Improved Two-Phase Stationary Frame EPLL to Eliminate Effect of Input Harmonics, Unbalance and DC offsets

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Abstract -- The effect of input harmonics, unbalance and DC offsets on the two-phase stationary frame enhanced phase locked loop (αβ-EPLL) is theoretically analyzed first. It indicated that the non-ideal grid conditions cause the periodic ripples in the estimated amplitude, frequency and phase angle. An improved αβ-EPLL by introducing two pure integrators and moving average filter (MAF) into both the amplitude and frequency estimation loops of αβ-EPLL is proposed to eliminate the effect of the aforementioned non-ideal grid conditions. An improved implementation of MAF to eliminate the effect of the variation of the grid frequency is proposed as well. The linear model of the improved αβ-EPLL is built and the design rule of the control parameters is determined. Detailed experimental results are presented to verify the validity and feasibility of the improved αβ-EPLL.

Index Terms – Enhanced phase-locked loop, αβ frame, DC offset, harmonics, unbalance

I. INTRODUCTION

In many grid-connected power conversion devices, the phase-locked loop (PLL) algorithm is usually used to online estimate the synchronous information of the grid to achieve a good control performance [1]-[6]. Nowadays, the synchronous reference frame PLL (SRF-PLL) [7]-[11] and three-phase stationary frame enhanced PLL (3P-EPLL) [12, 13] are most frequently used to estimate the phase angle and frequency of the three-phase grid voltage. 3P-EPLL is derived directly from the basic definition of the PLL algorithm and can estimate simultaneously not only the phase angle, the frequency but also the amplitude and has the merits of compact structure, robust to random noise and estimating the multiple variables at the same time. However, since 3P-EPLL is operated under the three-phase stationary frame, the calculation is still complicated.

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In [14], a two-phase stationary frame EPLL (αβ-EPLL) is proposed to estimate the synchronous information of the three-phase grid. Because the αβ-EPLL is performed under the two-phase stationary frame, the amount of trigonometric and multiplication operations is significantly reduced as compared to the standard 3P-EPLL and the calculation burden is reduced accordingly. As a result, it has widely potential application prospect.

Furthermore, it is well known that the effect of the non-ideal grid conditions, such as the harmonics, unbalance and even the DC offsets on the EPLL should be eliminated to ensure that the EPLL is able to apply in the actual system. Several improvement approaches of the 3P-EPLL to eliminate the effect of the non-ideal grid conditions on it have been proposed [15] - [18]. For example, in [15], a window-based filter is introduced in 3P-EPLL to eliminate the effect of the input harmonics and in [16, 17], three DC estimation integrators is introduced in 3P-EPLL to eliminate the effect of the input DC offsets. These approaches effectively improve the performance of 3P-EPLL. However, because the structure of αβ-EPLL is different from the three-phase one, these approaches successfully applied in it can not be directly applied in αβ-EPLL if without any modification or improvement. Up to now, no solution to eliminate the effect of the non-ideal grid conditions on αβ-EPLL is presented.

In this paper, the effect of non-ideal grid conditions on αβ-EPLL is theoretically analyzed for the first time and it proves that αβ-EPLL has no the ability to reject the effect mentioned above. Then by plugging two DC integrators and moving average filter (MAF) into αβ-EPLL , an improved αβ-EPLL ( I-αβ-EPLL ) is proposed to eliminate these effect. The linear model of I-αβ-EPLL is built and the design rule of the control parameters is determined. An improved implementation of MAF to eliminate the effect of the variation of the grid frequency is proposed as well. Because only additional two integrators and MAFs are used in I-αβ-EPLL , the calculation burden of the I-αβ-EPLL is not significantly increased as compared to the standard αβ-EPLL.

II. EFFECT OF NON-IDEAL GRID CONDITIONS ON αβ-EPLL

A. Overview of αβ-EPLL

The structure of αβ-EPLL is shown in Fig. 1, where the heavy line denotes vector calculation, thin line denotes algebraic operation and '·' means a dot product. The essential principle is briefly introduced as follows. The ideal three-phase grid voltages are expressed as
where \( U \), \( \omega \) and \( \phi \) denote the amplitude, frequency and initial phase angle of the input signals, respectively. They are transformed into two-phase stationary frame and the new signals can be expressed as

\[
\begin{align*}
u_a &= U \sin (\omega t + \phi) \\
u_b &= U \sin (\omega t + \phi - 120^\circ) \\
u_c &= U \sin (\omega t + \phi + 120^\circ)
\end{align*}
\]  

(1)

According to [14], the differential equations of the \( \alpha\beta \)-EPLL can be expressed as

\[
\begin{align*}
\dot{\mu}_a &= \mu_c \left[ e_{\alpha} \sin(\Delta \omega t + \phi) + e_{\beta} \sin(\Delta \omega t + \phi - 90^\circ) \right] \\
\dot{\mu}_b &= \mu_c \left[ e_{\alpha} \cos(\Delta \omega t + \phi) - e_{\beta} \cos(\Delta \omega t + \phi - 90^\circ) \right] \\
\dot{\mu}_c &= \Delta \omega + \mu_d \dot{\phi} + \omega_0 \\
\dot{e}_\alpha &= u_a(t) - y_a(t), \quad i = \alpha, \beta
\end{align*}
\]  

(3)

where \( y_\alpha, y_\beta, \dot{U}, \dot{\phi}, \dot{\phi} \) are the values of \( u_\alpha, u_\beta, U \), \( \omega \) and \( \phi \) estimated by \( \alpha\beta \)-EPLL. \( \mu_c \) is the integral coefficient of the amplitude estimation loop and \( \mu_d, \mu_c \) are the proportional and integral coefficients of the PI controller of frequency loop, respectively.

**B. Effect of input harmonics and unbalance**

The input signals of the \( \alpha\beta \)-EPLL containing the positive-, negative- and zero-sequence components and harmonics can be given by

\[
\begin{align*}
u_a &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^+ \sin \left( \omega h t + \phi_{n\alpha}^+ \right) + U_{n\beta}^- \sin \left( - \omega h t + \phi_{n\beta}^- \right) \right] \\
u_b &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^- \sin \left( - \omega h t + \phi_{n\alpha}^- \right) + U_{n\beta}^+ \sin \left( \omega h t + \phi_{n\beta}^+ \right) \right] \\
u_c &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^+ \sin \left( \omega h t + \phi_{n\alpha}^+ \right) + U_{n\beta}^- \sin \left( - \omega h t + \phi_{n\beta}^- \right) \right]
\end{align*}
\]  

(4)

where \( U_{n\alpha}^+, U_{n\beta}^-, U_{n\alpha}^- \) and \( \phi_{n\alpha}^+, \phi_{n\beta}^-, \phi_{n\alpha}^- \) are the amplitudes and initial phase angles of the positive-, negative- and zero-sequence \( h \)-order harmonics, respectively. When \( h = 1 \), all the variables denote the ones of the fundamental components.

According to 3s/2s transformation, the new input signals of the \( \alpha\beta \)-EPLL can be obtained as

\[
\begin{align*}
u_a &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^+ \sin \left( \omega h t + \phi_{n\alpha}^+ \right) + U_{n\beta}^- \sin \left( - \omega h t + \phi_{n\beta}^- \right) \right] \\
u_b &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^- \sin \left( - \omega h t + \phi_{n\alpha}^- \right) + U_{n\beta}^+ \sin \left( \omega h t + \phi_{n\beta}^+ \right) \right] \\
u_c &= \sum_{h=1}^{\infty} \left[ U_{n\alpha}^+ \sin \left( \omega h t + \phi_{n\alpha}^+ \right) + U_{n\beta}^- \sin \left( - \omega h t + \phi_{n\beta}^- \right) \right]
\end{align*}
\]  

(5)

Assuming that the \( \alpha\beta \)-EPLL tracks the positive-sequence components of input signals correctly, the input signals of frequency and amplitude loops are given by

\[
\begin{align*}
u_a &= U \sin (\omega t + \phi + \phi_0) \\
u_b &= U \sin (\omega t + \phi - 120^\circ + \phi_0) \\
u_c &= U \sin (\omega t + \phi + 120^\circ + \phi_0)
\end{align*}
\]  

where \( \phi_0 \) is the initial phase angle of the input signals.

**C. Effect of DC offsets**

The input signals containing DC offsets are defined as

\[
\begin{align*}
u_a &= U \sin (\omega t + \phi) + U_{dcA} \\
u_b &= U \sin (\omega t + \phi - 120^\circ) + U_{dcB} \\
u_c &= U \sin (\omega t + \phi + 120^\circ) + U_{dcC}
\end{align*}
\]  

(6)

The input signals in \( \alpha\beta \) frame can be derived as

\[
\begin{align*}
u_a &= U \sin (\omega t + \phi) - \tilde{U} \sin (\alpha \theta + \phi) + U_{dcA} - \frac{1}{2} (U_{dcB} + U_{dcC}) \\
u_b &= U \sin (\omega t + \phi - 90^\circ) + \tilde{U} \sin (\alpha \theta + \phi - 90^\circ) + \frac{\sqrt{3}}{2} U_{dcB} - \sqrt{3} U_{dcC} \\
u_c &= U \sin (\omega t + \phi + 90^\circ) + \tilde{U} \sin (\alpha \theta + \phi + 90^\circ) + \frac{\sqrt{3}}{2} U_{dcB} - \sqrt{3} U_{dcC}
\end{align*}
\]  

(7)

Then \( e_\alpha \) and \( e_\beta \) can be calculated as

\[
\begin{align*}
e_\alpha &= U \sin (\omega t + \phi) - \tilde{U} \sin (\alpha \theta + \phi) + U_{dcA} - \frac{1}{2} (U_{dcB} + U_{dcC}) \\
e_\beta &= U \sin (\omega t + \phi - 90^\circ) + \tilde{U} \sin (\alpha \theta + \phi - 90^\circ) + \frac{\sqrt{3}}{2} U_{dcB} - \sqrt{3} U_{dcC}
\end{align*}
\]  

(8)

The input signal of frequency loop can be given by

\[
\begin{align*}
u_a &= U \sin (\omega t + \phi - \alpha \theta - \phi) + (U_{dcA} - U_{dcB}) \cos (\alpha \theta + \phi) + \sqrt{3} (U_{dcB} - U_{dcC}) \cos (\alpha \theta + \phi - 90^\circ) \\
&= U \sin (\omega t + \phi - \alpha \theta - \phi) + U_{dcA} \cos (\alpha \theta + \phi - \phi_0)
\end{align*}
\]  

where

\[
\begin{align*}
u_a &= \sqrt{(U_{dcA} - U_{dcB})^2 + 3(U_{dcB} - U_{dcC})^2} \\
\phi_0 &= \arctan \left( \frac{\sqrt{3} (U_{dcB} - U_{dcC})}{2U_{dcA} - U_{dcB} - U_{dcC}} \right)
\end{align*}
\]  

(9)

(10)

(11)
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The input signal of the amplitude loop is derived as

\[ e_{\alpha} = U \cos(\omega t + \phi) - \tilde{U} + \left( \frac{1}{2} U_{\text{dc}} - \frac{1}{2} U_{\text{ac}} \right) \sin(\tilde{\omega} t + \tilde{\phi}) + \frac{\sqrt{2}}{2} \left( U_{\text{dc}} - U_{\text{ac}} \right) \sin(\tilde{\omega} t + \tilde{\phi} - 90^\circ) \] (12)

\[ = U \cos(\omega t + \phi - \tilde{\omega} t - \tilde{\phi}) - \tilde{U} + U_{\text{dc}} \sin(\tilde{\omega} t + \tilde{\phi} - \phi_{\text{dc}}) \]

Then when the estimation is completed, that is \( \tilde{\omega} = \omega \) and \( \tilde{\phi} = \phi \), the input signal of the frequency loop is

\[ e_{\omega} = U_{\text{dc}} \cos(\omega t + \phi_{\text{dc}}) \] (13)

And the angular frequency increment can be obtained as

\[ \Delta \omega = U_{\text{dc}} \left[ \mu_{\omega} \cos(\omega t + \phi_{\text{dc}}) + \frac{\mu_{\omega}}{\omega} \sin(\omega t + \phi_{\text{dc}}) \right] \] (14)

Likewise, the input signal of the amplitude loop is

\[ e_{\omega}^{a} = U_{\text{dc}} \sin(\omega t + \phi_{\text{dc}}) \] (15)

And the amplitude increment can be derived as

\[ \Delta U = -\frac{\mu_{\omega}}{\omega} U_{\text{dc}} \cos(\omega t + \phi_{\text{dc}}) \] (16)

It can be seen from (13) - (16) that, when the three-phase inputs contain the DC offsets, a periodic ripple at fundamental frequency is caused in both the frequency and amplitude outputs. When \( U_{\text{dc}} = U_{\text{ab}} = U_{\text{ac}} \), the periodic ripple becomes zero.

III. IMPROVED \( \alpha\beta\)-EPLL TO ELIMINATE EFFECT OF NON-IDEAL GRID CONDITIONS

The structure of the proposed \( 1-\alpha\beta\)-EPLL is shown in Fig. 2. Two DC offset estimation integrators are embedded in the \( \alpha\beta\)-EPLL to remove the DC offsets, where \( \mu_{\alpha\beta} \) denotes the integral coefficient of the DC integrator. The effect of the input harmonics and unbalance is eliminated by introducing MAF in both the amplitude and frequency loops. The details of \( 1-\alpha\beta\)-EPLL is described as follows.

A. Principle of DC Offset Elimination

The outputs of the two DC integrators are obtained as

\[
\tilde{U}_{\text{dc}} = \mu_{\alpha\beta} \int \left[ \begin{array}{c}
\text{Periodic components} \\
\text{DC components}
\end{array} \right] U \sin(\omega t + \phi) - \tilde{U} \sin(\tilde{\omega} t + \tilde{\phi}) + \left( \frac{1}{2} U_{\text{dc}} - \frac{1}{2} U_{\text{ac}} \right) \sin(\tilde{\omega} t + \tilde{\phi}) + \frac{\sqrt{2}}{2} \left( U_{\text{dc}} - U_{\text{ac}} \right) \sin(\tilde{\omega} t + \tilde{\phi} - 90^\circ) dt
\]

\[
\approx \mu_{\alpha\beta} \left( \frac{1}{2} U_{\text{dc}} - \frac{1}{2} U_{\text{ac}} \right) t
\]

\[
\tilde{U}_{\alpha\beta} = \mu_{\alpha\beta} \int \left[ \begin{array}{c}
\text{Periodic components} \\
\text{DC components}
\end{array} \right] U \sin(\omega t + \phi - 90^\circ) - \tilde{U} \sin(\tilde{\omega} t + \tilde{\phi} - 90^\circ) + \frac{\sqrt{2}}{2} \left( U_{\text{dc}} - U_{\text{ac}} \right) \sin(\tilde{\omega} t + \tilde{\phi} - 90^\circ) dt
\]

\[
\approx \mu_{\alpha\beta} \left( \frac{\sqrt{2}}{2} U_{\text{dc}} - \frac{\sqrt{2}}{2} U_{\text{ac}} \right) t
\]

Because the periodic component brings a zero average value to the output signal of the integrator, when \( \tilde{U}_{\text{dc}} \) and \( \tilde{U}_{\alpha\beta} \) are both equal to the input DC offsets, \( e_{\alpha} \) and \( e_{\omega} \) will become zero. The effect of the input DC offsets will be eliminated accordingly.

B. Elimination Effect of Harmonics and Unbalance Based on MAF

The s-domain transfer function of MAF is [19]-[21]

\[ G_{\text{MAF}}(s) = \frac{\tilde{x}(s)}{x(s)} = \frac{1 - e^{-T_{\text{sl}}s}}{T_{\text{sl}}s} \] (18)

where \( x(s) \) and \( \tilde{x}(s) \) are the input signal and its filtered one of MAF, respectively. \( T_{\text{sl}} \) is the window length of MAF. Then substituting \( s = j\omega \) into (18) yields

\[ G_{\text{MAF}}(j\omega) = \left| \frac{\sin(\omega T_{\text{sl}}/2)}{\omega T_{\text{sl}}/2} \right| \angle -\omega T_{\text{sl}}/2 \] (19)

It can be known from (19) that the gain of MAF at zero frequency is equal to 1. The gain at frequencies of \( n/T_{\text{sl}} \), \( n = 1, 2, \cdots \) is zero and the input periodic signals at these frequencies will be removed by MAF.

It can be known from (6) and (7) that, there are only even-order periodic ripples in the input signals of the amplitude and frequency loops, if setting \( T_{\text{sl}} = T_f / 2 \), the periodic ripples caused by the input harmonics and unbalance will be eliminated before they input the amplitude and frequency loops.

In the following, the implementation of MAF and the frequency adaptability is discussed. The z-domain transfer function of MAF can be expressed as

\[ G_{\text{MAF}}(z) = \frac{1 - z^{-N}}{N - z^{-1}} (N = \frac{f_s}{f_{\text{sl}}} = f_s T_{\text{sl}}) \] (20)

where \( f_s \) denotes the sampling frequency.

Because the actual grid frequency is usually not constant and varies in a certain range, if the window length of MAF keeps constant, the gain of MAF at the frequencies of \( 2n/T_f \), \( n = 1, 2, \cdots \) will not be zero and the periodic ripples will not be completely eliminated. In order to handle this matter, the frequency estimated by the EPLL is introduced to MAF after it is filtered by a low-pass filter (LPF). \( N \) will be tuned in real time as

\[ N = \text{Round}[f_s / 2T_f] \] (21)

where \( f_s \) is the filtered value of the estimated frequency.

A series of memorizers with the length of \( N \) are set in the microcontroller to implement MAF. Because \( N \) varies with the variation of the actual grid frequency, the choice of \( N \) and the implementation of MAF need to be carefully considered. \( N \) usually increases with the reduction of the...
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actual grid frequency, in order to ensure that the number of
the memorizers is large enough to store the sampling date
even when the actual grid frequency is equal to the smallest
value, the maximum $N$ should be set as

$$N_{\text{max}} = \text{Round}\left(\frac{f_s}{2f_{\text{min}}}ight)$$ (22)

In the following, the implementation of MAF when
considering the variation of the actual grid frequency is
discussed in detail. It should ensure that when $N$ varies,
the output of MAF $\tilde{x}(k)$ varies as smoothly as possible. The
iteration form of (18) can be expressed as

$$\tilde{x}(k) = \frac{1}{N'}[\tilde{x}(k-1) + x_i(0) - x_i(N')]$$ (23)

where $\tilde{x}(k-1)$ is the output of MAF in the last sampling
cycle, $x_i(0)$ is the latest sampling value of the input signal
of MAF and $x_i(N)$ is the oldest value of the data of the
memorizers. When the actual value of $N$ changes suddenly
to $N'$, if $N$ is replaced by $N'$ directly, $\tilde{x}(k)$ will change
suddenly. So (23) has to be modified to avoid this issue.
The case of $N'$ larger than the old $N$ is discussed first. In this
case $(N'-N)$ more memorizers will be used. In order
to reduce the peak of the sudden change of $\tilde{x}(k)$, when $N$
turns into $N'$, $\tilde{x}(k)$ is calculated as

$$\tilde{x}(k) = \frac{1}{N'}[\tilde{x}(k-1) + x_i(0) - x_i(N')]$$ (24)

At the same time the values of the additional $(N'-N)$
memorizers are all set as $\tilde{x}(k)$ . In the subsequent
calculation cycles, the calculation of $\tilde{x}(k)$ will use (23) by
replacing $N$ with $N'$ and it is turned back to a normal MAF.
When $N'$ is smaller than $N$, $\tilde{x}(k)$ will be calculated as

$$\tilde{x}(k) = \frac{1}{N'}[N\cdot\tilde{x}(k-1) - \sum_{i=N-N-1}^{N} x_i(i) + x_i(0)]$$ (25)

In the subsequent calculation cycles, the calculation of $\tilde{x}(k)$ will still use (23). It is worth noticing that, although
the similar issue is discussed in [22], because in [22] the
term of $1/N'$ is lost in the expression of $\tilde{x}(k)$, the result
is not correct and the general expression of $\tilde{x}(k)$ as
mentioned in (24) and (25) are not shown in [22].

C. Linear model and design of the control parameters

The differential equation after introducing DC offset
estimation loops in the $\alpha$-$\beta$-EPLL is turned into

$$\begin{cases}
e_i = u_t - y_t - \hat{U}_{dc}, \\
\hat{e}_i = \mu_{dc} e_i,
\end{cases}$$ (26)

where $\hat{U}_{dc}$ are the estimated DC offsets.

When the amplitude of input signal turns into $U_1$, $\hat{U}_{dc}$
can be derived as

$$\begin{cases}
\hat{U}_{dc,\alpha} = -\mu_{dc} (U_1 - U) \cos(\omega t + \phi) / \omega \\
\hat{U}_{dc,\beta} = -\mu_{dc} (U_1 - U) \cos(\omega t + \phi - 90) / \omega
\end{cases}$$ (27)

Substituting (27) to (26), the new error signals become

$$\begin{cases}
e_{i,\alpha} = (U_1 - U) \left[ \sin(\omega t + \phi) + \frac{\mu_{dc}}{\omega} \cos(\omega t + \phi) \right] \\
e_{i,\beta} = (U_1 - U) \left[ \sin(\omega t + \phi - 90) + \frac{\mu_{dc}}{\omega} \cos(\omega t + \phi - 90) \right]
\end{cases}$$ (28)

The new input signal of the amplitude loop is solved as

$$e_{i,\alpha} = \frac{3}{2} (U_1 - U) + \mu_{dc} \left[ \cos(\omega t + \phi) \sin(\omega t + \phi) \right] + \cos(\omega t + \phi - 90) \sin(\omega t + \phi - 90)$$ (29)

It means that the DC integrators do not change the
structure of the amplitude loop. Assuming that the phase
angle jumps to $\theta$, $\hat{U}_{dc}$ can be derived as

$$\begin{cases}
\hat{U}_{dc,\alpha} = -\mu_{dc} U \left[ \cos(\theta) - \cos(\theta) \right] \\
\hat{U}_{dc,\beta} = -\mu_{dc} U \left[ \cos(\theta - 90) - \cos(\theta - 90) \right]
\end{cases}$$ (30)

The new input signal of the frequency loop is solved as

$$e_{i,\alpha} = U (\sin(\theta - \theta) + \mu_{dc} U) \cos(\theta - \theta - 90) / \omega$$ (31)

The differential equation of $1-\alpha$-$\beta$-EPLL is expressed as

$$\begin{cases}
\dot{U} = -\mu \left[ \hat{U} - U \cos(\theta - \theta) \right] \\
\dot{\theta} = -\mu U \left[ \sin(\theta - \theta) + \mu_{dc} \left[ \cos(\theta - \theta) - 90 \right] / \omega \right]
\end{cases}$$ (32)

(32) is rewritten as

$$\begin{cases}
m = -\mu \left[ \text{Linear} \right] m - U \cos(\eta - 1) \eta \\
h = -\mu U \left[ \text{Linear} \right] \eta - \sin(\eta) + \mu_{dc} \left[ \cos(\eta - 1) - 1 \right] / \omega \eta
\end{cases}$$ (33)

where $m = U - \hat{U}$, $\eta = \theta - \theta$, $n = \omega - \omega$.

By replacing the trigonometric functions with their first-order
Taylor series around the stable operating point, the nonlinear terms of (33) can be regarded as zero and (33) can be simpliﬁed as

$$\begin{cases}
m = -\mu m, \quad \dot{n} = -\mu U \eta, \quad \dot{\eta} = n - \mu U \eta
\end{cases}$$ (34)

It can be seen that there is no $\mu_{dc}$ in (34), it means that
introducing the two DC integrators does not affect the
dynamic performance of both the amplitude and frequency
loops. Then $\mu_{dc}$ can be designed independently. The closed-loop
transfer function of the DC offset estimation loop is

$$G_{dc}(s) = \frac{\mu_{dc}}{s / \mu_{dc} + 1}$$ (35)

It means that a large $\mu_{dc}$ will obtain a fast estimation
speed. It can be known from (31) that a large $\mu_{dc}$ will
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enlarge the transient ripple amplitude of the frequency loop. So \( \mu_e \) is set as 50 to obtain a time constant of 20ms and a good compromise between the DC offset estimation speed and transient frequency estimation error.

It can be known from (34) that, the parameters of the equations contain \( U \), then by introducing the estimated amplitude into the frequency loop, the variable \( U \) can be removed, which has been verified in [13]. The linear transfer functions of the amplitude loop and frequency loop can be solved from (34) as

\[
G_a(s) = \frac{\bar{U}}{U} = \frac{K_b}{s + K_a}, \quad G_m(s) = \frac{\bar{\phi}}{\phi} = \frac{K_j}{s^2 + K_j s + K_j}
\]

The first-order form of \( G_{MFL}(s) \) can be expressed as

\[
G_{MFL}(s) = \frac{1}{T_s s / 2 + 1}
\]

Because MAF is plugged in either the feedforward path of the amplitude loop or the frequency loop, by replacing \( \mu \) with \( \mu G_{MFL}(s) \) and replacing \( \mu_n \) and \( \mu_o \) with \( \mu G_{MFL}(s) \) and \( \mu G_{MFL}(s) \) in (36), the new linear model of the I-\( \alpha\beta \)-EPLL is solved as

\[
\begin{align*}
G_{al}(s) &= s + \mu G_{MFL}(s) - \frac{2 \mu_s}{T_f} s^2 + 2s / T_f + 2 \mu_s / T_f, \\
G_{am}(s) &= \frac{2 \mu_o}{T_f} s + 2 \frac{\omega_o}{T_f} s + \frac{2 \mu_s}{T_f}
\end{align*}
\]

In the following, the design rule of the control parameters is discussed. The standard form of the transfer function of amplitude loop can be rewritten as

\[
G_{al}(s) = \frac{2 \mu_s}{T_f} s^2 + 2 s / T_f + 2 \mu_s / T_f, \quad \omega_o = \frac{2 \mu_s}{T_f}, \quad \omega_o = \frac{2 \mu_s}{T_f}
\]

By setting \( \xi = 1 \), it can be solved that \( \mu_s = 1 / (2T_f) \).

Because the characteristic equation of \( G_{al}(s) \) is 3-order and to simplify the design procedure, it is assumed that the characteristic equation possesses a real root and two conjugated roots with the same real part as the real root. \( G_{am}(s) \) is rewritten as

\[
G_{am}(s) = \frac{2 \mu_s}{T_f} \frac{1}{s + \alpha} (s + \alpha + j \beta) (s + \alpha - j \beta)
\]

By setting the coefficients of (38) equal to the ones of (40), the following equations yield

\[
\alpha = \frac{2 \mu_s}{3T_f}, \quad \beta = \frac{2 \mu_o}{T_f}, \quad 4 \beta^2 + \beta^2 = 3 \mu_o
\]

In theory, \( \mu_o \) and \( \mu_o \) can be solved as infinite solutions when \( \beta \) varies in a wide range. If setting \( \beta = 0 \), it can be solved as \( \mu_o = 2 / (3T_f) \) and \( \mu_o = 4 / (2T_f) \). In this case, there are three real roots of the characteristic equation with the same value and it will be always stable because the real root is less than zero.

IV. EXPERIMENTAL RESULTS AND COMPARISON

The I-\( \alpha\beta \)-EPLL is implemented in the experimental platform based on the DSP chip of TMS30F28335 to test its performance. First, the experimental results of the \( \alpha\beta \)-EPLL and its improved version suffering from the DC offset and input harmonics are tested and shown in Fig. 3. A 0.1 p.u. DC offset is added in phase \( a \) and \( b \) first and the 5th, 7th, 11th and 13th harmonics with the amplitude equal to 0.1 p.u. are added after 100 ms. Furthermore, after another 100ms, both the DC offset and harmonics are removed. It can be seen from Fig. 3 that, I-\( \alpha\beta \)-EPLL estimates the information of the input signals accurately under ideal conditions. However, when the input signal contains the harmonics or DC offset, the performance is deteriorated.

The proposed I-\( \alpha\beta \)-EPLL estimates the required information without the steady-state error and ripples under both harmonics and DC offset conditions. The settling time is less than 30 ms.

Second, the performance of the \( \alpha\beta \)-EPLL and improved one under unbalanced condition is tested and the results are shown in Fig. 4. The amplitude of phase \( a \) suddenly reduces to 0.1 p.u. and the phase angle of phase \( b \) jumps 80°. After 100ms, the input signals turn back to three-phase symmetric sine waves. It can be seen from Fig. 4 that, the amplitude, phase angle and frequency estimated by \( \alpha\beta \)-EPLL contain a periodic ripple at twice fundamental frequency. As for the I-\( \alpha\beta \)-EPLL, the amplitude, phase angle and frequency of the positive-sequence voltage components of three inputs are accurately estimated. Besides, there almost are no overshoots in the estimated amplitude and phase angle during the transient process. The maximum frequency error is 10Hz and the settling time is 30ms.

Finally, the dynamic response of the I-\( \alpha\beta \)-EPLL, including the amplitude step of ±40%, phase angle step of 40° and frequency step of ±10% of the three-phase input signals is respectively tested. The corresponding experimental waveforms of the original input signals and the ones in \( \alpha\beta \) frame and the amplitude, frequency and phase angle estimated by the I-\( \alpha\beta \)-EPLL and the estimation errors are all shown in Fig. 5. The dynamic response data of three classic PLL algorithm, including MAF based SRF-PLL in [19], 3P-EPLL, dual-second-order generalized integrator PLL (DSOGI-PLL) in [21] and I-\( \alpha\beta \)-EPLL are listed in Table I as well, where \( T_m \) is the settling time, \( \Delta A \), \( \Delta \phi \), \( \Delta \omega \) are the maximum errors of the estimated amplitude, phase angle and frequency, respectively.

It can be seen from Fig. 5 and Table I that, the amplitude, phase angle and frequency estimated by the I-\( \alpha\beta \)-EPLL are immediately updated when the actual variables change suddenly. The overshoots of the three estimated variables are very small and it ensures the excellent dynamic performance. The dynamic response times of the I-\( \alpha\beta \)-EPLL in the three cases are similar with the ones of MAF based SRF-PLL and 3P-EPLL. Furthermore, while only the I-\( \alpha\beta \)-EPLL is able to eliminate the effect of the input harmonics, unbalance and DC offsets simultaneously. The execution time of the I-\( \alpha\beta \)-EPLL is almost equal to the one of the 3P-EPLL, it indicates that although the introduction of the DC integrators and MAFs increases the execution time, because the I-\( \alpha\beta \)-EPLL is operated in \( \alpha\beta \) frame, the execution time is not increased obviously.
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DC offset
Harmonics
Removed

Input signals(0.4p.u./div)

Amplitude(0.4p.u./div)

Amplitude error(0.2p.u./div)

Frequency(20Hz/div)

Frequency error(2Hz/div)

Phase angle(180 /div)

Phase error(5 /div)

Time(40ms/div)

(a)

(b)

Fig. 3. Experimental waveforms of the PLLs suffering from the input DC offsets and harmonics, (a) αβ-EPLL, (b) Improved one.

V. CONCLUSION

The effect of the non-ideal grid conditions on the αβ-EPLL is originally analyzed in this paper and it proved that the standard αβ-EPLL does not achieve a correct estimation of the grid synchronous information under non-ideal grid conditions. Furthermore, an improved version is proposed to eliminate the effect of non-ideal input conditions simultaneously, including the harmonics, unbalance and DC offsets. It is verified by the experimental results that the proposed I-αβ-EPLL estimates the synchronous information of the grid correctly under both the ideal and the non-ideal grid conditions. The proposed I-αβ-EPLL possesses the optimum comprehensive performance and solves the contradictory between the estimation accuracy, dynamic response and execution time.

REFERENCES

Fig. 4. Experimental waveforms under unbalance conditions, (a) αβ-EPLL, and (b) Improved one.


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![Fig. 5. Dynamic experimental waveforms of I-αβ-EPLL. (a) Amplitude step, (b) Phase step, and (c) Frequency step.](image)

**TABLE I. Dynamic Data of Four PLLs**

<table>
<thead>
<tr>
<th>Dynamic Cases</th>
<th>Data</th>
<th>SRF-P LL</th>
<th>3P-EPLL</th>
<th>DSOGI-P LL</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (±40%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sw}$ (ms)</td>
<td>40</td>
<td>20</td>
<td>30</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>$ΔA_α$ (p.u.)</td>
<td>-</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>$Δφ_α$ (°)</td>
<td>15</td>
<td>11</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>$Δω_α$ (Hz)</td>
<td>17</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>Phase (40°)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sw}$ (ms)</td>
<td>40</td>
<td>20</td>
<td>50</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>$ΔA_α$ (p.u.)</td>
<td>-</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$Δφ_α$ (°)</td>
<td>7</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>$Δω_α$ (Hz)</td>
<td>2</td>
<td>0.5</td>
<td>0</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Frequency (±10%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sw}$ (ms)</td>
<td>40</td>
<td>20</td>
<td>50</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>$ΔA_α$ (p.u.)</td>
<td>-</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$Δφ_α$ (°)</td>
<td>7</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>$Δω_α$ (Hz)</td>
<td>2</td>
<td>0.5</td>
<td>0</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Harmonics rejection</td>
<td>yes</td>
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<td>yes</td>
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</tr>
<tr>
<td>Unbalance rejection</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>DC offset rejection</td>
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<td>no</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Execution Time (us)</td>
<td>-</td>
<td>3.04</td>
<td>7.9</td>
<td>3.05</td>
<td></td>
</tr>
</tbody>
</table>

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