An Experimentally Verified Criterion for Propagation Across Unbounded Frictional Interfaces in Brittle, Linear Elastic Materials

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Understanding how a propagating fracture interacts with existing fractures, lithologic boundaries and other material interfaces is essential to the interpretation of fracture network geometries. In this paper, a first order analysis of the stresses near a mode I fracture impinging upon a frictional interface oriented normal to the growing fracture results in a simple criterion that predicts whether a growing fracture will terminate at or cross the interface. The analysis uses the linear elastic fracture mechanics solution for the stresses near a fracture tip to determine the compressive stress required to prevent slip along the interface at the moment when the stress on the opposite side of the interface is sufficient to initiate a fracture. A series of experimental investigations designed to assess the conditions required for crossing are presented and shown to be consistent with the criterion. Using data from previously published experiments, the criterion is shown to accurately predict the occurrence of compressional crossing in nine different brittle materials including three types of synthetic materials and six types of natural rock.

INTRODUCTION

The behavior of natural fractures as they intersect other fractures or material interfaces such as sedimentary bedding planes is important for interpreting the chronology of natural fracture formation [1–9]. Understanding the behavior of induced hydraulic fractures as they intersect other fractures and lithologic boundaries is also important in designing many energy extraction systems [e.g. 10]. Accurate interpretations of, and models for, these fracture systems require an understanding of the mechanics of fracture intersection.

From our experimental work, we have identified two primary mechanisms by which two fractures intersect and cross each other. In the first, termed coincident initiation (Fig. 1), younger fractures initiate on existing fractures. When coincident initiation is simulated in the laboratory by straining a brittle coating applied to a plastic substrate (Fig. 2; [11]), it is observed that in many cases, the same flaw that caused the older fracture causes the younger one. As the younger fracture propagates away from its initiation point, it may intersect other fractures and increase the connectedness of the fracture network.

In this paper, we focus on a second mechanism of fracture crossing that may occur at greater depths. Fracture crossings develop by means of this mechanism, termed compressional crossing, under certain stress states that allow younger fractures to propagate across older fractures. Unlike coincident initiation, compressional crossing involves the propagation of a fracture tip across a material interface.

Previous studies of the mechanics of compressional crossing [12–14] have suggested that in environments with modest compression acting perpendicular to the interface, slip and opening along the interface as the fracture tip approaches reduces the degree of stress concentration at the tip of the fracture and results in the suspension of propagation. However, if either bonding and/or compressive stress acting normal to the interface sufficiently inhibits opening and slip along the interface, then it is possible for a younger fracture to cross the interface. Compressional crossing has been demonstrated in several experimental studies where hydraulic...
Fig. 1. Schematic diagram of coincident initiation. (a) tensile stress in the $y$ direction acts at a stress concentrating flaw and results in the growth of a fracture (b), (c) stress in the $y$ direction becomes compressive, causing fracture growth to stop and the fracture to close, tensile stress in $x$ direction builds and a new fracture grows from the same initial flaw in either one (d) or both (d') directions.

Fractures were observed to cross interfaces in layered sedimentary rocks when the interfaces were either bonded or subject to sufficient compression normal to the interface to enhance the frictional resistance to sliding [15-23]. Detailed analyses of fracture surfaces near sedimentary bedding interfaces also suggest that fractures can propagate across frictional interfaces [24, 25].

Some previous work has been done towards modeling the stresses generated by a fracture in contact with a frictional interface [26, 27]. Attempts to extend these analyses to predict the stress state required for compressional crossing are difficult to apply and/or untested against actual compressional crossing data [12, 19, 21, 28]. Predictive extensions of these analyses are difficult to derive because stresses near frictional interfaces not only depend on the applied load, but also on the history of loading. It is not obvious how the loading and deformation that may occur as the fracture approaches the interface influence the stress field of the fracture at the interface. Furthermore, while the stress perturbations due to small scale plastic and other inelastic deformation occurring during fracture propagation often are justifiably ignored in many applications to rock fracture mechanics, it remains to be shown that they can be neglected in compressional crossing problems. Therefore, the empirical verification of any proposed compressional crossing criterion is essential.

We present a simple model of compressional crossing for orthogonal intersections in linear elastic media. The crossing criterion which results from this model is then shown to be consistent with our laboratory experiments and with results from several previously published compressional crossing laboratory studies.

**CRITERION DEVELOPMENT**

We restrict our attention to orthogonal intersections between the approaching fracture and a frictional inter-
Assume that the fluid pressure within the approaching experimental observations discussed later. We further fracture. These assumptions are consistent with our own pressive stress. We also assume that the frictional inter-
propagate in a direction that is perpendicular to the least compresion principal stress [e.g. 35], we assume that the approaching fracture is perpendicular to the least side of the interface [22, 25, 35]. If crossing always crossed frictional interfaces also support reinitiation by single fracture, it is unclear what mechanism would be occurred by means of the continuous propagation of a fracture on the other side of the interface. Crossing in this model is a discontinuous process.

Since it is well known that isolated fractures will propagate in a direction that is perpendicular to the least compressive principal stress [e.g. 35], we assume that the approaching fracture is perpendicular to the least compressive stress. We also assume that the frictional interface does not alter the propagation direction of the fracture. These assumptions are consistent with our own experimental observations discussed later. We further assume that the fluid pressure within the approaching fracture is low enough that it is energetically more favorable for the fracture to cross the interface rather than to deflect into the interface. Requirements for this second condition have been described by He and Hutchinson [37]. A sufficient requirement for orthogonal crossing to be more energetically favorable than deflection is that the fluid pressure is less than the magnitude of the remote compressional stress acting across the interface.

Our criterion for compressional crossing can then be stated as follows: compressional crossing will occur if the magnitude of the compression acting perpendicular to the frictional interface is sufficient to prevent slip along the interface at the moment when the stress ahead of the fracture tip is sufficient to initiate a fracture on the opposite side of the interface. The interface is assumed to be unbonded (cohesionless) and to obey a linear friction law. For a frictional interface oriented parallel to the y-axis, this implies that slip along the interface will occur whenever:

$$|\sigma_{ys}| < -\mu \sigma_{ss}$$

where $\mu$ is the coefficient of friction for the interface. For many rocks, typical values of $\mu$ range between 0.1 and 0.9 [38]. Of course, many natural fractures contain secondary fillings and thus are not cohesionless. However, in the extreme case, cohesion results in a perfectly bonded interface which has no impact on the propagation of the fracture. That fractures are often seen to terminate at pre-existing, unfilled fractures and bedding interfaces suggests that many of these interfaces may have little to no cohesion.

The materials on either side of the interface are assumed to be linear elastic, homogeneous and isotropic. Thus the stresses near of the tip of the approaching fracture can be determined using results from linear elastic fracture mechanics. A concise review of linear elasticity with applications to geology is found in the text by Jaeger and Cook [38]. Numerous example applications of linear elastic fracture mechanics theory to rock fracture are reviewed in Atkinson [39] and Pollard and Aydin [40].

Consider the stress induced along the interface by an approaching fracture tip (Fig. 4). Dollar and Steif [26] have shown that unlike the stresses near the tip of an isolated fracture in a linear elastic medium, there is no mathematical stress singularity at a fracture tip in contact with a frictional interface if the fracture driving stress is tensile. The loss of the stress singularity at the fracture tip when it contacts the interface greatly reduces the stress concentration just ahead of the fracture tip. This suggests that reinitiation of the fracture on the opposite side of the interface may occur prior to contact when the stress singularity at the approaching fracture tip still exists. Therefore our analysis, as shown in Fig. 4, considers the fracture-interface geometry just prior to contact. In this situation the fracture tip is still some distance $\delta$ from the interface where, for an approaching fracture length of $2a$, $\delta \ll a$. It is assumed that at this distance no slip has occurred along the interface and the
mathematical stress singularity at the tip of the approaching fracture still exists. Irwin [41] has suggested that inelastic deformation at the fracture tip results in the stresses and deformation near the fracture tip behaving as if the mathematical stress singularity is slightly ahead of the fracture tip. Thus, the distance between the location of the mathematical stress singularity and the interface may be less than \( \delta \).

Since it is assumed that no slip or opening has occurred along the interface, the interface may be treated as if it were perfectly bonded. In this case, if the materials on either side of the interface are homogeneous, isotropic and have the same elastic moduli, the stresses near the tip are well known from linear elastic fracture mechanics theory [42]. For the fracture tip shown in Fig. 4, which is subject to predominantly opening mode stress, the stresses near the tip can be approximated as [e.g. 43]:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
\sigma'_{xx} \\
\sigma'_{yy} \\
\sigma'_{xy}
\end{bmatrix} + \begin{bmatrix}
\sigma''_{xx}(r, \theta) \\
\sigma''_{yy}(r, \theta) \\
\sigma''_{xy}(r, \theta)
\end{bmatrix} + \begin{pmatrix}
K_1 \\
\sqrt{r}
\end{pmatrix} \begin{bmatrix}
\cos \theta \left[ 1 - \sin \theta \sin \frac{3\theta}{2} \right] \\
\cos \theta \left[ 1 + \sin \theta \sin \frac{3\theta}{2} \right] \\
\sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}
\end{bmatrix} + \cdots \quad (2)
\]

Here \((r, \theta)\) are polar coordinates with the origin at the fracture tip, \(\sigma''_{ij}\) are the remotely applied stresses and \(K_1\) is a constant known as the opening mode stress intensity factor. We focus on the region near the fracture tip and thus neglect terms of order \(r^{1/2}\) and higher indicated by the \(+ \cdots\).

If the materials on either side of the interface do not have the same elastic moduli, then equations (2) are only accurate to the extent that \(\lambda \approx 1/2\), where \(\lambda\) satisfies [37, 44]:

\[
\cos \lambda \pi = \frac{2(\beta - \alpha)}{1 + \beta} (1 - \lambda^2) + \frac{\alpha + \beta^2}{1 - \beta^2} \quad (3)
\]

where \(\alpha\) and \(\beta\) are the Dundurs' parameters [45]:

\[
\begin{align*}
\alpha &= \frac{G_1(1 - v_1) - G_2(1 - v_2)}{G_1(1 - v_1) + G_2(1 - v_1)} \\
\beta &= \frac{G_1(1 - 2v_1) - G_2(1 - 2v_2)}{2[G_1(1 - v_1) + G_2(1 - v_1)]} \quad (4a)
\end{align*}
\]

\(G_1\) and \(v_1\) are the shear modulus and Poisson's ratio for the material containing the approaching fracture and \(G_2\) and \(v_2\) are the respective moduli for the material in which reinitiation occurs. For many rock types, \(0.1 \leq v \leq 0.3\) and \(1 \text{ GPa} \leq G \leq 100 \text{ GPa}\) [46]. For the cases where \(v_1 = v_2 = 0.1\) and \(v_1 = v_2 = 0.3\), Fig. 5 shows the variation in \(\lambda\) as a function of \(G_1/G_2\). Figure 5 suggests that for rocks having similar Poisson's ratios, equations (2) are approximately valid for \(G_1/G_2\) ratios between \(~0.4\) and \(~2.0\). Since our analysis is based upon equations (2), the criterion we derive is only valid for interfaces that do not
Fig. 6. Geometry and stresses of the fracture process zone. Stresses within this zone are assumed to be equal or less than the stresses calculated from the stress field equations (2) at a distance $r_c(\theta)$. The approximate maximum magnitude of the components of stress $\sigma_i(\text{max})$ acting along the interface can thus be calculated using the stress field equations (2) with $\theta = \pm \pi/2$ and $r = r_c(\pm \pi/2)$.

 involve strong contrasts in elastic properties across the interface (i.e. for cases where $\lambda \approx 0.5$).

Note that the stress components all display a $1/\sqrt{r}$ dependence and are singular at the fracture tip. In reality, inelastic deformation at and near the fracture tip prevent the stresses from becoming infinite. It follows then that there exists some critical radius $r_c(\theta)$ within which the stresses are of sufficient magnitude for inelastic fracture processes to operate. The existence of inelastic deformation within this area, referred to as the fracture process zone (Fig. 6), implies that the stress field equations (2) do not hold for distances less than $r_c(\theta)$. The stresses within this zone, and indeed, the size and shape of the zone itself, depend largely upon the micromechanical deformation mechanisms acting at the fracture tip. However, it is common in the fracture mechanics literature to assume that stresses within the process zone are equal to or less than the stresses at $r_c(\theta)$ [e.g. 47, 48].

Since the stresses within the fracture process zone are assumed to be equal to or less than the stresses defined by the stress field equations (2) with $r = r_c(\theta)$ and since $\delta \ll r_c$, the maximum magnitudes of the components of stress $\sigma_i(\text{max})$ that act along the frictional interface can be found by setting $\theta = \pm \pi/2$ and $r = r_c(\pm \pi/2)$ in the stress field equations (2). In particular:

$$
\sigma_{xx}(\text{max}) = \sigma'_{xx} + \sigma_{xx}[r_c(\pm \pi/2), \pm \pi/2],
$$

$$
\sigma_{yy}(\text{max}) = \sigma'_{yy} + \sigma_{yy}[r_c(\pm \pi/2), \pm \pi/2],
$$

$$
\sigma_{xy}(\text{max}) = \sigma'_{xy} + \sigma_{xy}[r_c(\pm \pi/2), \pm \pi/2].
$$

Substituting equations (3) into (1) implies that slip will not occur along the interface if:

$$
|\sigma_{xy}(\text{max})| < -\mu \sigma_{xx}(\text{max}).
$$

(6)

Since the approaching fracture is perpendicular to the least compressive principal stress, $\sigma'_{xy} = 0$ and equation (4) can be re-written as:

$$
\frac{K_i}{\sqrt{r_c(\pm \pi/2)}}\left(\sin \frac{\pi}{4} \cos \frac{\pi}{4} \cos \frac{3\pi}{4}\right) < -\mu \left[\sigma'_{xx} + \frac{K_i}{\sqrt{r_c(\pm \pi/2)}}\right] \left(\cos \frac{\pi}{4} \left(1 - \sin \frac{\pi}{4} \sin \frac{3\pi}{4}\right)\right)
$$

(7)

This can be rearranged to yield:

$$
\frac{\sigma'_{xx} \sqrt{r_c(\pi/2)}}{K_i} \cos \frac{\pi}{4} \left(1 - \sin \frac{\pi}{4} \sin \frac{3\pi}{4}\right) \left(\cos \frac{\pi}{4} \cos \frac{\pi}{4} \right) + \frac{1}{\mu} \left(\sin \frac{\pi}{4} \cos \frac{\pi}{4} \cos \frac{3\pi}{4}\right).
$$

(8)

To find the value of $\sqrt{r_c(\pi/2)}$, we assume that for a fracture to begin to propagate on the other side of the interface, the tensile stress induced by the approaching fracture must be equal to the tensile strength $T_o$ of the material on the opposite side of the interface:

$$
\sigma_{yy}(\text{max}) = T_o.
$$

(9)

Substituting equations (2) and (5) into equation (9) yields:

$$
\sigma'_{yy} + \frac{K_i}{\sqrt{r_c(\pm \pi/2)}} \left[\cos \frac{\pi}{4} \left(1 + \sin \frac{\pi}{4} \sin \frac{3\pi}{4}\right)\right] = T_o
$$

(10)

Thus $\sqrt{r_c(\pi/2)}$ is equal to:

$$
\sqrt{r_c(\pi/2)} = \frac{K_i}{T_o - \sigma'_{yy}} \left[\cos \frac{\pi}{4} \left(1 + \sin \frac{\pi}{4} \sin \frac{3\pi}{4}\right)\right].
$$

(11)
Substitution of equations (11) into (8) implies that in order for crossing to occur:

\[
\frac{-\sigma_{xx}'}{T_o - \sigma_{yy}'} > \frac{\cos \frac{\pi}{4} \left( 1 - \sin \frac{\pi}{4} \sin \frac{3\pi}{4} \right) + \frac{1}{\mu} \left( \sin \frac{\pi}{4} \cos \frac{\pi}{4} \cos \frac{3\pi}{4} \right)}{\cos \frac{\pi}{4} \left( 1 + \sin \frac{\pi}{4} \sin \frac{3\pi}{4} \right)}
\]

(12a)

or, evaluating the constants:

\[
\frac{-\sigma_{xx}'}{T_o - \sigma_{yy}'} > \frac{0.35 + 0.35}{1.06}
\]

(12b)

The threshold, as a function of the coefficient of friction, is plotted in Fig. 7.

EXPERIMENTAL DESIGN

To check the validity of the crossing stress ratio criterion, a series of experiments were designed to determine the stress required for compressional crossing to occur as a function of interface coefficient of friction and material tensile strength. The essential experimental design is shown schematically in Fig. 8(a) and discussed in greater detail below. Briefly, three molded planks of brittle material were subjected to compression normal to their longest dimension. A hole in the center plank was used to initiate a fracture which propagated towards the interfaces between the planks. The fracture was driven by a wedge device inserted into the hole in the center plank. The behavior of the fracture (stopping or crossing) as a function of the amount of compression applied across the interface was then recorded. By repeating this experiment using different brittle materials or lubricating the interface, the critical amount of compression required for compressional crossing, as a function of both material tensile strength and interface coefficient of friction, was determined.

Sample preparation

Samples were made of either anchoring cement (Pour Stone manufactured by Custom Building Products, Bell, Calif., U.S.A.) or a plaster of paris based patching material (Rock Hard manufactured by the The Donald Durham Company, Des Moines, Iowa, U.S.A.). The powder material was mixed with water at powder to water volumetric ratios of 3:1 for the anchoring cement and 9:4 for the Rock Hard. The mixture was well mixed and poured into acrylic molds to form 37.5 × 8.0 × 1.0 cm slabs. All molds, as shown in Fig. 9, were designed to create small, 0.64 cm dia holes located 1.0 cm in from either end of the sample. These holes were used to mount the sample for milling the interfaces and to grip the sample during the frictional pull-out test. Center plank molds also cast a centered, nearly rectangular hole in the sample that was used to initiate a fracture that propagated towards the frictional interface. The sample was allowed to set for one day before being removed from the mold. The anchoring cement samples were then immediately milled and tested while the Rock Hard samples were first allowed to dry under cement bricks for four days at room temperature. The porous nature of the bricks allowed the samples to dry while the weight of the bricks helped maintain their flatness.
Fig. 8. (a) Schematic diagram of the compressional crossing experiment. A hole in the center plank of brittle material is used to initiate a fracture which propagates from the hole's narrow ends towards the frictional interfaces between the planks. The fracture is driven by a wedge device which pries apart the long sides of the hole. The behavior of the fracture at the interfaces (stopping or crossing) vs the coefficient of friction of the interfaces and the amount of compression applied normal to the interfaces is recorded. (b) Schematic diagram of the pull-out test. The test is used to determine the coefficient of friction of the interfaces between the planks.

Once dried, the long edges of the samples were milled using a computer controlled milling table accurate to 0.0002 cm. The milling assured consistently smooth and parallel interfaces between samples. Final sample sizes were $37.5 \times 7.6 \times 1.0$ cm for the center planks and $37.5 \times 7.8 \times 1.0$ cm for the outer planks.

**Determination of coefficient of friction**

The interface coefficient of friction for each three-plank sample was determined using a pull-out test. The experimental design of the pull-out test is shown schematically in Fig. 8(b). Servo-controlled hydraulic pistons applied compression normal to the interfaces between the individual planks. Another servo-controlled hydraulic piston pulled the center plank in a direction parallel to the interfaces. The amount of force needed to cause slip along the interfaces as a function of applied force normal to the interfaces was recorded and used to determine the coefficient of friction of the interfaces. That the milling machine produced consistent frictional interfaces is demonstrated by the low standard deviation in the coefficient of friction for all like interfaces (Table 1). To simplify the analysis of the data, the coefficient of friction for all similar interfaces was assumed to be the same.

Fig. 9. Geometry of compressional crossing sample plank. Only the center planks contain the centered nearly rectangular opening shown above. Only one edge of the side planks is milled, resulting in a final width of 7.8 cm rather than the 7.6 cm shown. 0.64 cm holes are formed using 0.64 cm i.d. hexagonal nuts embedded in the sample.
same and equal to the mean coefficient of friction value from all tests on similar samples.

**Compressional crossing experiment**

Once the interface coefficient of friction was measured, the three plank samples were unloaded. A pre-determined amount of compression normal to the interface was then reapplied using servo-controlled hydraulic pistons and a wedge device inserted into the hole in the center plank (Figs 10(a) and (b)). By slowly tightening a bolt on this device, a constant opening displacement was applied along the long sides of the hole in the center plank (Fig. 10(c)), causing the sides of the center hole to be pried apart and resulting in fracture tips propagating from both narrow ends of the center hole towards the plank interfaces. The behavior of the propagated fracture tip as it intersected the interface (stopping or crossing) for each level of applied compression was recorded. If the fracture initially stopped at the interface, the loading at the center hole was increased until either the fracture crossed the interface or a net slip $\geq 0.5$ cm was observed to have occurred along the interface.

**Determination of tensile strength**

After completion of the compressional crossing test, the tensile strength of each plank was determined using a four point tensile strength test. The experimental design for these tests is shown in Fig. 11. Two or three tensile strength tests were performed on each plank with an averaged value taken to represent the plank strength. Loads on the metal plate, generated either by filling a hanging bucket with water or by using one of the servo controlled hydraulic pistons, were applied to the sample via rods in the acrylic frame shown in Fig. 11. The frame permitted vertical movement of the rods while maintaining approximately constant horizontal spacing of the rods between samples. Table 2 summarizes the average tensile strengths of the two plank types as well as the

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**Table 1. Summary of mean coefficients of friction and their standard deviations for various interface types**

<table>
<thead>
<tr>
<th>Interface type tested</th>
<th># Samples</th>
<th>Mean coefficient of friction</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchoring cement–anchoring cement</td>
<td>6</td>
<td>0.48</td>
<td>0.03</td>
</tr>
<tr>
<td>Anchoring cement–Rock Hard</td>
<td>8</td>
<td>0.62</td>
<td>0.05</td>
</tr>
<tr>
<td>Greased anchoring cement–greased anchoring cement</td>
<td>5</td>
<td>0.09</td>
<td>0.03</td>
</tr>
</tbody>
</table>

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Fig. 10. Schematic of the wedge device used to derive the fracture tips towards the frictional interfaces. (a) and (b) show side and front views of the device. By tightening the screw, top and bottom sections of the device are separated. To drive the fracture, the device is inserted into the rear of the center plank hole as shown in (c). The screw is tightened and the long sides of the center plank hole pried apart.
Table 2. Summary of mean tensile strengths and their standard deviations for the two plank types

<table>
<thead>
<tr>
<th>Material type</th>
<th># Samples</th>
<th>Mean tensile strength (MPa)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchoring cement</td>
<td>20</td>
<td>2.80</td>
<td>0.70</td>
</tr>
<tr>
<td>Rock Hard</td>
<td>8</td>
<td>0.76</td>
<td>0.13</td>
</tr>
</tbody>
</table>

standard deviations of these averages. The large variation in tensile strengths between samples, as indicated by the large standard deviations, suggests that there were differences in the composition of the dry powder used to form each sample and differences in the water contents of various samples at the time of testing. To account for these differences when analyzing the data, the mean tensile strength for each sample was used in the analysis of that sample rather than the mean tensile strength for all like samples that is given in Table 2.

Stress state in planks

When the three-plank sample is loaded with compression normal to the plank interfaces, the load is unevenly transferred from the load cell on the hydraulic piston to the sample planks by means of a stainless steel grip. To account for this complicated loading geometry, a steady-state, isoparametric finite element model [49] was used to determine the stresses near the frictional interfaces. Elastic moduli for the grips were assumed to be typical of those found for cold rolled stainless steel ($G = 75$ GPa, $\nu = 0.30$; [49]). The moduli for the anchoring cement and Rock Hard were determined using the servo-controlled hydraulic pistons to observe the stress-strain behavior of the material. The values of the elastic moduli for the anchoring cement are $G = 18$ GPa and $\nu = 0.12$, while for the Rock Hard, the values are $G = 0.68$ GPa and $\nu = 0.10$. Plane stress conditions were assumed in modeling the samples. However, the small Poisson's ratios of the samples result in the assumption of plane stress or plane strain having little influence on the results.

Figure 12 shows the finite element mesh used to model the stress distribution within the samples. Because the sample has two lines of symmetry, only one-quarter of...
the entire sample needs to be modeled. The distribution of the $\sigma_{xx}$ stress component within each of the two sample types is shown in Fig. 13. The magnitudes shown in Fig. 13 are normalized by $\sigma' = F_a/A_{int}$, where $F_a$ is the force applied by the piston and $A_{int}$ is the area of the frictional interface. In both sample types, the magnitude of the $\sigma_{xx}$ component of stress near the section of the frictional interface where crossing may occur is close to $1.25\sigma'$. The distribution of the other stress components within the samples are similar in the sense that stress gradients near the section of the frictional interface where crossing may occur are small. (Although there is a discontinuity in the $\sigma_{yy}$ component of stress across all interfaces of dissimilar materials.) In both the anchoring cement and Rock Hard Samples, the magnitude of the $\sigma_{yy}$ component of stress near where reinitiation may occur is approximately $0.3\sigma'$. Due to the symmetry of the sample, no shear stresses ($\sigma_{xy}$) exist along the frictional interface at the point where reinitiation may occur.

Three sets of experiments were run. In the first set, all three planks of the three-plank sample were made of anchoring cement. In the second set, the two outer planks were made of Rock Hard, while the inner plank was made of anchoring cement. In the last set, all three planks were made of anchoring cement, but the interfaces were coated with lithium soap to reduce the interface coefficient of friction.

**RESULTS AND DISCUSSION**

The results from the three sets of experiments are shown in Fig. 14. It can be seen from this figure that the criterion derived above, indicated in the figure as the theoretical threshold, is consistent with the laboratory data. Note that the criterion appears to be consistent even for the bi-material interface between the Rock...
is continuous across the interface while on the other side, the trace is clearly discontinuous. The narrow depth (1 cm) of the interface in these experiments results in the continuous or discontinuous nature of the crossing often being persistent throughout the entire depth. Discontinuous propagation across the interface was not uncommon in the experimental trials reported here and suggests that many of the fracture offsets observed at fracture intersections may represent a single discontinuous process.

Comparison to published crossing data

Several previous experimental studies of compressional crossing are available for comparison to the work described here [19–22]. While the experimental details of each of these studies are slightly different, the basic experimental design used in each study is similar. This design, shown in Fig. 16, involves compressing pre-cuts blocks of rock or molded blocks of hydrostone. A hole drilled into the center block allows fluid to be pumped into the center of the sample in order to propagate a hydraulic fracture towards the frictional interfaces. In one study [21], only two blocks were used. The essential data collected in each study was the amount of compression needed for the hydraulic fracture to propagate across the interface.

The different types of materials used in these studies and their tensile strengths are summarized in Table 3. Note that in addition to the synthetic brittle materials, Table 3 includes six different types of actual rock. Also note that where a range of tensile strengths was reported, only the lowest value in that range is shown in Table 3.

In order to compare the experimental results from these investigations, results from each study were changed into the format used in Fig. 14. While the conversion is straightforward where individual trial results are given [19–21], for some sample types, Anderson reports only a critical applied normal stress above which crossing occurred. This critical value was reported as a mean value “with a scatter of about 25%”, [22, p. 28]. To convert this into the individual trial format used here, 25% of the mean value was subtracted from the mean and assumed to be a “no crossing” data point and 25% of the mean was added to the mean for use as a “crossing” data point. Therefore, these single points actually represent more than one single experimental trial.

<table>
<thead>
<tr>
<th>Material type [Ref.]</th>
<th>Tensile strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchoring cement [this study]</td>
<td>2.5</td>
</tr>
<tr>
<td>Arizona sandstone [20]</td>
<td>8.0</td>
</tr>
<tr>
<td>Berea sandstone [20]</td>
<td>4.2</td>
</tr>
<tr>
<td>Hydrostone [19, 21]</td>
<td>3.1</td>
</tr>
<tr>
<td>Indiana limestone [22]</td>
<td>4.7</td>
</tr>
<tr>
<td>Lueders limestone [20]</td>
<td>3.5</td>
</tr>
<tr>
<td>Nugget sandstone [20]</td>
<td>3.7</td>
</tr>
<tr>
<td>Rock Hard [this study]</td>
<td>0.7</td>
</tr>
<tr>
<td>Tennessee sandstone [20]</td>
<td>10.8</td>
</tr>
</tbody>
</table>
Figure 17 summarizes the available compressional crossing data. The results from the different experimental designs, including the one presented above, are clearly consistent. That the data points in Fig. 17 represent nearly 100 individual experimental runs involving five different experimental designs and nine different materials suggests that some common physical process is involved in compressional crossing.

Shown with the data in Fig. 17 is the theoretical threshold developed earlier. The fit, even for the data from natural rock samples, is quite good. In contrast, if one assumes that the condition necessary for compressional crossing is simply that the interface have sufficient shear capacity to develop a tensile stress equal to the tensile strength of the material [17], the fit is not as good.

**SUMMARY AND CONCLUSIONS**

A simple criterion for predicting whether a fracture will propagate across a frictional interface oriented perpendicular to the approaching fracture has been derived. This model uses the linear elastic fracture mechanics solution for the stresses near a fracture tip to determine the stresses required to prevent slip along the interface at the moment when the stress on the opposite side of the interface is sufficient to reinitiate a fracture. Data from experiments designed to assess the conditions required for crossing to occur are consistent with the criterion, as are data from previously published experimental crossing studies.

As presented in this paper, our criterion is only valid for perpendicular intersections between the approaching fracture and the frictional interface and for interfaces where the elastic properties of the materials on either side of the interface are similar. Although these assumptions are often valid within natural fracture networks, many fracture intersections in geologic media involve cohesive, non-orthogonal interfaces between strongly dissimilar interfaces. Further work is required to extend the criterion to include these effects. In addition, more experimental data concerning the requirements for compressional crossing across cohesive, inclined interfaces and across interfaces between strongly dissimilar materials are needed.

**REFERENCES**
