Experimental and Numerical Investigation of Permeability Evolution with Damage of Sandstone Under Triaxial Compression

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Abstract A series of triaxial compression tests with permeability measurements was carried out under different confining pressure and pore pressure difference coupling conditions to investigate some mechanical properties and permeability evolution with damage of sandstone. It is found that the shapes of stress–strain curves, permeability evolution curves, and failure patterns are significantly affected by the confining pressure but are only slightly affected by the pore pressure difference. In addition, the corresponding numerical simulations of the experiments were then implemented based on the two-dimensional Realistic Failure Process Analysis-Flow (RFPA 2D-Flow) code. In this simulator, the heterogeneity of rock is considered by assuming the material properties of the mesoscopic elements conform to a Weibull distribution and a statistical damage constitutive model based on elastic damage mechanics and the flow–stress–damage (FSD) coupling model. The numerical simulations reproduced the failure processes and failure patterns in detail, and the numerical results about permeability–strain qualitatively agree with the experimental results by assigning different parameters in the FSD model. Finally, the experimental results about relationship between permeability evolution and volumetric strain are discussed.

Keywords Triaxial compression · Hydromechanical coupling · Permeability evolution · Failure process · Numerical simulation · Rock mechanics

List of symbols

- $A$: Cross-sectional area
- $D$: Damage variable
- $E$: Young’s moduli of damaged material
- $E_0$: Young’s moduli of undamaged material
- $f_c$: Uniaxial compressive strength
- $f_{cr}$: Compressive residual stress
- $f_{ij}$: Body force in the $j$th direction
- $f_t$: Tensile strength
- $f_{tr}$: Tensile residual stress
- $G$: Shear modulus
- $k$: Absolute permeability
- $K$: Hydraulic conductivity
- $K_0$: Initial hydraulic conductivity
- $L$: Sample height
- $m$: Homogeneity index
- $p$: Pore pressure
- $P(\alpha)$: Cumulative probability function of Weibull distribution
1 Introduction

Percolation and seepage control play an important role in the design and construction of rock engineering projects, including some conventional engineering activities, such as hydraulic and hydropower projects, oil and gas exploitation, underground mining extraction, and tunnel construction, and some newly emergent rock engineering activities, such as radioactive waste storage (Chen et al. 2014) and geothermal reservoir development (Olasolo et al. 2016; Ghassemi 2012). The coupling effect of ground stress and fluid pressure in the surrounding rock may increase the risk of geohazards and accidents, which may affect the regular construction and daily operations of rock engineering work. Therefore, discovering the mechanical properties and permeability characteristics of rock under hydromechanical coupling conditions would be of great benefit to the prevention and control of geological disasters and engineering accidents and the safety assessment of subsurface construction.

Over the last several decades, numerous scholars have researched the correlation between rock permeability and mechanical properties, i.e., stress, strain, strength, etc. Since the permeability of Westerly granite under high pressure was investigated through experimental testing by Brace et al. (1968), similar laboratory studies in this field have been extensively conducted in different types of rock, e.g., by Li et al. (1994), Wang and Park (2002), Hu et al. (2010), Luo et al. (2011), Zhang and Nemčík (2013) in sandstone; by Zoback and Byerlee (1975), Souley et al. (2001), and Oda et al. (2002) in granite; by Zeng et al. (2011) and Wang et al. (2013) in coal; and by Davy et al. (2007) in argillite. Although some differences in the experimental results were observed due to variations in experimental conditions, these researchers have reached some degree of consensus as follows: Under the combined action of load and fluid pressure, the pores and cracks in the rock will open or close, and the rock’s skeleton grains will shrink or expand, which will change its permeability characteristics, once the seepage field changes, which, in turn, will affect the stress field. The stress field and fluid pressure interact with, influence, and restrict each other, resulting in hydromechanical coupling. In addition to experimental research, some theoretical and numerical investigations on the hydromechanical coupling effects of rock have been widely reported. Yuan and Harrison (2006) reviewed previous research and summarized that the following three main methods were employed to model the progressive mechanical breakdown of, and associated fluid flow in, rock: discrete models based on fracture mechanics; continuum damage mechanics models; and statistical models. For example, Tang et al. (2002) set up a flow–stress–damage (FSD) coupling model for heterogeneous rocks based on a statistical model that considers the growth of existing fractures and the formation of new fractures. This model was coded into a two-dimensional Realistic Failure Process Analysis (F-RFPA2D) by Yang et al. (2004) to simulate hydraulic fracturing in permeable rock, and then it was extended to three dimensions by Li et al. (2012). It has been proven that the FSD model is a useful tool to simulate the failure process in permeable materials. In short, the results of previous research deepen the understanding of the mechanical properties and permeability characteristics of rock under hydromechanical coupling conditions and provide several theoretical methods for numerically simulating corresponding issues.

However, the essential mechanism of stress-to-seepage and seepage-to-stress coupling has not been completely discovered. Therefore, in an attempt to gain a better understanding of the mechanical properties and
permeability evolution with damage of rock in failure processes, a series of triaxial compression tests was conducted on sandstone samples in this research. Correspondingly, numerical simulations under similar hydromechanical coupling conditions were performed using the code RFPA2D-Flow. And the main purpose of this paper focuses on the following:

1. Investigating the coupling effects of the confining pressure and pore pressure difference on stress–strain curves and permeability evolution with damage of rock with triaxial compression tests.
2. Reproducing the failure processes of rock samples under hydromechanical coupling conditions and exploring the impacts on failure patterns subjected to confining pressure and pore pressure gradient with numerical simulation method.
3. Discussing the relationship between permeability and volumetric strain of triaxially compressive rock samples.

2 Experimental Investigations

2.1 Sample Preparation

To investigate the mechanical behavior and permeability characteristics of rock under hydromechanical coupling, sandstone material from Zigong City, Sichuan Province, China, was collected. The samples were carefully drilled from a large sandstone block without any macroscopic cracks. After cutting and polishing, the rock samples (Fig. 1) were machined into a cylindrical shape 50 mm in height and 25 mm in diameter.

Table 1 summarizes some of the physical parameters of the sandstone samples in this study. These samples were carefully selected by measuring their sizes and physical properties to ensure their uniformity and homogeneity for the triaxial compression tests. The physical parameters of the sandstone samples, including bulk density (average 2.28 g cm\(^{-3}\)), connected porosity (average 8.89%), \(S\)-wave velocity (average 1479.81 m s\(^{-1}\)), and \(P\)-wave velocity (average 2292.58 m s\(^{-1}\)), are considerably close to their average values.

2.2 Experimental Equipment and Method

All of the triaxial compression tests were carried out using the domestically designed TAW-1000 electrohydraulic servo-controlled rock mechanics testing system, at a maximum confining pressure of 60 MPa and a maximum axial loading capacity of 1000 kN, as shown in Fig. 2a. The system is also configured with an independent closed loop capable of applying a maximum pore pressure of 40 MPa inside the rock sample. This function can be used to measure absolute permeability during triaxial compression by applying pore pressure gradient between both end faces to make the fluid permeate through the rock sample. The axial and radial strain was measured simultaneously by the axial and radial displacement gauges with a measuring capacity of 0–4 mm and a reading accuracy of \(\pm 1\%\) at room temperature, as shown in Fig. 2b. The reading of the axial strain was also used as a feedback signal for the external load control. To reduce the end-friction effects on the measurements, these displacement gauges were placed at the middle height of the sandstone sample, and the other two steel anti-friction gaskets were placed between the end planes of the rock sample and the upper and lower loading plates.

For the purpose of this study, nine groups of triaxial compression tests were performed under three different confining pressure levels of 10, 20, and 30 MPa coupling with three pore pressure difference levels in the axial direction of 1.0, 4.0, and 7.0 MPa. First, hydrostatic stress

![Fig. 1 Sandstone samples achieved in the laboratory and a representative SEM photomicrograph. a The sandstone samples and b SEM photomicrograph](image-url)
was placed on the sandstone sample with confining pressure increasing by stress control at a constant rate of 0.06 MPa s\(^{-1}\) until the designated level was reached. The loading of axial deviatoric stress was then started by displacement control at a rate of 0.02 mm min\(^{-1}\), which converts to a strain rate of approximately 6.67 \(\times\) 10\(^{-6}\) s\(^{-1}\).

In triaxial tests on sandstone at strain rate of 2.5 \(\times\) 10\(^{-5}\) s\(^{-1}\) or less, it is found that the change in strength is of a much smaller magnitude (Sangha and Dhir 1975). Thus, the value of displacement rate was chosen in the experiments to minimize the effect of strain rate.

Permeability, which mainly depends on porosity, pore size, pore connectivity, pore geometry, and pore tortuosity, is one of the most significant characteristics of rock, and it has important implications for geological and geotechnical activities. Thus, it is crucial and necessary to understand the evolution of permeability under mechanical conditions (Berkowitz 2002). Generally, there are two main experimental methods used to measure permeability during triaxial compression: steady-state flow tests and transient (or pulse) tests (Brace et al. 1968; Oda et al. 2002; Davy et al. 2007). For rock with relatively high permeability—e.g., \(k \geq 10^{-19}\) m\(^2\)—steady-state flow tests are more suitable (Davy et al. 2007; Hu et al. 2010). In this study, the steady-state flow method was chosen, because the initial permeability of the sandstone samples is approximately 10\(^{-15}\) m\(^2\). Darcy’s law is usually assumed to be valid for steady-state flow permeability tests, which can calculate the absolute permeability \(k\) by a linear relation between the flow rate and the imposed pore pressure gradient. Here, Darcy’s law is described by

\[
k = \frac{Q \mu L}{\Delta \rho A}
\]

where \(Q\) denotes the volumetric water flow rate (m\(^3\) s\(^{-1}\)), \(\mu\) is the dynamic viscosity coefficient (1.005 \(\times\) 10\(^{3}\) Pa s at a

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Height/mm</th>
<th>Diameter/mm</th>
<th>Density/g cm(^{-3})</th>
<th>Porosity/%</th>
<th>S-wave velocity/m s(^{-1})</th>
<th>P-wave velocity/m s(^{-1})</th>
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<td>2.281</td>
<td>8.892</td>
<td>1479.812</td>
<td>2792.587</td>
</tr>
</tbody>
</table>
room temperature of 20 °C), L denotes the sample height (m), Δp is the pore pressure difference applied between both end planes of the rock sample (Pa), and A is the cross-sectional area (m²).

When absolute permeability was measured, the upper end of the saturated rock sample was connected to a water pump and a pressure of 1.1, 4.1 or 7.1 MPa was applied to this end. The lower end of the sample was opened to atmospheric pressure which is approximately equal to 0.1 MPa. Thus, a constant pore pressure difference of 1.0, 4.0 or 7.0 MPa was imposed between both ends of the sample, which would drive the water from one end to the other. After calculation, the derivative of the elevation head corresponding to these three levels of pore pressure difference is 2000, 8000, and 14,000, respectively.

It is notable that, according to geomaterial sign convention, positive signs are used to indicate compressive stress and strain, and negative signs for tensile stress and strain throughout this paper.

2.3 Experimental Results

2.3.1 Characteristics of Stress–Strain Curves

Figure 3 displays typical curves of deviatoric stress and permeability versus axial strain of the sandstone samples under three different confining pressures coupling with three different pore fluid pressure gradients. All of these stress–strain curves can be characterized by four phases (Wang and Park 2002; Yu et al. 2015):

1. Crack closure phase. At the initial loading stage, the curves are slightly concave upward. This nonlinear phase implies that the origin pores and microcracks in the rock sample are becoming compacted and closed because of triaxial compression.

2. Linear elastic deformation phase. In this stage, the slope of the stress–strain curve is constant until the yield phase. It is believed that, with the axial load increasing continuously, the secondary cracks in the samples are steadily forming, growing, and connecting.

3. Yield phase. The stress–strain responses become nonlinear again and convex upward until the peak stress reaches. In this phase, the rock sample is seriously damaged, and the growth of microcracks becomes unstable. A higher number of microcracks form, concentrate and connect in the yield phase than in the linear elastic deformation phase. The axial and volumetric strain begins to increase rapidly, resulting in the volume of the rock sample changing from compression to dilatation.

4. Peak stress and post-peak phase. It can be observed from the stress–strain curves in Fig. 3 that the confining pressure has a significant effect on the shapes of these curves after reaching peak stress. In the case of confining pressure of 10 or 20 MPa, the deformation after peak stress is characterized by strain softening. The stress–strain curves in the post-peak phase switch from strain softening (∂σ/∂ε < 0) at a confining pressure of 10 or 20 MPa to almost ideal plasticity (∂σ/∂ε = 0) at a confining pressure of 30 MPa, indicating that the brittleness of the rock sample is weakened and the ductility is improved when the confining pressure rises. In addition, as shown in Table 2, the axial strain corresponding to the peak stress also grows markedly with the confining pressure. These abovementioned phenomena in the peak and post-peak phases are known as “the brittle–ductile transition” (Wong and Baud 2012; Martin et al. 2013).

It is not easy, from Fig. 3, to clearly describe the effect of the pore pressure difference on the stress–strain curves. However, it can be seen from Table 2 that the peak stress increases slightly with increasing pore pressure difference when the confining pressure is set to either 10, 20, or 30 MPa. This occurs because when a higher, different pore pressure difference is manifested between the end planes of the sample, the pore pressure in the sample is also higher. According to the principle of effective stress, a higher pore pressure would reduce the effective confining pressure, resulting in a lower peak stress under triaxial compression.

2.3.2 Effect of Confining Pressure and Pore Pressure Gradient on Permeability of Sandstone

It also can be observed from the experimental results shown in Fig. 3 that the confining pressure and pore pressure difference have a significant influence on the permeability of sandstone. In the case of the confining pressure of 10 MPa, permeability evolution during triaxial compression has some features in common with that under the condition of three different pore pressure gradients: In the crack closure phase, the permeabilities decrease slightly with increasing axial strain, which is generally considered to be mainly caused by the closure of the origin microcracks and voids. They then continue to decrease in the linear elastic deformation phase, reaching their lowest points before the yield phase. After the yielding occurs, the permeability–strain curves increase dramatically owing to the connected secondary cracks and primary cracks becoming the main flow channels in this phase. With the stress–strain curves reaching their peak stress points, the permeabilities begin to show different change tendencies in the post-peak phase—namely a slight decrease and continual increase—which make the entire permeability
evolution curves different shapes of ‘S’ or ‘\( \sqrt{ } \)’ types. In this phase, a macrocrack forms and becomes the preferential seepage channel. And the permeability is controlled by the macrocrack width, which may become wider or narrower under triaxial loading. These abovementioned experimental results offer good consistency with those of previous research (Li et al. 1994; Souley et al. 2001; Wang and Park 2002; Paterson and Wong 2005; Wang et al. 2013).

When a confining pressure of 20 or 30 MPa is set for the experimental series, the permeability evolution has the same variation trend before sample yielding occurs. After reaching the post-peak phase, the permeability–strain curves begin to fluctuate to a small degree (\( \pm 2 \times 10^{-16} \text{ m}^2 \)).
at a confining pressure of 20 MPa. By contrast, the permeability curves decrease constantly at a confining pressure of 30 MPa in the same phase.

The reason for the differences in the permeability evolution curves under these three levels of confining pressure is their different failure modes after compression, which is shown in Fig. 7. Shear single failure, which is also a main characteristic reflecting the range of brittleness in the samples, occurs at relatively low confining pressure. After the onset of yielding, coalescence of microcracks plays a critical role in flow seepage in the rock sample, resulting in the increase in permeability until the shear band forms. Because the brittle–ductile transition occurs with the increase in confining pressure, the macroscopic failure mode becomes a local compaction band. This type of failure has been reported in many previous studies (Olsson 1999; Wong et al. 2001; Vajdova et al. 2004; Baud et al. 2004, 2015). The drop in post-peak permeability at high

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Confining pressure/MPa</th>
<th>Hydraulic pressure difference/MPa</th>
<th>Peak stress/MPa</th>
<th>Strain corresponding to peak stress/%</th>
<th>The first value on the permeability curve/10^{-15} m^2</th>
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<td>0.364</td>
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</table>
confining pressure occurs owing to porosity reduction during compactive strain localization failure, which is evidenced by observing microstructures in the rock (Vajdova et al. 2004; Ji et al. 2012). Interestingly, at confining pressures in the upper part of the brittle range but below the brittle–ductile transition range—namely 20 MPa—the rock sample fails in a mode that mixes a high-angle shear band with local compaction bands. The permeability growth induced by a macroscopic high-angle shear band may balance the decreasing trend caused porosity reduction, which finally leads to a fluctuation in permeability after the yield phase.

The first point on the permeability–strain curve was measured after hydrostatic pressure was applied but before the axial loading. As shown in Table 2, the first value on the permeability curve increases with the pore pressure difference \( \Delta p \) when the confining pressure is constant. Generally speaking, the absolute permeability is not supposed to change with the hydraulic gradient, but the absolute permeability does not change only when the internal structure of the rock sample does not change. In this research, a relatively large gradient—1, 4, or 7 MPa—is applied between both end faces of the sample; thus, the internal structure change caused by the high pore pressure cannot be neglected, leading to the first values on the permeability evolution curves change. It also can be seen from Table 2 that, when the pore pressure difference is set at the same level, the initial permeability decreases with increasing confining pressure. Serving as the seepage channels, the connected original microcracks and pores mainly determine the initial permeability. After the confining pressure is applied, they decrease in size and play a negative role in permeability.

3 Numerical Simulation

3.1 Brief Description of RFPA\textsuperscript{2D}-Flow

In this research, the simulator used was the numerical code RFPA\textsuperscript{2D}-Flow that can simulate the damage and failure process of materials under hydromechanical coupling conditions. In RFPA\textsuperscript{2D}-Flow, finite element method is used to calculate and analyze the stress field, and heterogeneity is considered by assigning different material properties to the mesoscopic elements that constitute the solid or structure model. Based on elastic damage mechanics, the constitutive relationships of these mesoscale elements in tensile and shear modes are adopted differently to determine whether these elements reach their damage thresholds as judged by the maximum tensile criterion and the Mohr–Coulomb criterion, respectively. A negative exponential function based on Biot’s theory of consolidation (Biot 1941) is employed to establish the permeability–stress relationship in the code.

3.1.1 Heterogeneity Consideration

It is known that rock is a type of heterogeneous geological material composed of several different mineral crystals and micropores at the mesoscopic level. Owing to the mesoscopic heterogeneity and discontinuity, rock exhibits different mechanical properties in terms of spatial distribution. To consider the heterogeneity, all mechanical properties and physical parameters of each mesoscale element, including elastic modulus, Poisson’s ratio, strength, and density, are assumed to conform to the Weibull distribution in the RFPA\textsuperscript{2D}-Flow code. Here, the probability density function \( \phi(\alpha) \) is given by

\[
\phi(\alpha) = \frac{m}{\alpha_0} \left( \frac{\alpha}{\alpha_0} \right)^{m-1} e^{-\left( \frac{\alpha}{\alpha_0} \right)^m}
\]

and the cumulative probability function is written as

\[
P(\alpha) = 1 - e^{-\left( \frac{\alpha}{\alpha_0} \right)^m}
\]

where \( \alpha \) is a given material property (such as the elastic modulus or strength), \( \alpha_0 \) is a scale parameter denoting the average value of the material property, and \( m \) is a shape parameter that defines the shape of the distribution function. In the RFPA\textsuperscript{2D}-Flow code, \( m \) represents the degree of material homogeneity and is thus called the homogeneity index.

Figure 4 shows Weibull distribution for material properties with different homogeneity indices \( m \). It is clear that when the homogeneity index \( m \) has a relatively high value, the mechanical properties and physical parameters of mesoscale elements in the rock gather in a very small range, which reflects that the rock is comparatively homogeneous. In contrast, when the rock’s properties are...
widely distributed with a relatively lower \( m \) index, the inhomogeneity of the rock’s mesoscale element properties is larger. The homogeneity index has been considered an effective approach to assign material properties for inhomogeneous rock and concrete (Tang 1997; Tang et al. 1998, 2000; Liu et al. 2004; Zhu and Tang 2004, 2006; Zhu et al. 2005; Wang et al. 2011, 2012; Tang and Tang 2012).

### 3.1.2 Statistical Damage Constitutive Model

Based on the principle of strain equivalence and the concept of effective stress (Lemaitre 1985; Lemaitre and Desmorat 2005), the constitutive model for one-dimensional linear elasticity of a damaged material can be described by the following formula:

\[
\sigma = E\varepsilon = E_0\varepsilon(1 - D)
\]

and Young’s modulus of damaged materials can be used to describe the degradation process as follows:

\[
E = E_0(1 - D)
\]

where \( \sigma \) is stress; \( E \) and \( E_0 \) are Young’s moduli of the damaged and undamaged material, respectively; \( \varepsilon \) is the elastic strain; and \( D \) is the damage variable.

It is assumed that each element of the solid or structure is said to be linear elastic, isotropic, and undamaged before uniaxial or multiaxial loading. Figure 5 illustrates the elastic constitutive law of a mesoscopic element under uniaxial compressive and tensile loading in the RFPA 2D-Flow code. On the tension side of the model shown in the first quartile of Fig. 5, the damage variable \( D \) can be expressed as (Tang et al. 2002; Yang et al. 2007)

\[
D = \begin{cases} 
0 & \varepsilon \leq \varepsilon_0 \\
1 - \frac{f_t}{E_0\varepsilon_0} & \varepsilon_0 \leq \varepsilon \leq \varepsilon_u \\
1 & \varepsilon \geq \varepsilon_u
\end{cases}
\]

Similarly, on the compression side of the model shown in the first quartile of Fig. 5, the damage variable \( D \) can be expressed as (Tang et al. 2002; Yang et al. 2007)

\[
D = \begin{cases} 
0 & \varepsilon \leq \varepsilon_{t0} \\
1 - \frac{f_t}{E_0\varepsilon_{t0}} & \varepsilon_{t0} \leq \varepsilon \leq \varepsilon_c \\
1 & \varepsilon \geq \varepsilon_c
\end{cases}
\]

where \( f_t \) is the compressive residual stress and \( \varepsilon_{t0} \) is the maximum principal strain at which damage starts to occur.

When the mesoscale element is under shearing or multiaxial stress conditions, the Mohr–Coulomb criterion is employed to describe the second damage threshold, which can be expressed as

\[
F = \sigma_1 - \sigma_3 \geq f_c
\]

where \( \phi \) is the friction angle of the element, \( f_c \) is the uniaxial compressive strength, and \( \sigma_1 \) and \( \sigma_3 \) denote the major and minor principal stresses, respectively. Based on the generalized Hooke’s law and the Mohr–Coulomb criterion, the maximum principal strain \( \varepsilon_{c0} \) can be deduced as

\[
\varepsilon_{c0} = \frac{1}{E_0} \left[ f_c + \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 - \nu (\sigma_2 + \sigma_3) \right]
\]

where \( \sigma_2 \) denotes the intermediate principal stress and \( \nu \) is Poisson’s ratio.

It is assumed that the damage evolution of each element under a three-dimensional stress state is controlled only by the maximum compressive principal strain \( \varepsilon_c \). By substituting Eq. (9), the damage equation under uniaxial compression—i.e., Eq. (7)—can be extended to describe the damage under the triaxial stress condition. Here, the extended equation is given by

\[
D = \begin{cases} 
0 & \varepsilon_1 \leq \varepsilon_{t0} \\
1 - \frac{f_t}{E_0\varepsilon_{t0}} & \varepsilon_{t0} \leq \varepsilon_1 \leq \varepsilon_c \\
1 & \varepsilon_1 \geq \varepsilon_c
\end{cases}
\]

### 3.1.3 Flow–Stress–Damage (FSD) Coupling Model (Tang et al. 2002; Yang et al. 2004)

In the past several decades, many theoretical and experimental studies have been performed to establish a coupling equation of fluid and stress to characterize the hydromechanical coupling behavior of geomaterials (Biot...
Biot’s theory of consolidation was one of the most widely accepted, and it can be extended to include permeability varying with stress (Tang et al. 2002; Yang et al. 2004). The governing equations are written as follows.

Equation of equilibrium:

\[ r_{ij} + f_j = 0 \]  

where \( r_{ij} \) denotes the components of the Cauchy stress tensor and \( f_j \) is the body force in the \( j \)th direction.

Geometrical equation:

\[ e_{ij} = \frac{1}{2} \left( \dot{u}_{ij} + \dot{u}_{ji} \right) \]  

where \( e_{ij} \) is the components of the Cauchy strain tensor and \( u_i \) is the displacement in the \( i \)th direction.

Constitutive equation:

\[ \sigma'_{ij} = \sigma_{ij} - \beta p \delta_{ij} = \lambda \delta_{ij} \epsilon_v + 2G\epsilon_{ij} \]  

where \( \sigma'_{ij} \) is the components of the effective stress tensor; \( p \) is the pore pressure; \( \delta_{ij} \) is the Kronecker delta; \( \epsilon_v \) is volumetric strain, which is defined as \( \epsilon_v = \epsilon_{ii} \); \( G \) is the shear modulus of the material; and \( \beta \) and \( \lambda \) are the pore pressure coefficient and Lamé coefficient, respectively.

Storage equation:

\[ K \nabla^2 p = \frac{1}{Q_B} \frac{\partial p}{\partial t} - \beta \frac{\partial \epsilon_v}{\partial t} \]  

where \( Q_B \) is Biot’s constant, \( t \) is the time, and \( K \) is the hydraulic conductivity. It is noted that the hydraulic conductivity \( K \) is not the same as the absolute permeability \( k \). The relation between them is given by the following equation:

\[ K = \frac{k_H}{\mu} \]  

where \( \mu \) is the fluid dynamic viscosity coefficient and \( \gamma \) is the unit weight of the fluid.

Hydraulic conductivity function based on the stress and damage (Louis 1974; Yang et al. 2011):

\[ K(\sigma, p) = \xi K_0 e^{-\eta(\sigma_{ij}/3 - \beta p)} \]  

where \( K_0 \) is the initial hydraulic conductivity, \( \xi \) is the damage factor of the hydraulic conductivity, and \( \eta \) is a material constant.

In the RFPA2D-Flow code, the FSD coupling model of each mesoscopic element during the entire damage evolution process is written by (Tang et al. 2002; Yang et al. 2007)

\[ K = \begin{cases} K_0 e^{-\eta(\sigma_{ij}/3 - \beta p)} & D = 0 \\ \xi K_0 e^{-\eta(\sigma_{ij}/3 - \beta p)} & 0 < D \leq 1 \end{cases} \]  

### 3.2 Mechanical Model and Parameters

In this research, all laboratory triaxial compression tests were conducted blind, and the failure process of the rock sample was invisible because the rock samples were sealed in an opaque confining pressure cylinder. In an attempt to more fully understand the process of crack formation, propagation, and coalescence, a series of numerical simulations corresponding to the laboratory experiments was performed. As presented in Fig. 6, the sandstone samples were simplified to a plane axisymmetric model (biaxial) of dimensions 50 × 25 mm². A total of 500 × 250 = 12,500 quadrilateral elements in the same size were employed for meshing the model. Young’s modulus

\[ E = \frac{9.370 \times 10^9}{25} \]  

\[ E = \frac{7.028 \times 10^9}{25} \]  

\[ E = \frac{4.685 \times 10^9}{25} \]  

\[ E = \frac{2.343 \times 10^9}{25} \]  

\[ E = \frac{0.000 \times 10^9}{25} \]  

where \( \mu \) is the fluid dynamic viscosity coefficient and \( \gamma \) is the unit weight of the fluid.
modulus and the uniaxial compressive strength of these mesoscopic elements were assumed to follow the Weibull distribution—i.e., Equation (2). Confining pressure and axial loading were applied in the horizontal and vertical directions of the model, respectively. And fully coupled method was employed, indicating that both axial stress and confining pressure are completely computed together. The pore pressure difference was imposed by setting pervious boundary conditions of fluid pressure on the top-end plane of the model and on the bottom-end plane. The left- and right-hand sides of the model were impervious boundaries. The distribution of pore pressure in the model was assumed to be linear in the vertical direction. The input parameters in the model are summarized in Table 3.

### Table 3 Material properties of sandstone samples for numerical model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneity index ( m )</td>
<td>3</td>
</tr>
<tr>
<td>Mean of Young’s modulus/GPa</td>
<td>49.2</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.123</td>
</tr>
<tr>
<td>Mean of uniaxial compressive strength/MPa</td>
<td>200</td>
</tr>
<tr>
<td>Ratio of compressive and tensile strength</td>
<td>10</td>
</tr>
<tr>
<td>Damage factor of coefficient of permeability ( (\xi) )</td>
<td>5, 1.2, 0.3</td>
</tr>
<tr>
<td>Pore pressure coefficient ( (\beta) )</td>
<td>1</td>
</tr>
<tr>
<td>Material constant ( (\eta) )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

3.3 Numerical Results

#### 3.3.1 Progressive Failure and Associated Failure Pattern

In the process of calculation, the finite elements are damaged following the statistical damage constitutive model given in Sect. 3.1.2, and then the stress on the elements will be iteratively re-calculated based on the current boundary conditions at this loading step. The code RFPA\(^{2D}\)-Flow would not go on to the next loading step until no damaged elements are newly formed for the iterative step of this loading step. When employing FSD coupling model, it is assumed that there is always a linear pressure gradient along the length direction of the numerical model at all loading steps.

It is noteworthy that, from the numerical results, the crack behavior of the model is slightly subject to the pore pressure gradient but is significantly affected by the confining pressure. When varying the pore pressures coupled with a constant confining pressure imposed on the model, the failure processes are highly similar. Limited by space, only typical detailed mesoscopic failure processes at several selected loading steps under four different confining pressures are presented in Fig. 7. In the figure, the distribution of Young’s modulus, the associated acoustic emission source locations, and the distribution of maximum shear stress of the rock sample are displayed. The color of Young’s modulus and maximum shear stress in the figure indicates their relative magnitude at a certain loading step. The center and diameter of the circle are the location of the acoustic emission and the relative magnitude of the released elastic strain energy, respectively. The white circles indicate the acoustic emission source caused at a certain loading step, and the red ones are generated at all previous steps.

Figure 7a illustrates the failure process of the numerical model under uniaxial compression. Both step 30 and step 80 are in the linear elastic deformation phase. At loading step 30, the distribution of Young’s modulus is almost the same as before loading, which indicates that there are few damaged mesoscopic elements. Accordingly, only a small number of acoustic emission source locations randomly appear in the model. The maximum shear stress at this step is distributed uniformly. With the axial loading increasing until step 80, a large number of acoustic emission events caused by the release of elastic strain energy are recorded in the model, which reflects that many elements are close to being damaged. However, the distribution of the maximum shear stress is still uniform at this step, and it is not obvious where the macrocracks will form. At loading step 105, a shear nucleation zone in the rock sample caused by the heterogeneity has formed. The distribution of the acoustic emission is mostly clustered around the shear nucleation zone, and the maximum shear stresses of the mesoscopic elements in the model are relatively higher around the zone than in the other region. When the axial load continues to increase until loading step 113, dispersed cracks in the shear nucleation zone gradually propagate, coalesce, and connect, leading to a distinct shear band. At the same time, an associated crack has formed across the shear band. The acoustic emission resource locations are more concentrated along the shear band and associated crack. At loading step 120, the rock sample loses its bearing capacity. The macrocrack aperture grows, and the shear band is inclined at 28° to the axial direction.

As presented in Fig. 7b, when the numerical model is under a confining pressure of 10 MPa, the cracking process is somewhat different. At loading step 30, the maximum shear stresses of the elements distributed in the lower part of the model are higher than in the upper part. This is due to the linear assumption regarding the pore pressure in the model, which results in the effective confining pressure being trapezoidly distributed instead of rectangularly distributed in the height direction. With increasing axial load, a macrofracture inclined at 44° to the axial direction finally forms at the lower part of the model.
It can be observed is Fig. 7c that not only a shear band but also a nucleation zone is generated in the model when the confining pressure is set to 20 MPa. The shear band is inclined at approximately 42° in the axial direction, and the nucleation zone is approximately parallel to the lateral pressure direction. This type of nucleation zone also can be called a local compaction band. In Fig. 7d, at a confining pressure up to 30 MPa, only a local compaction band gradually appears in the lower part of the model, which is clearly reflected by the distribution of the associated acoustic emission source locations.

Figure 8 shows the comparison of laboratory experimental fracture patterns after compression and numerically simulated results. The simulated failure patterns according to RFPA2D-Flow offer good consistency with experimental results. In the uniaxial compression experiment, the shear fracture occurs at an angle of approximately 24° to the direction of the axial compressive load, which is very close to the principal crack angle of approximately 28° in the simulated results. Shear fracture also takes place at a confining pressure of 10 MPa, and the fracture angle in these cases is relatively higher than that in unconfined compression, approximately 39° to the direction of the maximum compressive principal stress. Its simulated fracture pattern is also similar to the experimental crack mode. At a confining pressure of 20 MPa, the ductility of the rock sample increases, and its failure shows up as a mixed pattern consisting of a high-angle shear band and a zone of intense deformation. This type of failure pattern can be a precursor for brittle–ductile transition. As the brittle–ductile transition is approached with increasing confining pressure to 30 MPa, the failure pattern tends to become a compactive strain localization band, and fine-scale fracturing, rather than a shear fracture, occurs. All of these numerical simulation failure patterns based on RFPA2D-Flow are in good agreement with their experimental results.
3.3.2 Simulated Permeability Evolution with Damage

Figure 9 illustrates the comparison of permeability evolution with axial strain obtained from triaxial experiments and the biaxial numerical simulation according to RFPA 2D-Flow. As mentioned in Sect. 3.1.3, the permeability is calculated by Eq. (17), in which the total flow rate of the 250 elements in the bottom line of the numerical model was taken, to correspond to experimental macroscopic results. Based on a great deal of trial calculations, the damage factor of coefficient of permeability $n$ in the numerical models was finally assigned values of 5, 1.2, and 0.3 at the confining pressure of 10, 20, and 30 MPa, respectively.

It can be observed from Fig. 9a–c that the permeability–axial strain curves have the same change tendency under all of these confining pressure and pore pressure coupling conditions—that is, to remain stable at first and then increase dramatically. Thus, the numerical tendencies of permeability are qualitatively similar to the experimental results at the confining pressure of 10 MPa when the parameter $n = 5$. The abrupt growth in permeability after damage occurs in the FSD coupling mesoscopic model was initially established by Tang et al. (2002) and Yang et al. (2004) based on some laboratory experimental research on damage-induced permeability growth in brittle rock (Schulze et al. 2001; Souley et al. 2001; Wang and Park 2002). In Fig. 9a–c, the numerical results agree well with the experimental results; thus, the validity of the FSD coupling model in calculating variations in permeability of brittle rock was also re-verified.

In Fig. 9d–i, there are consistencies between the numerical solution and experimental results as well by picking the parameter $n = 1.2$ and 0.3 at confining pressures of 20 and 30 MPa. In Sect. 2.3.2, it is known that the effect of brittle–ductile transition makes the permeability evolution with damage of the sandstone samples showing different change tendency—decreasing, fluctuating, or remaining steady after failure of rock samples, indicating that the degree of ductility of the rock sample determined...
by the confining pressure has significant influence on permeability change. This phenomenon of permeability evolution of rock sample with ductile failure has also widely reported (Zhu and Wong 1997; Vajdova et al. 2004; Hu et al. 2010; Han et al. 2013); however, this type of permeability evolution is difficult to simulate by using a same value of $n$ as in the process of brittle failure in the FSD coupling model. After theoretical analysis and trail calculations, it is found that the parameter $n$ is related to degree of ductility. That is the reason why different values of the parameter $n$ are assigned in the numerical simulation.

In this research, the FSD coupling model of the mesoscopic elements was established based on macroscopic experiments, and then the mesoscopic model was employed to calculate the results of the macroscopic experiments. Although there is no quantitative coincidence between macroscopic experiments and simulated results, the FSD coupling model can still be a good tool to predict or estimate the permeability evolution in the failure process of rock.

4 Discussion

In recent years, many experimental and theoretical studies to characterize permeability evolution laws of rock have been reported. These research results show that the permeability of rock is mainly related to porosity, temperature, stress, and failure models, etc. (Ma 2015; Zhu and Wong 1997; Morris et al. 2003; Ma and Wang 2016). However, there have been only a few reports on the research into the relationship between permeability and volumetric strain under triaxially compressed conditions. In this section, the relationship will be explored preliminarily in light of the experimental results.

Based on the theory of elasticity, in this paper, the volumetric strain is calculated by the following equation:

$$\varepsilon_v = \varepsilon_{ax} + 2\varepsilon_{rad}$$

where $\varepsilon_v$, $\varepsilon_{ax}$, and $\varepsilon_{rad}$ are the volumetric strain, axial strain, and radial strain, respectively.
Affected by a set of factors, such as the inhomogeneity of the rock sample, machining precision, end-friction effects, hydrostatic pressure, and strain localization, the real value of radial strain cannot be accurately measured by the gauge, resulting in the fact that volumetric strain also cannot be calculated accurately. For example, as shown in Fig. 10, the radial strain gauge is placed at the middle height of the sample, whereas the position of strain localization occurs in the lower part. Certainly, the data recorded by the radial strain gauge are not very precise, so the radial strain measured by the gauge is called nominal radial strain by some scholars.

Although the nominal radial strain is not very accurate, especially when the data are obtained after the linear elastic phase, the calculated volumetric strain according to Eq. (18) still has some regularity. By sorting out and analysis of the experimental results, the permeability–volumetric strain curves are plotted in Fig. 11, in which arrows represent the permeability–volumetric strain evolution direction with increasing external loading. In the early stage of all of these curves, the volumetric strain increases in the positive direction, and the permeability decreases with increasing volumetric strain, which indicates that the volume reduction in rock samples caused by the combination of axial pressure, confining pressure, and pore pressure downsized or blocked the seepage channels. After reaching their maximum values, the volumetric strains begin to decrease. As known in rock mechanics, the inflexion point of volumetric strain means that the volume of the rock samples starts to dilate. At a confining pressure of 10 MPa, the permeability increases remarkably with increasing dilatancy. In contrast, the curves remain steady at a confining pressure of 20 MPa, and they decrease significantly under the condition of 30 MPa confining pressure.

As summarized in Fig. 12, with consultation of the literature, it is found that the previous experimental results of permeability–volumetric strain relationships were different from each other and also different from ours. It is found that permeability decreases with increasing volume of the
Fig. 8 Comparison of fracture patterns after experiments and numerical simulation.

a $P_c = 0$ MPa (uniaxial compression), b $P_c = 10$ MPa, c $P_c = 20$ MPa, and d $P_c = 30$ MPa
rock, and it then remains constant after the peak strength is reached and only increases shortly before the shear band develops (Heiland and Raab 2001; Heiland 2003). In the study conducted by Wang et al. (2014), the permeability–volumetric strain curve was divided into four phases. The permeability decreases in the first phase, then remains stable and nearly constant in the second phase, increases to the peak in the third phase, and decreases in the fourth phase. Of these phases, the first three correspond to volumetric compaction, and the fourth corresponds to volumetric dilatation. The work by Souley et al. (2001) presents a different tendency. The permeability decreases first and then remains stable with increasing volumetric strain. When the volumetric dilatation begins, the permeability increases remarkably and then gradually becomes flat. In addition to these tendencies, it has been reported that the volume strain and permeability have a rather highly corresponding relationship with the axial strain when freezing-thawing cyclic treated sandstone is triaxially compressed (Yu et al. 2015).

It can be observed in the abovementioned experimental results that the evolution of permeability subjected to volumetric strain is quite complex, and a recognized relationship between them has not been completely clarified. It may depend on some factors, such as lithological characters, loading rate (Heiland 2003), confining pressure (Wang et al. 2014), and differential stress (Heiland and Raab 2001). Other factors should be sought, and all of these factors should be explored deeply in further studies. Apparently, it is necessary to design some new methods for
measuring volumetric strain directly and accurately in the future. This is the only way to avoid the influence of strain localization.

5 Conclusions

To gain a better understanding of the mechanical properties and permeability of sandstone in the failure process, a series of triaxial compression tests with different confining pressures coupled with different pore pressure differences was carried out. Correspondingly, the numerical simulations were conducted using the RFPA2D-Flow code, in which the heterogeneity of rock was considered by assuming that the material properties of its mesoscopic elements conform to a Weibull distribution, incorporating a statistical damage constitutive model based on elastic damage mechanics and an FSD coupling model based on Biot’s theory of consolidation. The following conclusions can be drawn:

1. All experimental stress–strain curves can be divided into four phases: crack closure phase, linear elastic deformation phase, yield, and peak stress and post-peak phases. With increasing confining pressure, the stress–strain curves have the same shape before the peak stress is reached, but they switch from strain softening to almost ideal plasticity in the post-peak phase. The brittle–ductile transition occurs at a relatively high confining pressure. The peak stress increases slightly with increasing pore pressure difference at all confining pressure levels.

2. In the triaxially compressive process under hydromechanical coupling, the permeability decreases with increasing axial strain in the first three phases.

However, after reaching the post-peak phase, the permeability curves vary with increasing confining pressures; i.e., they increase, remain constant, and decrease. In addition, the pore pressure difference has no significant effect on the shape of the permeability evolution. However, when it is constant, the first
values on the permeability evolution curves decrease with increasing confining pressure.

3. Mainly subjected to confining pressure, failure patterns under triaxial compression are characterized by four typical types: single shear band, high-angle shear band, a combination failure of high-angle shear band and compaction band, and compaction band only. Numerical simulation based on RFPA2D-Flow code reproduced the failure processes well, and all failure patterns obtained by numerical simulation are in good agreement with their corresponding experimental results.

4. By assigning different values of parameter $\xi$, which is related to degree of ductility, the numerical results about permeability evolution with damage agree well with the experimental results; thus, the FSD coupling model can be a good tool to predict or estimate the permeability evolution in the failure process of rock.

5. There may be some relationships between volumetric strain and permeability that are strongly affected by the confining pressure. However, owing to the inaccuracy of the measured radial strain, the relationships cannot be clarified completely. Additionally, by consulting the literature, it was found that the evolution of permeability subject to volumetric strain is quite complex but that it is not yet governed by a universally accepted law. More accurate volumetric strain measurement methods are required for future studies.

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