Chapter 19
Composite Slab: A Unilateral Contact Problem

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Abstract. Unilateral contact problems, where the contact and no contact regions between bodies are not a priori known, are complex and highly nonlinear problems. The interaction between concrete and profiled steel sheeting in composite slabs can be classified as a unilateral contact problem. The analysis presented in this paper is divided into a numerical, mathematical and experimental treatment of the physical problem under consideration. Therefore, a series of full-scale tests of composite slabs has been carried out, under the rules of Eurocode 4. Furthermore, a 3-d finite element computational simulation of the composite slab has been realized using the ANSYS software package. The interface has been simulated by the use of contact elements and the application of unilateral contact conditions. The numerical approach of the problem under consideration is completed by a 2-d model which describes in mathematical terms the existing contact and frictional phenomena. The comparative evaluation of the results leads to useful conclusions.

Keywords: Composite slabs, profiled steel sheeting, unilateral contact, non-monotone laws, hemivariational inequalities.

19.1 Experimental Analysis

19.1.1 Preparation and Arrangement

The composite slabs used in the experiment have been cast in fully supported condition. This is the least favourable condition for the experimental analysis according to Eurocode 4 [1, 2]. The profiled steel sheeting has been used as formwork

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and tension reinforcement [3, 4]. It is of trapezoidal shape, of S320 type and 1 mm in thickness. The concrete used for the casting has been of C25/30 grade. The average 28-day cylinder strength of the concrete under compression has been found, by testing cylinder specimens, equal to 24.3 MPa. The concrete has not been reinforced. A total number of 9 composite slab specimens were cast. Electronic strain gauges connected to computer software have been placed on critical positions of the specimens.

The nine composite slabs have been split into three groups of three specimens according to geometry. The dimensions of the slabs were: length 2200 mm, width 600 mm and height 175 mm (first group) or 140mm (second group) or 110 mm (third group). The same profiled steel sheeting has been used for each group.

One slab of each group has only been subjected to static monotonic loading until failure. The failure did not occur in less than one hour and the calculated failure loads determined the level of the cyclic load for the next test.

![Fig. 19.1 Test arrangement](image)

The other six slabs (two of each group) have first been subjected to a sequence of loadings for three hours. This was a sinusoidal cyclic load applied at an initial stage aiming to severe the chemical bond between the concrete and the profiled steel sheeting at the interface of the composite slab. The frequency of the cyclic load has been 0.5 Hz [4] and its amplitude ± 3.3 kN from an initial offset of 6.6 kN. This cyclic loading sequence has been applied in 5000 cycles. The next loading sequence has been of a monotonic nature. The subsequent static test lasted one hour.
19.1.2 Results

In Figures 19.2 and 19.3 the obtained deflection response, from two different sequences of tests, for specimen 1 of the first group is depicted. The response of both the cyclic and the monotonic sequences of tests are plotted.

Fig. 19.2 Load-deflection curves for specimen 1 of the first group

Fig. 19.3 Bending moment-mid deflection curves for specimen 1 of the first group
In Figure 19.4 the load-slip curve at the interface of the composite slab is depicted.

![Load-slip curve](image)

**Fig. 19.4** Load-slip curve for specimen 1 of the first group

In Figure 19.5 the failure of the composite slab accompanied by a major failure crack at the position of loading, at the L/4 on the span length, is shown.

![Failure of the composite slab](image)

**Fig. 19.5** Failure of the composite slab

In Figure 19.6 the bending moment-mid deflection curves from each group is depicted.
19.2 Computational Analysis

19.2.1 Description of the Finite Element Model

A composite slab of the first group (according to the experimental analysis) has been modeled. The steel sheeting profile exhibits very small thickness compared to its other dimensions and is therefore considered as a thin-walled element [5, 6]. A three-dimensional eight-node shell element has been selected for the profiled steel sheeting. For modelling purposes it has been decided for the steel sheeting thickness to coincide with the shell element thickness.

A three-dimensional eight-node isoparametric structural element has been selected for the concrete. No reinforcement has been used, to coincide with the experimental study.

The profiled steel sheeting-concrete interface has been modelled with three-dimensional contact surface-surface elements in order to achieve composite action. Steel and concrete surfaces have been assumed to be deformable. The contact elements overlay the elements used for the simulation of the steel sheeting and concrete. The contact pair has been constructed by using area to area contact finite elements. ANSYS detects contact at the Gauss points of the interface of the composite slab.
The analysis performed is geometrically nonlinear with stress stiffening, large deflections and small strains characteristics. The selected finite elements support geometric and boundary nonlinearities. ANSYS uses the Newton-Raphson method as an incremental-iterative solution process. The tangential stiffness matrix is updated after each iteration. The convergence procedure is force-based and thus considered absolute.

In Figure 19.7 a pre-stressed finite element model is depicted.

### 19.2.2 Material Laws

The constitutive material law selected for steel has been multilinear elastoplastic-strain hardening using the von Mises yield criterion, as seen in Figure 19.8. Steel has been assumed to be homogenous. The yield stress for the structural steel in tension has been determined at 320 MPa, the ultimate strength at 480 MPa and the modulus of elasticity at 210 GPa.
For the concrete, the non-linear material simulation of the ANSYS software program has been used [7]. This includes both cracking and crushing failure modes. ANSYS uses the William-Warnke criterion as the failure criterion. The characteristic yield stress in compression has been determined at 25 MPa and the modulus of elasticity at 30.5 GPa. The ultimate tensile stress has been determined at 2.6 MPa [8]. Both materials have been considered isotropic [9]. The Poisson ratio for structural steel and concrete has been assumed as 0.3 and 0.2 respectively.

The friction which develops in the steel-concrete interface has not been deemed negligible; instead a constant friction coefficient of 0.30 has been considered. ANSYS software program uses the Coulomb friction model which is adequate in most problems [10].

### 19.2.3 Results

In the following Figures 19.9 to 19.11 the computational results are presented. In Figure 19.9 the deflection of the composite slab at the time of the maximum vertical load is plotted.

![Fig. 19.9 Deflection of the composite slab under maximum vertical load](image)

![Fig. 19.10 Load-deflection curve](image)
In Figure 19.11 the load-deflection diagram for the middle section of the composite slab is presented. In Figure 19.11 the longitudinal shear strength $\tau_u$ at the interface of the composite slab is depicted. It was found equal to 0.22 MPa.

19.3 Mathematical Analysis

19.3.1 Description of the Problem

The problem of the composite slab is highly nonlinear. During loading, the concrete slab cracks introducing nonlinearities at the formulation of the problem. Furthermore, the sliding at the steel-concrete interface results in a constant change of the boundary conditions. The contact regions at the interface are typically unknown prior to analysis, resulting in significant changes of both normal and tangential stiffness. These nonlinearities are described in the following by the use of nonmonotone constitutive laws.

In this case, the hemivariational inequality that describes the problem, expresses the principle of virtual work for a composite slab with profiled steel sheeting.

19.3.2 Mathematical Formulation of the Problem

In the following, the problem of the composite slab will be treated as a unilateral contact problem with friction. In the normal direction to the interface of the composite slab unilateral contact conditions, relations (19.3.1) and (19.3.2), are applied. For the case of two deformable bodies [11], it is valid:
If $\bar{u}_N > 0$ then $S_N = 0$  \hfill (19.3.1)

If $\bar{u}_N = 0$ then $S_N \geq 0$, \hfill (19.3.2)

where: $\bar{u}_N$ is the quantity $u_{N1} + u_{N2} + h - u_0$, $u_{N1}$ is the displacement of the first body, $u_{N2}$ is the displacement of the second body, $h$ is the distance between the two bodies and $u_0$ is the relative displacement of the bodies due to rigid body motion (rigid body displacement). Moreover, $S_N$ is the normal contact force at the interface of the two bodies. This case is different from the Signorini-Fichera [12] boundary conditions because small boundary displacements have been taken into account.

In the direction tangential to the steel-concrete interface, a nonmonotone friction law has been applied, of the form depicted in Figure 19.12 [13, 14].

![Fig. 19.12 The applied nonmonotone friction law](image)

From the analysis of the friction law it is obtained that:

If $|S_T| < \mu |S_N|$ then $u_T = 0$ \hfill (19.3.3)

If $\mu |S_N| \leq |S_T| < \mu |S_N| + \beta |S_N|$ and $u_T \neq u_{sl}$, then $u_T = 0$ \hfill (19.3.4)

If $\mu |S_N| \leq |S_T| < \mu |S_N| + \beta |S_N|$ and $u_T \neq 0$, then $u_T = u_{sl}$ \hfill (19.3.5)

If $|S_T| = \mu |S_N| + \beta |S_N|$ and $u_T \neq u_{sl}$, there exists $\lambda \geq 0$ such that $u_{T_i} = -\lambda S_{T_i}$, with $i=1,2,3$. \hfill (19.3.6)
Here $S_T$ are the tangential forces at the interface of the composite slab, $S_N$ are the normal forces at the interface of the composite slab, $u_T$ is the tangential displacement, $u_{sl}$ is the displacement at the point when the sticking between the bodies at the interface of the composite slab excesses into sliding, $\mu > 0$ symbolizes the coefficient of friction and $\beta$ is the coefficient of cohesion, which takes two values $\beta = 1$ (existence of cohesion) or $\beta = 0$ (lack of cohesion).

As depicted in Figure 19.12, there is a sudden decrease of the shear forces, developed at the interface of the composite slab, at the time when the maximum value of the shear strength is reached. This decrease denotes the transition from the phase of sticking to the phase of relative sliding between concrete and steel sheeting due to lack of cohesion at their interface. After the initiation of the phase of relative sliding, the shear forces at the interface remain constant, as a result of a constant coefficient of friction. The vertical branches of the nonmonotone friction law describing the tangential direction of the contact interface denote that the problem must be treated according to nonsmooth and nonconvex mechanics. In the case where cohesion is not taken into account, $\beta = 0$, the monotone Coulomb law of friction [11, 12] is obtained

If $|S_T| < \mu |S_N|$ then $u_T = 0$ \hspace{1cm} (19.3.7)

If $|S_T| = \mu |S_N|$, there exists $\lambda \geq 0$ such that $u_{T_i} = -\lambda S_{T_i}$, with $i=1,2$ \hspace{1cm} (19.3.8)

where $\mu > 0$ is the coefficient of friction. Relation (19.3.7) describes the case of sticking contact and relation (19.3.8) the case of sliding contact.

Let us now consider a system of two deformable solid bodies $\Omega_1$ and $\Omega_2$ in a global Cartesian coordinate system $Ox_1x_2x_3$ of $\mathbb{R}^3$.

![Fig. 19.13 The system of two deformable bodies](image)
Furthermore, let us denote by $\Gamma$ the common interface of the two bodies under consideration, by $F$ the boundary of the system of the bodies where external loading is applied and by $C$ the boundary of the system of the bodies where the kinematical conditions are applied. These boundaries are not overlapping one another.

If $\overline{P}_i$ is the vector of the external forces applied in the system of the two bodies and $\overline{P}_i$ corresponds to the unknown forces which are developed at the interface (boundary) of the bodies, the equilibrium condition can be written

$$\overline{P}_1 = -\overline{P}_2. \quad (19.3.9)$$

For each body separately the total vector of forces can be written as

$$P_1 = \begin{pmatrix} \overline{P}_1 \\ P_1 \end{pmatrix} \quad \text{and} \quad P_2 = \begin{pmatrix} \overline{P}_2 \\ -P_2 \end{pmatrix}. \quad (19.3.10)$$

The following equilibrium equations can now be formulated

$$G_1s_1 = P_1 \quad \text{and} \quad G_2s_2 = P_2, \quad (19.3.11)$$

where $G_i$ is the equilibrium matrix and $s_i$ is the strain vector for each body. The compatibility equations for the bodies are

$$e_1(u) = G_1^T u_1 \quad \text{and} \quad e_2(u) = G_2^T u_2, \quad (19.3.12)$$

where $e_i, u_i$ are the strain and displacement vectors for each body respectively. $X$ is used here to denote the field of the kinematically admissible displacements $u \in X$. The relative displacement (slip) at the common boundary of the two bodies [12] is

$$[u_j] = u_{j,1} - u_{j,2}, \quad (19.3.13)$$

where $j=1, k$ is the number of the nodes at the interface.

The behaviour of the interface of the two bodies under loading can be described by the use of the following nonlinear expressions of constitutive cohesion and friction laws

$$-\bar{P}_{jN} \in \partial\Phi_{jN}(u_{jN}),$$

$$-\bar{P}_{jT} \in \partial\Phi_{jT}(u_{jT}). \quad (19.3.14)$$
where $\Phi_{jN}, \Phi_{jT}$ are superpotential functions and $\widetilde{\partial}$ is the generalized gradient of Clarke-Rockafellar [15]. The aforementioned constitutive laws can be written in inequality form

$$-\bar{P}_{jN} \leq \Phi_{jN}(u^*_j - u_{jN})$$

$$-\bar{P}_{jT} \leq \Phi_{jT}(u^*_j - u_{jT}),$$

(19.3.15)

$\forall \ u^*_j$ and $u^*_j \in \mathbb{R}^3$, where $u^*_j$ is the vector of the perturbed displacements.

It is assumed that functions $\Phi_N, \Phi_T : \mathbb{R}^3 \rightarrow \mathbb{R}$ are locally Lipschitz. This leads to a set of equilibrium equations

$$(G_1s_1)^T = P_1^T u_1 \text{ and } (G_2s_2)^T = P_2^T u_2.$$  

(19.3.16)

Taking into consideration the compatibility and equilibrium equations, the principle of virtual work for the problem can be formulated

$$s_1^T e_1(u^* - u) + s_2^T e_2(u^* - u) = P_1^T (u^*_1 - u_1) + P_2^T (u^*_2 - u_2).$$  

(19.3.17)

The following relations also hold:

$$\bar{P}_N(\bar{u}_{1N} - \bar{u}_{2N}) = \sum_{j=1}^{k} \bar{P}_{jN}(\bar{u}^*_j),$$

$$\bar{P}_T(\bar{u}_{1T} - \bar{u}_{2T}) = \sum_{j=1}^{k} \bar{P}_{jT}(\bar{u}^*_j).$$  

(19.3.18)

From relations (19.3.17) and (19.3.18) the following equality is obtained

$$s_1^T e_1(u^*_1 - u_1) + s_2^T e_2(u^*_2 - u_2) - \bar{P}_N(\bar{u}^*_1 - \bar{u}_{1N}) - \sum_{j=1}^{k} \bar{P}_{jN}(\bar{u}^*_j) =$$

$$\sum_{j=1}^{k} \bar{P}_{jT}(\bar{u}^*_j - \bar{u}_{jT}) =$$

$$\bar{P}_1^T (\bar{u}_{1} - \bar{u}) + \bar{P}_2^T (\bar{u}_2 - \bar{u}_2).$$

(19.3.19)

The problem is finally described by a hemivariational inequality [11, 16], where the unknowns are the kinematically admissible displacements $u \in X$ such that
\[
s_1^e (u_1^* - u_1) + s_2^e (u_2^* - u_2) + \sum_{j=1}^{k} \Phi_{jn} (\begin{bmatrix} u_{jN}^* \\ u_{jT}^* \end{bmatrix} - \begin{bmatrix} u_{jN} \\ u_{jT} \end{bmatrix}) + \\
\sum_{j=1}^{k} \Phi_{jT} (\begin{bmatrix} u_{jT}^* \\ u_{jT} \end{bmatrix}) \geq \Phi_{jT} (\begin{bmatrix} u_{1T}^* \\ u_{1T} \end{bmatrix}) + \Phi_{jT} (\begin{bmatrix} u_{2T}^* \\ u_{2T} \end{bmatrix}) \forall u^* \in X.
\]

(19.3.20)

The previous inequality is called hemivariational inequality [11] due to the appearance of terms of superpotential functions.

### 19.3.3 Mathematical 2-d Treatment of the Problem

For simplicity reasons, both the concrete and the profiled steel sheeting of the composite slab have been considered as two-dimensional orthogonal deformable bodies and modelled with triangular finite elements. The simulation and the analysis of the model have been performed using the MATLAB software package. The material laws and the loads used during this model coincide with those used in ANSYS.

In the normal to the interface direction, unilateral contact conditions have been applied [16, 17]. In the tangential to the interface direction, the Coulomb friction law has been applied. The coefficient of friction has been set to \( \mu = 0.3 \) and considered constant during the analysis. In Figure 19.14 the deformed shape of the finite element model is shown.

**Fig. 19.14** Finite element grid and deformed shape of the composite slab

In the following Figure 19.15 the normal contact forces are depicted.
The problem under consideration is a unilateral contact problem with Coulomb friction. Through the dual formulation, the problem is simplified and transformed into a quadratic minimization problem of the potential energy with simple constraints \cite{18}. The potential energy of the system is described by the following quadratic form:

$$\Pi = \frac{1}{2} \lambda^T K \lambda - P^T \lambda,$$

and the corresponding minimization problem is expressed as:

$$\min \{ \Pi = \frac{1}{2} \lambda^T K \lambda - P^T \lambda | \lambda \geq 0 \},$$

where $K$ denotes the dual stiffness matrix, $P$ is the dual load matrix, containing the external and the contact forces, and $\lambda$ are the Lagrange multipliers. The physical meaning of the Lagrange multipliers is that of nodal contact displacements at the interface of the composite slab.

The problem is solved by using the method of successive approximations \cite{18}. The minimization of the above quadratic form is realized by the conjugate gradient method \cite{19}. It has been proved \cite{20} that the next expression minimizes the equation (19.3.22):

$$K \lambda = P.$$
The iterative process terminates when the residual $r_i$ is smaller than a predefined quantity:

$$r_i = P - K\lambda_i \leq \varepsilon,$$

(19.3.24)

where $i=1,n$ are the iterative steps and $\varepsilon < 10^{-3}$ is the allowed error tolerance. The allowed error tolerance is usually a fraction of the initial residual one.

The following Figures 19.16 and 19.17 depict the load-middle deflection curve and the load-slip curve as measured during the analysis.

**Fig. 19.16** Load-middle deflection curve

**Fig. 19.17** Load-slip curve
19.4 Conclusions

The comparison, between the results of the experimental and the computational approach, lead to the following outcomes:

- The failure mode of all composite slab specimens was brittle. This behaviour was initiated by the slip (separation) of the profiled steel sheet from the concrete and was accompanied by a drastic decrease of the bearing capacity when severe flexural cracks developed at the concrete part.
- After the achievement of the maximum load, during the experiment, the load decreased suddenly but increased again almost up to the maximum level due to the resistance of the mechanical bond between the materials. This mechanical interlock originates from the interaction between the embossments of the profiled steel sheeting and the concrete.
- The load–deflection curve produced by the numerical approach of the problem is in close agreement with the load–deflection curve produced during the experiment and the slope of the curves is identical. In Figure 19.18 the results of both analyses are presented on the same graph.

![Load-mid deflection curve](image)

**Fig. 19.18** Experimental and numerical load-deflection curves

- The numerical model proposed in this paper is relatively simple and describes accurately the interaction of concrete and profiled steel sheeting in a composite slab.
- The 2-d model, as described in the mathematical approach of the problem, can be used as a design tool for composite slabs. The material characteristics of the profiled steel sheeting and the concrete used in the mathematical model coincide with the ones used in the experimental and the numerical approach of the problem.
The results of the mathematical model should not be directly compared to 3-d ones as they are outcomes of a simplified analysis. Nevertheless, the diagrams produced by the 2-d model are in close agreement with the experimental and the numerical ones and they depict with adequate accuracy the behaviour of the composite slab under examination.

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**References**


