the shaft output power, comparing the performance of the optimal control with a fixed voltage control. Fig. 9 shows the results of one such test on the inset magnet motor at 60 Hz, with the fixed voltage value set equal to the optimum voltage at maximum load. The plot shows clearly that fixed-voltage operation results in excessive losses, especially at light load.

One advantage of the optimizing control is that it does not depend on an accurate model of the loss phenomena in the machine but requires only that there be a minimum in the loss as a function of the voltage. The control is therefore robust in the sense of being insensitive to motor parameter values. To demonstrate this, a load test was performed on a different type of PM motor, a ten-pole surface magnet motor, using the same inverter and control as for the inset magnet motor. The motor parameters are given in the Appendix. The results of the test are given in Fig. 10 and once again demonstrate the performance of the optimizing control in reducing the losses, especially at light load.

Stabilizer Loop: The previous results do not show explicitly the performance of the dc link current feedback used to stabilize the drive. The need for this loop is illustrated in Fig. 11, which shows the commanded speed and measured speed of the motor during acceleration from standstill to 1500 r/min at no load, without the stabilizer. Without the stabilizer loop the motor enters a limit cycle. Fig. 12 shows the commanded and actual speed with the stabilizer in effect. The modulation of the frequency command at low speeds can be noted on the figure, and the improvement in performance is obvious.

The effect of the feedback gain on the damping is demonstrated in Figs. 13 and 14. Fig. 13 shows the dc link current and frequency command in response to a sudden load change, with the feedback gain equal to twice the nominal value, or about 10 rad/s/A. In Fig. 14 the same test is performed but with half the nominal gain. The reduction in damping is quite clear. It can be noted also that the oscillation frequency is not appreciably affected, in accordance with the predictions of the small-signal model.

One important consequence of making the frequency-modulation calculations in software is that there will be a
threshold oscillation necessary to produce a calculated frequency modulation greater than the least significant bit of the D/A converter. This effect can be seen in Figs. 7 and 8, where a small oscillation in the input power can be seen at no load, which corresponds to small oscillations in the rotor velocity. An increase in the gain will reduce the magnitude of rotor oscillation needed to excite the stabilizer loop.

The threshold oscillation phenomenon is more clearly illustrated by computer simulation. The inverter is modeled as an ideal dc–ac converter, and the digital control algorithm is simulated exactly as implemented in the actual drive. Fig. 15(a) shows the rotor speed at 60 Hz, no load, with a feedback gain of 20. Fig. 15(b) shows the computed frequency modulation generated by the digital control algorithm. After an initial transient (imposed to excite the oscillations) the rotor settles into a sustained oscillation. It can be noted that the frequency modulation is nonzero only for brief periods, and is not the continuous function modeled in the small signal analysis. Figs. 16(a) and 16(b) show the rotor speed and frequency modulation for the same operating point, but with a
inverter dc link current in order to provide active damping. Quasi-steady-state analysis of the PM motor small-signal dynamics provides guidance in selecting the feedback gain for the stabilizer.

A digital control algorithm performs all primary control functions, including the frequency modulation and efficiency optimization, using only the filtered dc link current as an input signal from the inverter. No ac side current sensors or speed sensors are required. Use of the dc link current as a power signal assures that the combined inverter-motor efficiency will be optimized.

The overall system described in this paper is a true synchronous drive, not a brushless dc drive, and therefore does not operate at a fixed torque angle. This feature allows independent control of the frequency and voltage, necessary for the efficiency optimizing control. At the same time, the dynamic performance is not as good as that of a torque-controlled PM drive, since the efficiency control will not permit the rapid voltage changes necessary to achieve rapid torque response. This drive is therefore more suited to applications where efficiency and simplicity are more important than high dynamic performance, notably in pump, fan, and compressor drives.

**Appendix**

**System Equations**

The PM motor is modeled by the standard $d-q$ axis synchronous machine model, with no damper windings on the rotor, the permanent magnets considered to produce a constant component of stator flux linkage $\lambda_{mf}$, and the stator voltage amplitude is $V$. The stator voltage equations are

$$V \cos \delta = (r_s + pL_q)i'_d + \omega_L(\lambda_{mf} + L_di'_d)$$

$$- V \sin \delta = (r_s + pL_d)i'_q - \omega_L L_qi'_q.$$  

(A.1)  

(A.2)

The mechanical subsystem equations are

$$\frac{2}{n} J \omega_\tau = \frac{3}{2} \left[ \lambda_{mf} i'_q + (L_d - L_q)i'_d i'_d \right] - T_i$$

$$p\delta = \omega_\tau - \omega_r.$$  

(A.3)  

(A.4)

where $T_i$ is the load torque, $J$ is the inertia, and $n$ is the number of poles. The power balance equation gives the dc link current

$$V_{bus} i_d = V(i'_q \cos \delta - i'_d \sin \delta).$$  

(A.5)

**Motor Data**

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<th>Four-Pole</th>
<th>Ten-Pole</th>
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| Open Ckt. | Voltage  | 100 | 50 | $V @ 60 Hz$
|         | Rated current | 3.6 | 4.3 | A
|         | Rated torque | 4.1 | 4.8 | N·m
| $\lambda_{mf}$ | 281 | 108 | mV·s
| $L_q$ | 109 | 4.9 | mH
| $L_d$ | 35 | 4.9 | mH
| $R_s$ | 2.4 | 0.75 | $\Omega$
| $J$ | 2.9 | 2.0 | N·mm·s²
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Dr. Colby is a member of the IEEE, Eta Kappa Nu, and Sigma Xi.

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