Armature-Reaction Magnetic Field Analysis for Interior Permanent Magnet Motor Based on Winding Function Theory
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As the magnets are embedded in a rotor for the interior permanent magnet (IPM) motor, the distribution of the armature reaction magnetic field is different from that of the surface permanent magnet motor (SPM) type. Various methods have been used for the magnetic field solution, including analytical methods based on the winding function and the Laplacian–Poisson equation. However, in the IPM motor, the boundary condition is too complicated for using the Laplacian–Poisson equation. What is more, the equivalent air-gap inverse function presented in literature is not suitable for the IPM motor. A new armature reaction magnetic field model is proposed for the IPM motor, considering the effect of the embedded magnet in the rotor, which is named a pole-cap effect in this paper. The proposed model is derived from the winding function theory, and the rotor magnetic motive force (MMF) function is employed to model the so-called pole-cap effect. The proposed model is used to predict the armature reaction field under a different type of stator MMF, such as different orientation of the excitation current as well as various MMF wavelengths. The calculation result is validated by the finite element analysis (FEA) and shows remarkable advantages over the traditional method. The new model proposed in the paper is quite useful for evaluating various IPM motor performances in an accurate and time-effective manner, such as inductance, stator core losses, and magnet eddy current losses. Complete demonstration of the method to calculate the aforementioned performance indices will be presented in a separate paper, and inductance calculation of a primitive winding is given as an example at the end of this paper.

Index Terms—Armature reaction magnetic field, interior permanent magnet (IPM) motor, pole-cap effect, winding function.

LIST OF SYMBOLS

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(N(\gamma))</td>
<td>Winding function.</td>
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<tr>
<td>(I(zt))</td>
<td>Current excitation.</td>
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<td>(F_s)</td>
<td>Stator MMF generated by current excitation.</td>
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<td>(F_{r,\text{arm}})</td>
<td>Rotor MMF induced by (F_s).</td>
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<td>(F_{g,\text{arm}})</td>
<td>Air-gap MMF in response to (F_s).</td>
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<td>(g^{-1})</td>
<td>Air-gap inverse function.</td>
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<td>(\gamma)</td>
<td>Angular position in the stator reference frame measured from the axis of phase (\alpha).</td>
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<tr>
<td>(B_{g,\text{arm}})</td>
<td>Air-gap flux density induced by (F_s).</td>
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<tr>
<td>(g_e)</td>
<td>Air-gap height.</td>
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<tr>
<td>(h_m)</td>
<td>Magnet height.</td>
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<tr>
<td>(w_m)</td>
<td>Magnet width.</td>
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<tr>
<td>(L)</td>
<td>Effective length.</td>
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<tr>
<td>(\alpha_p)</td>
<td>Magnet pole-arc coefficient.</td>
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<tr>
<td>(U_{\text{amp}})</td>
<td>Magnet voltage drop of the pole cap.</td>
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<tr>
<td>(r)</td>
<td>Air-gap radius.</td>
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<tr>
<td>(D_g)</td>
<td>Air-gap diameter.</td>
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<tr>
<td>(P)</td>
<td>Pole number.</td>
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<tr>
<td>(\mu_r)</td>
<td>Relative permeability of a magnet.</td>
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<tr>
<td>(\mu_v)</td>
<td>Permeability of a vacuum.</td>
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<tr>
<td>(sw)</td>
<td>Square wave.</td>
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<tr>
<td>(R_q)</td>
<td>Air-gap reluctance in the pole-arc range.</td>
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<tr>
<td>(R_{q,1})</td>
<td>Air-gap reluctance in the magnet pole-arc range.</td>
</tr>
<tr>
<td>(R_m)</td>
<td>Magnet reluctance.</td>
</tr>
<tr>
<td>(\Phi_{\text{arm}})</td>
<td>Flux flowing into the pole cap.</td>
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I. INTRODUCTION

In recent decades, an interior permanent magnet (IPM) motor has been widely used in high-performance applications, due to its high efficiency, wide speed range, and low size and weight for a given torque. Accurate evaluation of the armature reaction magnet field is of great importance because it forms the foundation to get the parameter and performance indices of an IPM motor, such as inductances, iron loss, and torque ripple. Different methods have been proposed to make this possible. These methods are roughly divided into two types: analytical methods based on the winding function [1]–[18] and the Laplacian–Poisson equation [19]–[25].

As magnets are embedded in the rotor of the IPM motor, the boundary condition is too complicated to get the armature reaction field distribution based on the Laplacian–Poisson equation. Therefore, this type of method has only been applied to obtain the field distribution in a surface permanent magnet motor (SPM) [19], [20], and a surface inset permanent magnet (PM) motor [24], [25]. Based on the field distribution, performance index, such as eddy-current loss in SPM, has been obtained [21]–[23].
Hence, when it comes to the armature reaction magnetic field analysis in the IPM motor, an analytical method based on the winding function is an attractive candidate. In this method, the winding function is employed to describe the spatial distribution of the stator magnetic motive force (MMF) in response to the instantaneous current excitation. The air-gap inverse function defined to represent the magnetic reluctance variation seen in the stator frame [11]. The air-gap magnetic field density distribution can be derived from the winding function and the air-gap inverse function.

This type of method was first introduced in 1965, and used to analyze induction motors [1]. Since 1992, this theory has been applied to study the performance of the induction motor fault analysis [2]–[6], when the air-gap inverse function is a fixed-amplitude straight line. Up to now, the winding function method has been extended to analyze the salient synchronous and synchronous reluctance motor [7]–[12], the surface permanent motor [13], [14], and the IPM motor [15]–[18].

In [14], a modified air-gap inverse function is proposed for armature reaction field evaluation in the surface inset PM motor. Unlike the original air-gap inverse function, the amplitude of the modified function in [14] is not constant as the result of the existence of the magnet. This work presents a simple method to calculate the modified equivalent air-gap inverse function: the air-gap height in the partition where the magnet mounted is seen as the summation of the actual air-gap height and the height of the magnet, and the remaining partition is seen only as the actual air gap.

However, in the IPM motor, the situation is different, making the methods in literature not working well. The nearly infinite permeance pole cap above the magnet makes the upper surface of the magnet an equal-potential area, and this is named a pole-cap effect in this paper. As a result, in the IPM motor, the equivalent air-gap inverse function cannot be expressed as the form of the synchronous motor [7]–[12] or the surface inset permanent motor [14].

In [15]–[18], a rotor MMF function is used instead of a modified air-gap inverse function to model the pole-cap effect. However, the rotor MMF is shown as an indistinct expression, and the finite element analysis (FEA) and the magnetic equivalent circuit (MEC) are employed to obtain the rotor MMF function for torque ripple and eddy loss optimization.

In this paper, an analysis method is proposed to predict the armature reaction magnetic field for the IPM motor, based on the winding function and the rotor MMF. To calculate the rotor MMF function, the relationship between the rotor MMF and the stator MMF is established.

The proposed model is used to predict the armature reaction field under a different type of stator MMF, such as different orientation of the excitation current as well as various MMF wavelengths. The calculation result is validated by the FEA and shows remarkable advantages over the traditional method. The new model proposed in the paper is quite useful for evaluating various IPM motor performances in an accurate and time-effective manner, such as inductance, stator core losses, and magnet eddy current losses. Complete demonstration of the method to calculate the aforementioned performance indices will be presented in a separate paper, and the inductance calculation of a primitive winding is given as an example at the end of this paper.

II. TRADITIONAL METHOD BASED ON THE WINDING FUNCTION

A. Winding Function and Stator MMF

In this section, some important physical concepts of the PM machine as well as their mathematical expressions are reviewed.

The winding function is formally defined to represent any actual winding configuration, and can be written as

$$
N(\gamma) = n(\gamma) - \langle n(\gamma) \rangle
$$

(1)

where function $n(\gamma)$ is called the turns function, while $\langle n(\gamma) \rangle$ is the average value of the turns function $n(\gamma)$ [11]. $\gamma$ is the angular position in the stator reference frame measured from the axis of phase $a$.

According to [11], the winding function $N(\gamma)$ of phase $a$ is even symmetric about the phase axis and the winding functions of phases $b$ and $c$ are identical but displaced by $2\pi/3$ and $4\pi/3$ electrical radians (elec.rad), respectively. So the winding function of a three-phase motor can be expanded into a Fourier series as

$$
\begin{align*}
N_n(\gamma) &= \sum_{h=1,3,5,\ldots} N_h \cos (h\gamma) \\
N_k(\gamma) &= \sum_{h=1,3,5,\ldots} N_h \cos \left( h\gamma - \frac{2}{3}\pi \right) \\
N_\omega(\gamma) &= \sum_{h=1,3,5,\ldots} N_h \cos \left( h\gamma - \frac{4}{3}\pi \right)
\end{align*}
$$

(2)

where $N_h$ is the amplitude of the $h$th-order harmonic.

Assuming the three-phase winding current is given by

$$
\begin{align*}
I_a(\omega t) &= I_1 \cos (\omega t - \psi_1) \\
I_b(\omega t) &= I_1 \cos \left( \omega t - \frac{2}{3}\pi - \psi_1 \right) \\
I_c(\omega t) &= I_1 \cos \left( \omega t - \frac{4}{3}\pi - \psi_1 \right)
\end{align*}
$$

(3)

where $\psi_1$ is the current phase measured from the $d$-axis, $\omega t$ is the instantaneous rotor angular position.

For the given set of phase currents as (3), the stator MMF $F_s(\gamma, \omega t)$ is given by

$$
F_s(\gamma, \omega t) = N_a(\gamma)I_a(\omega t) + N_k(\gamma)I_b(\omega t) + N_\omega(\gamma)I_c(\omega t)
$$

$$
= \sum_{h=1,3,5,\ldots} \kappa_h N_h I_1 \cos \left( (h\gamma + \omega t) - \psi_1 \right) + \sum_{h=1,3,5,\ldots} \kappa_h N_h I_1 \cos \left( (h\gamma - \omega t) + \psi_1 \right)
$$

$$
= \sum_{h=1,3,5,\ldots} F_s, h \cos \left( (h\gamma \pm \omega t) \mp \psi_1 \right)
$$

(4)

where $\kappa_{h \pm 1}$ is a stator MMF coefficient, and is defined as follows:

$$
F_s, h = \kappa_{h \pm 1} N_h I_1
$$

(5)

$$
\kappa_{h \pm 1} = \begin{cases} 
\frac{h}{3}, & h \pm 1 = 3m, \quad m = 1, 2, 3, \ldots \\
0, & h \pm 1 \neq 3m
\end{cases}
$$

(6)

The air-gap inverse function is defined to represent the magnetic reluctance variation seen in the stator frame. Taking ac-
count of PM distribution in the surface inset PM motor, the modified air-gap inverse function is defined by [14]

$$g^{-1}(\gamma, \omega t) = \begin{cases} \frac{1}{g_e} & -\frac{\pi}{2} \leq \gamma - \omega t \leq -\frac{\alpha_p \pi}{2} \\ \frac{1}{g_e + \frac{b_m}{\mu_r}} & -\frac{\alpha_p \pi}{2} \leq \gamma - \omega t \leq \frac{\alpha_p \pi}{2} \\ \frac{1}{g_e} & \frac{\alpha_p \pi}{2} \leq \gamma - \omega t \leq \frac{\pi}{2} \end{cases}$$

(7)

where $g_e$ is the air-gap height, $h_m$ is the magnet height, and $\alpha_p$ is the magnet pole-arc coefficient.

Fig. 1 shows a surface inset PM machine cross section in a pole pitch.

A generalized air-gap inverse function for any type of PM machine can be expressed as the Fourier series form, shown in

$$g^{-1}(\gamma, \omega t) = \delta_0 + \sum_{m=2,4,6}^{\infty} \delta_m \cos(m(\gamma - \omega t - \theta_m))$$

(8)

where $\theta_m$ is the angle between the $d$-axis and phase $n$ in the initial position.

B. Armature Reaction Field

With the help of the winding function and the air-gap inverse function, the armature-reaction air-gap magnetic field can be expressed as

$$B_{g,\text{attr}}(\gamma, \omega t) = \mu_0 g^{-1}(\gamma, \omega t) F_x(\gamma, \omega t).$$

(9)

The model presented in (9) can be used to calculate the motor parameter such as inductance, expressed in

$$L_{XY} = \frac{\lambda_{XY}}{I_Y} = \frac{r L \int_0^{2\pi} B_{g,\text{attr}}(\gamma, \omega t) N_X(\gamma) d\gamma}{I_Y}$$

= \frac{\mu_0 r L \int_0^{2\pi} g^{-1}(\gamma, \omega t) N_Y(\gamma) I_Y N_X(\gamma) d\gamma}{I_Y}$$

= \frac{\mu_0 r L \int_0^{2\pi} g^{-1}(\gamma, \omega t) N_X(\gamma) N_Y(\gamma) d\gamma}{I_Y}$$

(10)

where $X$ and $Y$ mean any phase of a three-phase motor.

III. ARMATURE REACTION ANALYSIS MODEL FOR IPM

From the investigation of (9), it can be seen that, the effect of the magnet reluctance in the magnet circuit is modeled as the so-called air-gap inverse function. The traditional model implies that all harmonic components in the stator MMF would confront the same magnet reluctance distribution. But this is certainly not true, especially in the IPM motor.

Certain assumptions have to be made for the new model proposed in this paper.

- Stator is smooth, thus slot effect is ignored.
- Saturation in the lamination core is neglected.
- Only the single-layer winding is considered.
- Flux passing through magnet bridges is disregarded.

According to [1], the winding function is derived upon the assumption of a smooth stator, so in the proposed model, the slot effect is ignored.

In most conditions, except overload, the magnet reluctance of a core is far smaller than the air-gap reluctance and the magnet reluctance, so generally the saturation in the core is neglected.

A single-layer winding is the basic type of winding; the double-layer-type and multilayer-type windings can be derived from a single-layer-type winding, so only a single-layer winding is considered.

In the IPM, the flux entering the pole cap is equal to the flux through the magnet plus the flux passing out through magnet bridges. The flux passing through magnet bridges is far smaller than the flux passing through the magnet. So in the proposed model, the flux passing through a magnet bridge is disregarded, and the flux entering the pole cap is assumed equal to the flux passing through the magnet.

Given the pole-cap effect in the IPM motor, an instantaneous spatial distribution of a rotor armature reaction MMF $F_{r,\text{arm}}(\gamma, \omega t)$ is introduced, and the air-gap MMF $F_{g,\text{arm}}(\gamma, \omega t)$ is expressed as

$$F_{g,\text{arm}}(\gamma, \omega t) = F_x(\gamma, \omega t) - F_{r,\text{arm}}(\gamma, \omega t).$$

(11)

Therefore, the armature reaction magnetic field can be expressed as

$$B_{g,\text{arm}} = \frac{\mu_0}{g_e} F_{g,\text{arm}}(\gamma, \omega t)$$

= \frac{\mu_0}{g_e} [F_x(\gamma, \omega t) - F_{r,\text{arm}}(\gamma, \omega t)].$$

(12)

A. Rotor MMF

The nearly infinite permeance pole cap above the magnet makes the upper surface of the magnet an equal-potential area. The waveform of the rotor MMF induced by the stator MMF represents that the magnetic voltage drop (MVD) crossing the magnet is constant, no matter what kind of excitation and magnet reluctance distribution there is. $F_{r,\text{arm}}(\gamma, \omega t)$ can be expressed as a piecewise function in electrical radians

$$F_{r,\text{arm}}(\gamma, \omega t) = \begin{cases} \frac{U_{\text{arm}}}{2} & \frac{\alpha_p \pi}{2} < \gamma - \omega t < \frac{\alpha_p \pi}{2} \\ \frac{\alpha_p \pi}{2} & \frac{\alpha_p \pi}{2} < \gamma - \omega t < \frac{\pi}{2} \end{cases}$$

(13)
where \( U_{\text{amp}} \) is the MVD crossing the magnet. Therefore, the rotor MMF \( F_{r,\text{arm}}(\gamma, \omega t) \) is a periodic function, and can be expressed in

\[
F_{r,\text{arm}}(\gamma, \omega t) = U_{\text{amp}} \cdot \text{sw}(\gamma - \omega t, \alpha_p)
\]

and \( \text{sw}(\gamma - \omega t, \alpha_p) \) is a unity square-wave function, rotating synchronously with the rotor, as shown in Fig. 2.

The square-wave \( \text{sw}(\gamma - \omega t, \alpha_p) \) can be expanded in a Fourier series

\[
\text{sw}(\gamma - \omega t, \alpha_p) = \frac{4}{\pi} \sum_{j=1,3,5,...} \sin \left( \frac{2j\pi}{P} \right) \cos \left( \frac{j(\gamma - \omega t)}{P} \right).
\]

The MVD crossing the magnet is determined in this section. Determining the magnetic potential of the pole-cap \( U_{\text{amp}}, F_{r,\text{arm}}(\gamma, \omega t) \) can be figured out as follows.

The flux passing the pole cap is obtained by integrating

\[
\Phi_g = \frac{\alpha_p}{2\pi} \int_{-\alpha_p/2}^{\alpha_p/2} \frac{F_s(\gamma, \omega t) - F_{r,\text{arm}}(\gamma, \omega t)}{g_e} r \, d\gamma
\]

where \( \gamma_m \) is the spatial angle measured in mechanical radians, \( \gamma \) is that in electrical radians, \( r \) is the air-gap radius, \( D_g \) is the air-gap diameter, \( L_e \) is the effective length, \( R_g \) is the air-gap reluctance in the pole-arc range, and \( \lambda_g \) is the air-gap reluctance in the magnetic pole-arc range.

Neglecting the leakage flux through the magnetic bridge, the flux \( \Phi_g \) entering the pole cap is equal to the flux entering through the magnet. Thus

\[
U_{\text{amp}} = \Phi_g R_m
\]
Substituting (18) into (16), the MVD of the pole cap \( U_{u.r.p} \) can be obtained
\[
U_{u.r.p} = \frac{R_m R_g}{R_g (R_m + R_g)} \times \sum_{h=1,5,7,\ldots} \frac{1}{h} \frac{F_s \cos((h \pm 1)\omega t + \psi_1) \sin \left(\frac{h\alpha_p \pi}{2}\right)}{h}\,
\]
(20)

In (21), \( \sin c(h, \alpha_p) \) is defined as the pole-cap coefficient, related to the stator MMF harmonic order \( h \) and the magnet pole arc \( \alpha_p \) as
\[
\sin c(h, \alpha_p) = \frac{\sin \left(\frac{h\alpha_p \pi}{2}\right)}{h\alpha_p \pi / 2}.
\]
(21)

In general, the higher the stator MMF harmonic order \( h \) is, the weaker the rotor MMF becomes

The armature reaction rotor MMF \( F_{r,arm}(\gamma, \omega t) \) is as follows:
\[
F_{r,arm}(\gamma, \omega t) = \frac{R_m}{R_m + R_{g1}} \sum_{h=1,5,7,\ldots} \sin c(h, \alpha_p) F_{s,\gamma h} \times \cos((h \pm 1)\omega t + \psi_1) \cdot s\omega(\gamma - \omega t, \alpha_p).
\]
(22)

\subsection{B. Armature Reaction Field}

Substituting (22) into (12), the armature reaction air-gap density becomes
\[
B_{g,arm} = \frac{\mu_0}{g_e} \sum_{h=1,5,7,\ldots} F_{s,\gamma h} \cos(\gamma \pm \omega t \mp \psi_1) \times \cos((h \pm 1)\omega t + \psi_1) \cdot s\omega(\gamma - \omega t, \alpha_p).
\]
(23)

In order to facilitate the comparison of the proposed analysis model and the traditional model, the traditional model shown in (8)–(9) is transformed to
\[
\frac{1}{g_e} \left(1 - \frac{R_m}{R_{g1} + R_m} s\omega(\gamma - \omega t, 1) \right) \times s\omega(\gamma - \omega t, \alpha_p).
\]
(24)
Fig. 6. IPM FEA model cross section.

Fig. 7. Different current sheet arrangements: (a) $d$-axis MMF with the first-order dominant; (b) $q$-axis MMF with the first-order dominant; and (c) MMF with the third-order dominant.

From (23), it can be seen that different harmonic in the stator MMF will face different magnet reluctance caused by the pole cap represented by the rotor MMF. This makes the proposed method essentially different from those in (10).

\[
F_{r-\alpha\mu}(\gamma, \omega t) = \frac{R_m}{R_m + R_{k1}} \sum_{h=1}^{\infty} F_{s-h} \times \cos((h\gamma \pm \omega t) \mp \psi_1) \\
\times \sin(\gamma - \omega t, 1) \cdot \sin(\gamma - \omega t, \alpha_p) \quad (25)
\]

According to (22)–(26), the differences between the proposed model and the traditional model are as follows.

- In the proposed model, each stator MMF harmonic component will generate a square-wave rotor MMF. In the magnet pole-arc range, the rotor MMF is constant, no matter what kind of excitation and magnet shape the is, while in the traditional model the rotor MMF in the magnet pole-arc range is determined by the waveform of the stator MMF and the shape of the magnet slot.

- According to the proposed model, the armature-reaction air-gap density caused by the fundamental stator MMF is related to the current phase and the pole-cap effect coefficient $\sin(1, \alpha_p)$. However, the traditional model cannot predict this characteristic. Fig. 3 shows the comparison of the armature reaction rotor MMF and resulting air-gap flux.
density distribution predicted by two models in the $d$- and $q$-axis fundamental stator MMF, respectively, at time 0. The rotor MMF in the proposed model is a square wave, while in the traditional model, it is varied with electrical radians. What is more, as a result of the pole-cap effect, when the current phase is $\pi/2$, the rotor MMF is equal to zero in the proposed model. However, the rotor MMF is still varied with space in this condition, according to the traditional model.

- According to the proposed model, the rotor MMF induced by the $h$th-order stator MMF is weakened by pole-cap coefficient $\sin c[h, \alpha_p]$. In general, the higher the stator MMF harmonic order $h$ is, the weaker the rotor MMF becomes. Fig. 4 shows the spectrum of the pole-cap coefficient. However, the traditional model cannot predict this phenomenon. Fig. 5 shows the armature reaction field comparison of the proposed model, the traditional model with the fifth stator MMF and the seventh stator MMF at time 0. In this case, current phase $\psi_1$ is equal to 0, so the stator MMF is collinear with the rotor MMF. The results from the proposed model point out that under higher order of harmonic excitation, the magnet seems to disappear, and the resulting air-gap flux density distribution is quite similar to the no-salient pole machine, where only the mechanical

![Comparison of air-gap density wave and spectrum obtained by three methods at time 0: (a) air-gap density wave and spectrum in the $d$-axis MMF with the first-order dominant; (b) air-gap density wave and spectrum in the $q$-axis MMF with the first-order dominant; and (c) air-gap density wave and spectrum in the MMF with the third-order dominant.](image-url)
IV. VALIDATION WITH FEA

To validate the proposed analysis model, the armature reaction fields obtained by the proposed model and the traditional model are compared with the FEA results. In the FEA model, the stator is smooth, winding is modeled by the current sheet in the air gap, and the length and the height of the magnet bridge are close to zero. Fig. 6 shows the cross section of the IPM FEA model; the coils are equivalent to an infinitesimal current sheet in the air gap.

To validate the pole-cap effect in the IPM motor, different current sheet placements are employed to structure different stator MMF harmonic components. A square-wave stator MMF is employed to take the place of the sinusoidal MMF. With different current sheet placements, the dominant harmonic component of the total MMF is changed.

Fig. 7 shows three current sheet arrangements, which, respectively, lead to the \( d \)-axis MMF with the first-order dominant, the \( q \)-axis MMF with the first-order dominant, and the \( d \)-axis MMF with the third-order dominant.

In Fig. 7, the circle in the air gap represents the current sheet. The red circle means the current sheet flow out of the plane, while the black circle means the current sheet flow into the plane. In the real IPM FEA model, the current sheet is modeled as a very small component in the air gap.

In Fig. 7(a), the current direction flows in the two coils in the air gap are different, while the current direction flows in the two coils in the air-gap are the same in Fig. 7(b). Therefore, the current sheet placement in Fig. 7(a) leads to the \( d \)-axis MMF with the first-order dominant, shown in Fig. 8(a). The current sheet placement in Fig. 7(b) leads to the \( q \)-axis MMF with the first-order dominant, shown in Fig. 8(b).

The MMF waves generated by the different current sheet placements are shown in Fig. 8.

The proposed model, the traditional model, and FEA are employed to obtain air-gap density in a pair pole range. Fig. 9 shows the comparison results under different MMF at time 0.

It can be observed that the agreement between the proposed model and FEA is very good, while the error between the traditional model and FEA ranges between 20% and 80%, and in most cases, it is around 50%.

The comparison of the air-gap density obtained by the three methods illustrates that the proposed model can accurately predict the armature reaction field, while the traditional model not only underestimates the main air-gap density harmonic but also overrates some harmonic components which should not exist, as shown in Fig. 9(c).

Fig. 10 shows the air-gap density variation with time on a point in the air gap, obtained from the three methods. The comparisons show that the proposed model can be competent in predicting the armature reaction field distribution verse time for the IPM, agreeing well with the FEA result. However, the traditional model will misestimate the air-gap density spectrum.

The winding inductance, iron loss, and torque ripple, which are caused by armature reaction, can be derived from the model of the armature-reaction magnetic field distribution. So IPM motor parameter prediction, iron loss, and torque ripple optimization can be processed, based on the proposed model. Winding inductance calculation for the IPM model is shown in Fig. 11.

Fig. 11 shows the comparison of the wave form and spectrum of the primitive winding inductance obtained by the three methods. The comparisons show an acceptable agreement between FEA and the proposed model, while the error of winding inductance constant term between FEA and the traditional model is about 40%, as well as in high-order terms.

V. CONCLUSION

A new armature reaction magnetic field model based on the winding function theory for the IPM motor has been proposed.
in this paper. A rotor MMF function that is employed to deal with the pole-cap effect explicitly exists in the IPM motor. The proposed model shows that the armature-reaction rotor MMF is determined by the pole-cap coefficient and the current phase, which cannot be predicted by the traditional method. FEA is used to validate the proposed model. The results show that the proposed model can be competent in the armature-reaction field analysis for the IPM motor, while the traditional model will misestimate the air-gap flux density spectrum. Based on the proposed model, the self- and mutual inductance, torque ripple, and iron loss caused by armature reaction can be evaluated, and useful torque ripple and iron loss minimization methods will be presented in future work.

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