Competitive nonlinear pricing with product differentiation

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Abstract

Researchers have shown that nonlinear pricing always yields greater profit than uniform pricing. However, most of these studies on nonlinear pricing have been done under monopoly setting. Unlike the previous studies, we examine the effect of nonlinear pricing in a competitive environment. We provide a game-theoretical explanation on the choice of a pricing structure between uniform and two-part pricing for the competing firms. We find that the degree of competition between the two firms (or the degree of differentiation between the two products) plays an important role in deciding the optimal pricing structure due to a strategic interaction of the two firms. Specifically, we find that both firms can choose uniform pricing as an equilibrium in a highly competitive environment. © 2002 Elsevier Science Inc. All rights reserved.

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1. Introduction

Most of the previous theoretical studies on nonlinear pricing tend to assume the monopoly situations. In this paper, we pay our attention to the nonmonopolistic situation and find that
the strategic interaction between firms matters in their pricing decisions in a differentiated oligopoly market setting. In particular, we compare the effects of two pricing structures (i.e., uniform and two-part pricing policy) on the firm’s profit with respect to the various levels of product differentiation.

We confirm the findings of previous studies that the firm would prefer two-part pricing policy to uniform pricing policy in order to extract more profits from consumers in the form of a fixed fee if there is no strategic interaction. However, we also find that the uniform pricing policy can also be a Nash equilibrium strategy if the strategic interaction exists. Specifically, our result shows that one firm chooses the uniform pricing if the other firm has the uniform pricing policy when the two products become very little differentiated (i.e., highly substitutable).

This paper is organized in six parts. In Section 2, we briefly review the related studies. In Section 3, we lay out the model assumptions and then present a two-stage game model to derive the equilibrium solutions. Results are summarized in Section 4, and discussions of the results are presented in Section 5. Section 6 has the concluding remarks.

2. Literature review

A nonlinear (nonuniform) price schedule is the price schedule in which the average price paid is dependent on the quantity purchased. Common examples of such price schedules are two-part pricing, two-block pricing, and all-units quantity discount. Thus, it is not surprising that nonlinear price schedule has been studied by many researchers (Buchanan, 1953; Dolan, 1987).

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1 For a general summary on nonlinear pricing structure, one can refer to Dolan’s (1987) extensive review of literatures in marketing and economics.

2 The total charge for an amount $q$, $R(q)$, is as follows:

Two-part pricing:

$$R(q) = \begin{cases} F + pq & \text{if } q > 0, \\ 0 & \text{if } q = 0, \end{cases}$$

where $F > 0$.

Two-block pricing:

$$R(q) = \begin{cases} p_h q & \text{if } 0 \leq q \leq z, \\ p_h z + p_l (q - z) & \text{if } q > z, \end{cases}$$

where $p_h > p_l$.

All-units quantity discount:

$$R(q) = \begin{cases} p_h q & \text{if } 0 \leq q < z, \\ p_l q, & \text{if } q \geq z, \end{cases}$$

where $p_h > p_l$. 

Faulhaber & Panzar, 1977; Gabor, 1955; Leland & Meyer, 1976; Littlechild, 1975; Murphy, 1977; Oi, 1971; Schmalensee, 1981). However, interestingly, we find that most of the research on the nonlinear pricing limits their analyses to monopoly markets.

The researchers at marketing and economics approach the pricing under competition with the game-theoretic framework (Choi, 1991; Dixit & Stiglitz, 1977; Ingene & Parry, 1995; McGuire & Staelin, 1983; Moorthy, 1988; Spence, 1976). We note that the scope of most of these game-theoretic studies is limited to the uniform price schedule. For example, Mitchell and Vogelsang (1991, p. 112) point out that the rareness of studies on nonlinear pricing under competitive market situations is surprising, considering the widespread popularity of nonlinear pricing or quantity discounts in real-life oligopolies.

In fact, only a few studies theoretically explain the nonuniform or nonlinear pricing decisions in the oligopoly setting. Oren, Smith, and Wilson (1983) investigate a smooth nonlinear price schedule in a symmetric Cournot oligopoly and show that quantity discounts are also optimal in competition among a finite number of identical firms if they are optimal in monopoly. Another research by Calem and Spulber (1984) examines the two-part pricing in a differentiated oligopoly and shows no corresponding tendency for competition to reduce fixed fees although competition tends to lower unit prices. However, their results suffer from limited implications. First, they do not consider the exclusion effect of fixed fee, a major influence of the two-part price schedule, since every consumer is assumed to buy both products (nonexclusion). Second, they do not consider the degree of product interdependence, i.e., product differentiation. Moreover, their result cannot explain which pricing structure (two-part vs. uniform) is better for a given degree of product differentiation since both firms are assumed to adopt two-part pricing. Finally, they do not explicitly consider the relationships between unit price and fixed fee. Thus, the equilibrium fixed fee is not uniquely determined within the context of their model.

In this paper, by overcoming such limitations in the nonlinear pricing literatures, we show that the two-part pricing cannot always be the better solution over the uniform pricing in the oligopolistic environment.

3. The model

3.1. Assumptions

We start with an assumption of a profit-maximizing firm as most researchers do. We also assume that the firm strategically decides its optimal price (including fixed fee for the case of two-part pricing policy). In addition, for simplicity, we assume that the marginal costs of both firms are constant and normalized to be zero. Besides these fundamental assumptions, we need the following four additional conditions, (a)–(b), to be assumed in our model.

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3 Smooth nonlinear price schedule is an extreme case of nonlinear price schedule in which marginal price changes continuously with the quantity purchased.
(a) Two firms producing substitutable products compete without collusion

If the two firms collude, they would maximize their aggregate profit instead of maximizing the profit of each firm. In this case, the firms’ behaviors and the market equilibrium should be different from those of individual profit maximizers. This collusive case is not in our interest. Moreover, in many cases, collusion is prohibited by the law.

(b) No resale of merchandise is allowed between consumers

Under two-part pricing policy, it is possible for smart consumers who have already paid fixed fee to buy up a lot of the products and to resell them to other consumers without imposing fixed fees. With (b), we assume no perfect arbitrage (resale without transaction costs) as in most studies on nonlinear pricing or price discrimination.

(c) There are heterogeneous consumers and their demand curves are not crossing

We assume that consumers are heterogeneous in the market with different tastes for the products we are concerned with. Unlike the uniform price schedule that only needs the information about the whole market demand curves, the information about individual demand curves is necessary to analyze the two-part price schedule since some consumers are driven out by the fixed fee. However, it is impossible for a firm to know the individual demand curves of all customers. The conventional approach to this problem is assuming that the individual demand curves do not cross, and consumers can be characterized by a single dimensional type parameter (Mitchell & Vogelsang, 1991, p. 73). Following this approach, we represent the consumer heterogeneity by the type parameter $\theta$ in our model.

(d) Consumers have a quasilinear quadratic utility function

Under a two-part price schedule, consumers buy the product only if the total utility from purchase is greater than the fixed fee. The assumption of quasilinear utility function enables us to exactly measure this total utility by the consumer surplus (Shy, 1995, p. 53).4

With these assumptions, we start our model with a quadratic utility function of Eq. (1) suggested by Dixit (1979) and Singh and Vives (1984).

\[ U(q_0, q_i, q_j) = q_0 + \alpha_i q_i + \alpha_j q_j - \frac{1}{2} (\beta_i q_i^2 + 2\gamma q_i q_j + \beta_j q_j^2). \]  

(1)

where $q_0$ denotes the amount of a numeraire good consumed, $q_i$ the amount of product $i$ consumed for $i = 1, 2$, $\gamma$ the degree of product substitutability and $\alpha_i, \beta_i (i = 1, 2)$ are positive constants. We further assume that the two products have symmetric effects to the utility of

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4 This assumption also implies that there is no income effect, an assumption frequently made in nonlinear pricing studies and applied welfare economics, since the utility level is quasilinear with respect to income $m$ (or the numeraire good). Income effects of nonlinear price schedules are treated by Goldman, Leland, and Sibley (1984). They point out that, in the presence of income effects, optimal pricing takes on the role of optimal redistributive taxation.
consumer (i.e., $\alpha_i = \alpha_j = \theta$, $\beta_i = \beta_j = \beta$). Then, we have the following linear inverse demand function (Eq. (2)) by equating $\frac{\partial U}{\partial q_i}$ to $p_i$, a price of firm $i$’s product, for $i = 1, 2$

$$p_i = \theta - \beta q_i - \gamma q_j \quad i j = 1, 2, \quad i \neq j.$$  \hspace{1cm} (2)

where $\gamma < \beta$ is required to insure that the price of a brand is more sensitive to a change in the quantity of its own than to that of the competing brand. Normalizing the own quantity effect of $\beta$ to be 1, the inverse demand function becomes

$$p_i = \theta - q_i - \gamma q_j \quad i j = 1, 2, \quad i \neq j.$$  \hspace{1cm} (3)

where $0 \leq \gamma < 1$.

The product substitutability increases as $\gamma$ approaches unity, where the products are maximally substitutable or virtually homogeneous. Note that in Eq. (3), $\theta$ denotes the marginal willingness to pay of the consumer of type $\theta$ for the first unit of the product.

By inverting the two equations in Eq. (3), we have the direct demand of the individual consumer of type $\theta$ as

$$q_i = D_i(\theta) = \frac{\theta}{1 + \gamma} - \frac{1}{1 - \gamma^2} p_i + \frac{\gamma}{1 - \gamma^2} p_j \quad i j = 1, 2, \quad i \neq j.$$  \hspace{1cm} (4)

Eq. (4) is a linear demand function with a downward slope and is denoted as $D_i(\theta)$ hereafter. Note that this particular form of demand allows us to analyze the situations where the two substitutable products are both purchased. For example, small offices own a copier despite the fact that a fax machine is sufficient for copying. Another example can be your two phones: one connected to the wall by line and the other a cellular one.

In addition, we assume that $\theta$ is uniformly distributed over the interval $[0, 1]$: $g(\theta) \sim [0, 1]$.\footnote{We later relax this distributional assumption and discuss the effect of this assumption.} Also, the own-price demand effect $1/(1 - \gamma^2)$ is assumed to be the same for all $\theta$ and so is the cross-price demand effect $\gamma/(1 - \gamma^2)$. Finally, we assume a nondiscriminatory price schedule, since a truly discriminatory two-part price schedule is difficult to implement due to the lack of information and possible illegality.\footnote{For example, Robinson–Patman Act.} In other words, the firm imposes the same price schedule to every consumer whether it is two-part or uniform.

### 3.2. Marginal consumer of firm $i$

A critical role in the analysis of nonlinear pricing is played by the marginal consumer who is indifferent between buying and not buying at a given price schedule. With the assumption
of noncrossing demand curves, the marginal consumer can be assigned a unique $\theta$. In our model, this marginal consumer is characterized by $\hat{\theta}_i$. Specifically, $\hat{\theta}_i^{TP}$ represents the marginal consumer for firm $i$ under two-part pricing, and $\hat{\theta}_i^{U}$ denotes the marginal consumer for firm $i$ under uniform pricing.

Calem and Spulber (1984) suggest a way to solve $\hat{\theta}_i^{TP}$ in oligopoly. With the constraints of $\alpha_i = \alpha_i = \theta$ and $\beta_i = \beta_j = 1$ as discussed above, the direct utility function of Eq. (1) becomes (Eq. (5))

$$U(q_0, q_i, q_j; \theta) = q_0 + \theta q_i + \theta q_j - \frac{1}{2} (q_i^2 + 2\gamma q_i q_j + q_j^2).$$

Then, define the consumer’s net surplus (exclusive of the fixed fees) for the purchase of one or both products as follows:

$$S(p_i, p_j; \theta) \equiv \max_{q_i, q_j} [U(q_0, q_i, q_j; \theta) - p_i q_i - p_j q_j]$$

(6)

$$S_i(p_i; \theta) \equiv \max_{q_i} [U(q_0, q_i, q_j; \theta) - p_i q_i]$$

(7)

$$S_j(p_j; \theta) \equiv \max_{q_j} [U(q_0, q_i, q_j; \theta) - p_j q_j].$$

(8)

Let $q_i(p_i, p_j; \theta)$ ($i, j = 1, 2, i \neq j$) solve Eq. (6), $\tilde{q}_i(p_i; \theta)$ solve Eq. (7), and $\tilde{q}_j(p_j; \theta)$ solve Eq. (8). Then, we have the following solutions.

$$\tilde{q}_i(p_i; \theta) = 0 - p_i \text{ and } S_i(p_i; \theta) = \frac{1}{2} (\theta - p_i)^2 \text{ for } i = 1, 2.$$  

$$q_i(p_i, p_j; \theta) = \frac{\theta}{1 + \gamma} - \frac{1}{1 - \gamma^2} p_i + \frac{\gamma}{1 - \gamma^2} p_j \text{ and}$$

$$S(p_i, p_j; \theta) = \frac{1}{2(1 - \gamma^2)} [(\theta - p_i)^2 - 2\gamma (\theta - p_i)(\theta - p_j) + (\theta - p_j)^2] \text{ for }$$

$$i = 1, 2, i \neq j.$$  

The gains from trade from purchasing product $i$ in addition to product $j$ will be

$$S(p_i, p_j; \theta) - S_j(p_j; \theta) = \frac{1}{2(1 - \gamma^2)} [(\theta - p_i) - \gamma (\theta - p_j)]^2 = \frac{(1 - \gamma^2)^2}{2} q_i(p_i, p_j; \theta)^2.$$  

(9)
Intuitively, Eq. (9) measures the monopolistic power of firm \(i\) that comes from the product’s differentiation. For example, in case of \(\gamma = 0\), firm \(i\) has a great monopoly power and \(S(p_i, p_j; \hat{\theta}_i) - S_j(p_j; \hat{\theta})\) will be equal to \(S_i(p_i; \hat{\theta})\) since the consumers consider firm \(i\)’s product to be really different from firm \(j\)’s product. However, if \(\gamma \approx 1\), the consumers can dispense with one of the two products and \(S(p_i, p_j; \hat{\theta}_i) - S_j(p_j; \hat{\theta})\) will be nearly equal to zero.

Calem and Spulber (1984) show that, in case of demand-substitutable products, the fixed fee \(f_i\) will be equal to the \(S(p_i, p_j; \hat{\theta}_i) / C_0 S_j(p_j; \hat{\theta}_j)\) of the marginal consumer of type \(\hat{\theta}_i\). Thus, we can obtain \(\hat{\theta}_i\) by solving the following equation:

\[
S(p_i, p_j; \hat{\theta}_i) - S_j(p_j; \hat{\theta}_i) = \frac{(1 - \gamma^2)}{2} q_i(p_i, p_j; \hat{\theta}_i^U) = f_i. \tag{10}
\]

Likewise, we can obtain \(\hat{\theta}_i\) by solving the following equation since the marginal consumer under uniform pricing can be defined by setting \(f_i\) to be zero:

\[
S(p_i, p_j; \hat{\theta}_i^U) - S_j(p_j; \hat{\theta}_i^U) = \frac{(1 - \gamma^2)}{2} q_i(p_i, p_j; \hat{\theta}_i^U) = 0 \text{ or } q_i(p_i, p_j; \hat{\theta}_i^U) = 0. \tag{11}
\]

3.3. Profit and total sales quantity of firm \(i\)

3.3.1. Under uniform pricing

From Eqs. (4) and (11), we get the marginal consumer \(\hat{\theta}_i\) for firm \(i\) under uniform pricing as

\[
\hat{\theta}_i^U = \frac{1}{1 - \gamma} (p_i - \gamma p_j). \tag{12}
\]

The profit function for firm \(i\) under uniform pricing is obtained as

\[
\Pi_i^{U} = \int_{\theta_i^U}^{1} p_iD_i(\theta')g(\theta')d\theta' = \frac{1}{2} (1 + \gamma) p_i [D_i(\theta = 1)]^2. \tag{13}
\]

The total sales quantity of firm \(i\) under uniform pricing is obtained as the sum of the demands of all consumer types from \(\hat{\theta}_i\) to \(\theta = 1\).

\[
Q_i^{U} = \int_{\theta_i^U}^{1} D_i(\theta')g(\theta')d\theta' = \frac{1}{2} (1 + \gamma) [D_i(\theta = 1)]^2. \tag{14}
\]
3.3.2. Under two-part pricing

From Eqs. (4) and (10), we get the marginal consumer's offer curve \( \hat{q}_{TP} \) for firm \( i \) under two-part pricing as

\[
\hat{q}^i_{TP} = \frac{1}{1 - \gamma} \left( \sqrt{2f_i(1 - \gamma^2)} + p_i - \gamma p_j \right).^7
\]

The profit function for firm \( i \) under two-part pricing is obtained as

\[
\Pi^i_{TP} = \int_{\hat{q}^i_{TP}}^{1} [p_iD_i(\theta') + f_i]g(\theta')d\theta'
\]

\[
= \frac{1}{2} (1 + \gamma) \left[ p_i[D_i(\theta = 1)]^2 + 2f_i \left( D_i(\theta = 1) - \frac{1}{1 - \gamma^2} p_i \right) - \sqrt{\frac{8}{1 - \gamma^2} f_i^3} \right]. (16)
\]

The total sales quantity of firm \( i \) is obtained by summing the demands of all consumer types from \( \hat{q}^i_{TP} \) to \( \theta = 1 \) as shown in Eq. (17),

\[
Q^i_{TP} = \int_{\hat{q}^i_{TP}}^{1} D_i(\theta')g(\theta')d\theta' = \frac{1}{2} (1 + \gamma) \left[ D_i(\theta = 1)]^2 - \frac{2}{1 - \gamma^2} f_i \right]. (17)
\]

3.4. Equilibrium solutions

We derive the equilibrium solutions based on the two-stage game framework. In the first stage, each firm chooses its pricing structure. Then, in the second stage, each firm decides the level of the strategic variables such as unit price and fixed fee to maximize its own profit under the pricing structure chosen in the first stage.\(^8\)

In fact, we solve this two-stage game backwards. First, for the second stage, by differentiating the profit function of firm \( i \) with respect to the strategic variables such as unit price and fixed fee, we obtain the first-order conditions for profit maximization under each pricing policy. We next derive the Nash equilibrium profits for each of the three game structures: UU (both firms adopt uniform pricing policies), TT (both firms adopt two-part pricing policies) and UT or TU (the first adopts uniform and the other chooses two-part pricing policy, or vice versa). The details for the derivation of the equilibrium solutions are in Appendix A.

Then, back to the first stage, we determine the Nash equilibrium of pricing policy by comparing the equilibrium profits of each pricing policy at a given substitutability level of \( \gamma \),

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^7 The other root is \( \hat{q}^i_{TP} = \frac{1}{1 - \gamma} \left( -\sqrt{2f_i(1 - \gamma^2)} + p_i - \gamma p_j \right) \), but this is associated with negative demand and thus rejected.

^8 We thank the anonymous reviewer who suggested presenting the timing of the game formally. The two-stage game framework can be also found at Hamilton and Slutsky (1990).
which varies in the region of \([0,1)\). (We hereafter denote two-part pricing policy as T and uniform pricing policy as U.)

4. Results

The equilibrium results of the second stage are summarized in Table 1. Note that all the results in Table 1 are in a simple form with only one parameter, \(\gamma\), the degree of substitutability between the two firms’ products.

For easy comparison of the equilibrium profits in the first stage, we plot the four profit functions of \(\Pi_{\text{UU}}(\gamma)\), \(\Pi_{\text{UT}}(\gamma)\), \(\Pi_{\text{TU}}(\gamma)\), and \(\Pi_{\text{TT}}(\gamma)\) in Fig. 1. For a better representation of the relative magnitude of the equilibrium profits, we rescale the profits in Table 1 by dividing with the same value of \((1 - \gamma)/(1 + \gamma)\) and plotting them in Fig. 2.

\[
\begin{array}{cccccc}
\text{Game structure} & \text{Unit price} & \text{Fixed fee} & \text{Marginal consumer} & \text{Total sales quantity} & \text{Profit} \\
\hline
\text{UU} & \frac{1-\gamma}{3-\gamma} & - & \frac{1-\gamma}{3-\gamma} & \frac{2}{(3-\gamma)^2(1+\gamma)} & \frac{2(1-\gamma)}{(3-\gamma)^2(1+\gamma)} \\
\text{TT} & \frac{1-\gamma}{5-\gamma} & \frac{2(1-\gamma)}{(5-\gamma)^2(1+\gamma)} & \frac{3-\gamma}{5-\gamma} & \frac{6}{(5-\gamma)^2(1+\gamma)} & \frac{10(1-\gamma)}{(5-\gamma)^2(1+\gamma)} \\
\text{UT} & \frac{(5-\gamma)(1-\gamma)}{15-\gamma^2} & - & \frac{5-2\gamma-\gamma^2}{15-\gamma^2} & \frac{2(5+\gamma)^2}{(15-\gamma^2)^2(1+\gamma)} & \frac{2(5+\gamma)^2}{(15-\gamma^2)^2(1+\gamma)} \\
\text{TU}^a & \frac{(3+\gamma)(1-\gamma)}{15-\gamma^2} & \frac{2(3+\gamma)^2(1-\gamma)}{(15-\gamma^2)^2(1+\gamma)} & \frac{9-2\gamma-\gamma^2}{15-\gamma^2} & \frac{6(3+\gamma)^2}{(15-\gamma^2)^2(1+\gamma)} & \frac{10(3+\gamma)^2}{(15-\gamma^2)^2(1+\gamma)} \\
\end{array}
\]

\(^a\) Read the first firm two-part pricing policy, the second firm uniform pricing policy.

Fig. 1. Profits for different game structures.
From Fig. 1 (or Fig. 2), we find that UU dominates TT for $\gamma > 0.183$.\(^9\) Note that, for $\gamma < 0.741$, one can make even greater profits by using two-part pricing policy if its competitor adopts uniform pricing policy, i.e., $\Pi_{TU} > \Pi_{UU}$. This implies that a firm has an incentive to deviate from U to T if $\gamma < 0.741$.\(^10\) Thus, TT is the unique Nash equilibrium in this region. Since both firms are better off with UU when $\gamma > 0.183$, the firm faces the problem of classical prisoner’s dilemma in choosing an optimal pricing structure when $0.183 < \gamma < 0.741$.

In contrast, for $0.741 < \gamma < 0.957$, two Nash equilibria exist since no one wants to change his pricing policy first if the initial structure is either TT or UU.\(^11\) Note that UU is dominant (or Pareto superior) with higher profits than TT.

Note that, for $\gamma > 0.957$, there is one Nash equilibrium, i.e., UU. Since $\Pi_{UT} > \Pi_{TT}$, there exists an incentive for a firm to deviate from two-part to uniform pricing policy. We summarized our findings discussed above in Table 2.

5. Discussion

In this section, we discuss several implications of our results. First, we derive the following proposition about the effects of product differentiation.

\(^9\) From Table 1, the TT and UU structures are equally profitable when $2/(3 - \gamma)^3 = 10/(5 - \gamma)^3$. The critical value of $\gamma = 0.183$ is the root of this equation for $\gamma$ in the interval [0, 1).

\(^10\) From Table 1, a firm is indifferent between TU and UU given that its competitor adopts uniform pricing strategy when $2/(3 - \gamma)^3 = (10(3 + \gamma)^3)/(15 - \gamma)^3$. The critical value of $\gamma = 0.741$ is the root of this equation for $\gamma$ in the interval [0, 1).

\(^11\) From Table 1, the TT and UT structures are equally profitable when $10/(5 - \gamma)^3 = (2(5 + \gamma)^3)/(15 - \gamma^2)^3$. The critical value of $\gamma = 0.957$ is the root of this equation for $\gamma$ in the interval [0, 1).
Proposition 1: As the two products become less differentiated, i.e., more substitutable, 1) both unit price and fixed fee of each firm decrease; 2) the number of consumers of each firm increases; 3) the total sales quantity of each firm does not necessarily increase; and 4) the profit of each firm decreases.

Proposition 1 can be graphically explained by Figs. 1, 3, 4, and 5.

Fig. 3 shows the decreases in both unit prices and fixed fees of each firm regardless of the game structure. The decrease of unit price with the increase of competition between the two firms is appealing to our intuition. This result shows that our demand model does not suffer from the weakness of the conventional linear demand function, i.e., the increase of equilibrium outputs (price and profit) with the decrease of product differentiation (Choi, 1991). Unlike the previous studies, in our demand function of Eq. (4), \( \gamma \) affects not only the cross-price sensitivity but also the own-price sensitivity. The intercept term of Eq. (4) also depends on \( \gamma \). Thus, the market potential is now decreasing with \( \gamma \), whereas the own-price sensitivity is increasing with \( \gamma \).

Intuitively, the gains from purchasing product \( i \) in addition to product \( j \) will decrease if the two products become more substitutable. Thus, the fixed fee \( f_i \) in Eq. (10) should decrease as the two products become more substitutable. For example, in the extreme case of \( \gamma \approx 1 \), the two products are almost homogeneous and only one of the products is sufficient for the consumer to fulfill his/her needs. Therefore, the two firms will compete head to head to capture the consumers, and in the mean time, the fixed fee will fall near zero.

Decreasing the unit price and fixed fee with the increase of \( \gamma \) implies that more people can consume the products. Fig. 4 shows the decrease of the marginal consumer type \( \hat{\theta}_i \), which implies that the number of consumers of each firm increases.

However, the decrease in price (including fixed fee for the case of two-part pricing policy) does not necessarily result in increasing total sales quantity. As shown in Fig. 5, we find that the total sales quantity increases only after a certain degree of \( \gamma \) is

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12 The conventional linear demand has the following functional form. \( D_i(\theta) = \theta - p_i + \gamma p_j, i, j = 1, 2, i \neq j \), where \( p_i \) denotes the price charged by firm \( i \), and \( \gamma \) the degree of substitutability between the two products.
achieved. This is because the increase in $\gamma$ has two different types of effects on total sales quantity. Specifically, if $p_i = p_j \equiv p$, the total sales quantity for firm $i$ is

$$Q_i = \int_{\hat{q}_i}^{1} D_i(\theta) g(\theta) \, d\theta = \int_{\hat{q}_i}^{1} \left( \frac{\theta}{1 + \gamma} - \frac{1 - \gamma}{1 - \gamma^2} p \right) \, d\theta = \frac{1}{1 + \gamma} \int_{\hat{q}_i}^{1} (\theta - p) \, d\theta. \quad (18)$$

Thus, as $\gamma$ increases, the denominator of Eq. (18) works negatively on total sales quantity, $Q_i$. Meanwhile, the decreased price along with the smaller lower limit of the definite integral in Eq. (18) works positively. (The lower limit $\hat{q}_i$ means the marginal consumer and Fig. 4 shows the decrease of $\hat{q}_i$ with the increases of $\gamma$.) For UU, the positive second effect exceeds the negative first effect above a certain degree of $\gamma$. Thus, the total sales quantity has the U shape as shown in Fig. 5. However, for TT, the second effect never overwhelms the first effect since $\hat{q}_i$ is still too big ($\approx 0.5$) even when $\gamma \approx 1$ as

13 For UU game structure: $\frac{\partial Q_i}{\partial \gamma} = \frac{2(3 \gamma - 1)}{(3 - \gamma)^3 (1 + \gamma)^2} = 0$ at $\gamma = \frac{1}{3}$. For UT game structure: $\frac{\partial Q_i}{\partial \gamma} = \frac{2(-225 + 130 \gamma + 170 \gamma^2 + 42 \gamma^3 + 3 \gamma^4)}{(15 - \gamma^2)^3 (1 + \gamma)^2} = 0$ at $\gamma = 0.778$. For TU game structure: $\frac{\partial Q_i}{\partial \gamma} = \frac{6(-45 + 66 \gamma + 78 \gamma^2 + 26 \gamma^3 + 3 \gamma^4)}{(15 - \gamma^2)^3 (1 + \gamma)^2} = 0$ at $\gamma = 0.430$. However, for TT game structure: $\frac{\partial Q_i}{\partial \gamma} = -\frac{18(1 - \gamma)}{(5 - \gamma)^3 (1 + \gamma)^2} < 0$ for $0 \leq \gamma < 1$. 

Fig. 3. Prices for different game structures.
Therefore, the overall effect of increasing $\gamma$ on the total sales quantity is always negative in the case of TT and, consequently, the total sales quantity decreases monotonically.

Decreasing profits with respect to $\gamma$ shown in Fig. 1 implies that differentiation is an essential factor for better profits whatever pricing policy a firm chooses. This is consistent with our intuition, as well as with the previous studies’ results on product differentiation. For example, by modifying Hotelling’s (1929) model, d’Aspremont, Gabszewicz, and Thisse (1979) show that each firm should locate its product at each end of the market, i.e., a firm should maximally differentiate its product from its competitor’s product.

**Proposition 2:** As the two products become less differentiated, uniform pricing can be an equilibrium.

Proposition 2 is supported if $\gamma > 0.741$ as shown in Table 2. In particular, we find that uniform pricing becomes the only equilibrium if the two products are very little differentiated (i.e., $\gamma > 0.957$).

In case of monopoly ($\gamma = 0$), if a firm changes its pricing structure from uniform to two-part, the firm’s profit is affected by the following three effects:

1. **Fixed fee effect (+):** Extra cash comes from charging the fixed fee.
2. **Own price effect ($-\frac{C}{p_i}$):** The unit price is reduced with the introduction of the fixed fee. Thus, the revenue from the unit price ($p_i Q_i$) decreases despite the increase in the total sales quantity with reduced unit price.\(^{15}\)
3. **Exclusion effect ($-\frac{C}{p_i}$):** A number of light users are driven out due to the fixed fee.

---

\(^{14}\) Since the two-part price schedule has a fixed fee part, some consumers are excluded from consumption despite the falling of fixed fee and unit price with the increase of $\gamma$.

\(^{15}\) By setting $\gamma = 0$ in Table 1, we can see that $p_i^{\text{TP}} Q_i^{\text{TP}} = (1/5) \times (6/5^2) < p_i^{\text{U}} Q_i^{\text{U}} = (1/3) \times (2/3^2)$ whereas $p_i^{\text{TP}} = 1/5 < p_i^{\text{U}} = 1/3$ and $Q_i^{\text{TP}} = (6/5^2) > Q_i^{\text{U}} = (2/3^2)$. Thus, the firm has 88/3375 less profit from unit price under two-part pricing policy.
In general, (1) is known to be greater than the sum of (2) and (3) for a monopoly firm. In other words, two-part pricing yields greater profit than uniform pricing under monopoly (Leland & Meyer, 1976). Our result is consistent with the previous studies’ results that two-part pricing gives greater profit than uniform when $\gamma$ is set to be zero.

However, under oligopoly, two firms now have strategic interactions. Despite the decrease in unit price under two-part pricing, a firm’s demand does not increase enough since the pricing decisions of two demand-substitutable firms are strategically complementary and the competitor reduces its price.\(^{16}\) That is, due to the strategic interaction, the negative effect of own price becomes greater under oligopoly since $Q_i$ does not increase sufficiently, whereas $p_i$ decreases with the introduction of two-part pricing. Meanwhile, the positive fixed fee effect becomes smaller since $q_i(p_i, p_j)^{TP}$ in Eq. (10) does not increase enough due to the strategic interaction. At a high value of $\gamma$, our result indicates that these negative effects of strategic interaction become even greater. (More detailed analyses are in Appendix B.)

In sum, two-part pricing cannot be the equilibrium when the two products are highly substitutable (e.g., $\gamma > 0.957$). Furthermore, Table 3 indicates that the attractiveness of uniform pricing increases if there are many light users in the market since two-part pricing drives out more light users.\(^ {17} \)

6. Conclusion

In this paper, we suggest that the practices of two-part pricing cannot be an equilibrium if the competition between the two products becomes intense in the market. This can be

\(^{16}\) Strategic complementarity means $(\partial^2 \Pi_A/\partial p_A \partial p_B) > 0$, whereas demand complementarity means $(\partial D_N/\partial p_B) < 0$. For strategic substitutability and demand substitutability, the signs are just the opposite (Moorthy 1988).

\(^{17}\) We considered these three different kinds of distribution functions to represent the situation where light users, medium users, and heavy users are large proportions, respectively. For $g(\theta) = 60(1 - \theta)$ and $g(\theta) = 20$, we could not get the closed-form solutions, thus, we used Newton–Raphson algorithm to find out the solutions.
(indirectly) supported by some examples. In the early days, when there are only a few video rental stores, they used to have the membership fee and be charged for each videotape rental. Nowadays, you probably pay no membership fee since you have choices among several stores located in your neighborhood. The Korea Telecom, the almost monopolistic local telephone company, adds an option for no installation fee at the time of opening a new telephone number at home as the mobile phone market grows rapidly. In addition, as the access to Internet becomes easier at work, as well as at other places, many on-line home PC communication service providers in Korea (such as Chonlian, Unitel, and Hitel) begin to remove the access fee to the Internet. We interpret these phenomena as the firms’ strategic transition from two-part to uniform pricing policy in an environment of increasing competition (or decreasing product differentiation).

The findings in this paper are new to those who study nonlinear pricing in a monopolistic industry setting. We believe that our results are robust and consistent in a competitive market environment. Thus, our study provides insights for understanding firms’ pricing behaviors under competition and expands the research horizon by introducing competition in the nonlinear pricing studies. However, despite our interesting findings, we must caution our readers that our results are based on the assumptions that we made. One of the assumptions that we made in this paper is that a consumer has both products. However, in reality, one can choose either one of the two substitutable products. For example, some have both mobile and traditional phones, whereas others have either one of them. In the future research, we will provide more general model for explaining this situation. In addition, we need to empirically confirm our findings with the data in the future research. In addition, one needs to explore many other (or generalized) functional forms of demands. One can also investigate the nonlinear pricing policy for more than two competing firms. Moreover, one can study the effect of the other nonlinear pricing practices such as two-block pricing and multipart pricing.

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<th>g(θ)</th>
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<th>Nash equilibrium</th>
<th>Pareto superior</th>
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Appendix A. Deriving the equilibrium solutions

When firm $i$ adopts uniform pricing policy, its first order condition for profit maximization is given by Eq. (A.1).

$$\frac{\partial \Pi_i^U}{\partial p_i} = \frac{1}{2(1-\gamma)^2(1+\gamma)}(1 - \gamma - p_i + \gamma p_j)(1 - \gamma - 3p_i + \gamma p_j) = 0. \quad (A.1)$$

For firm $j$, it is given by Eq. (A.2).

$$\frac{\partial \Pi_j^U}{\partial p_j} = \frac{1}{2(1-\gamma)^2(1+\gamma)}(1 - \gamma - p_j + \gamma p_i)(1 - \gamma - 3p_j + \gamma p_i) = 0. \quad (A.2)$$

When firm $i$ adopts two-part pricing policy, its first-order conditions are given by Eqs. (A.3) and (A.4).

$$\frac{\partial \Pi_i^{TP}}{\partial p_i} = \frac{1}{2(1-\gamma)^2(1+\gamma)}[(1 - \gamma - p_i + \gamma p_j)(1 - \gamma - 3p_i + \gamma p_j) - 4f_i(1 - \gamma^2)] = 0 \quad (A.3)$$

$$\frac{\partial \Pi_i^{TP}}{\partial f_i} = \frac{1}{2(1-\gamma)}[2(1 - \gamma - 2p_i + \gamma p_i) - 3\sqrt{2f_i(1 - \gamma^2)}] = 0. \quad (A.4)$$

For firm $j$, they are given by Eqs. (A.5) and (A.6).

$$\frac{\partial \Pi_j^{TP}}{\partial p_j} = \frac{1}{2(1-\gamma)^2(1+\gamma)}[(1 - \gamma - p_j + \gamma p_i)(1 - \gamma - 3p_j + \gamma p_i) - 4f_j(1 - \gamma^2)] = 0 \quad (A.5)$$

$$\frac{\partial \Pi_j^{TP}}{\partial f_j} = \frac{1}{2(1-\gamma)} \left[2(1 - \gamma - 2p_j + \gamma p_j) - 3\sqrt{2f_j(1 - \gamma^2)} \right] = 0. \quad (A.6)$$

The Nash equilibrium for UU game structure can be obtained by solving Eqs. (A.1) and (A.2) simultaneously. For TT, it can be obtained by solving Eqs. (A.3), (A.4), (A.5), and (A.6) simultaneously.\(^{18}\) We show our method of analysis for the asymmetric game structures

\(^{18}\) We solve this without the constraint of $p_i = p_j$ or $f_i = f_j$. That is, symmetry is not imposed on our equilibrium solutions a priori. However, since the two products are assumed to have symmetric effects to the utility of consumer, the equilibrium outcomes become symmetric a posteriori for the symmetric game structures of TT or UU.
of TU or UT structure in this appendix. It is analogous to the other two game structures of TT and UU. For the convenience of analysis, assume that firm $i$ takes uniform pricing policy and firm $j$ takes two-part pricing policy. Then, we have the Nash equilibrium by solving Eqs. (A.1), (A.5), and (A.6) simultaneously.

By solving Eq. (A.6) for $f_j$, we have

$$f_j = \frac{2}{9(1-\gamma^2)}(1 - \gamma - 2p_j + \gamma p_i)^2.$$  \hfill (A.7)

Substituting $f_j$ into Eq. (A.5) and rearranging, we get

$$\frac{1}{18(1-\gamma)^2(1+\gamma)}(1 - \gamma - 5p_j + \gamma p_i)(1 - \gamma + p_j + \gamma p_i) = 0.$$  \hfill (A.8)

Since $1 - \gamma + p_j + \gamma p_i > 0$, Eq. (A.8) is reduced to

$$1 - \gamma - 5p_j + \gamma p_i = 0.$$  \hfill (A.9)

Since $1 - \gamma - p_i + \gamma p_j \neq 0$ in Eq. (A.1), we get

$$1 - \gamma - 3p_i + \gamma p_j = 0.$$  \hfill (A.10)

Solving Eqs. (A.9) and (A.10) simultaneously, we get

$$p_i = \frac{(5 + \gamma)(1 - \gamma)}{15 - \gamma^2},$$  \hfill (A.11)

$$p_j = \frac{(3 + \gamma)(1 - \gamma)}{15 - \gamma^2}.$$  \hfill (A.12)

Substituting Eqs. (A.11) and (A.12) into Eq. (A.7), we get

$$f_j = \frac{2(3 + \gamma)^2(1 - \gamma)}{(15 - \gamma^2)^2(1 + \gamma)}.$$  \hfill (A.13)

Substituting equilibrium prices of Eqs. (A.11), (A.12), and (A.13) into Eqs. (13) and (16), we obtain $\Pi_U = (2(5 + \gamma)^3(1 - \gamma))/((15 - \gamma^2)^3(1 + \gamma))$, $\Pi_{TP} = (10(3 + \gamma)^3(1 - \gamma))/((15 - \gamma^2)^3(1 + \gamma))$.

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19 $1 - \gamma - p_i + \gamma p_j$ when divided by $1 - \gamma^2$ represents the demand of the largest consumer type for firm $i$ and therefore cannot be zero.
Similarly, from Eqs. (14) and (17), total sales quantity is given by \( Q_i^{U} = \frac{(2(5 + \gamma)^2)/(15 - \gamma^2)^2(1 + \gamma)}{((15/C_0 g)^2(1 + g))}, \)
\( Q_i^{TP} = \frac{(6(3 + \gamma)^2)/(15 - \gamma^2)^2(1 + \gamma)}{((15/C_0 g)^2(1 + g))} \).

Marginal consumer can be obtained from Eqs. (12) and (15) as \( \hat{q}^{U}_i = \frac{(5/C_0 g)^2}{(15/C_0 g)^2}, \)
\( \hat{q}^{TP}_i = \frac{(9/C_0 g)^2}{(15/C_0 g)^2} \).

Appendix B. Analysis of the strategic interaction

When firm \( i \) deviates unilaterally from uniform pricing (UU) to two-part (TU) and changes its unit price from \( p_{iU} \) to \( p_{iT} \), firm \( i \) might expect that firm \( j \) would maintain its unit price at \( p_{jU} \). Thus, firm \( i \) expects that its demand for the consumer of type \( \theta \) will become \( D_i^E(p_i^{TU}, p_j^{UU}; \theta) \). However, with the strategic interaction, firm \( j \) also changes its unit price from \( p_{jU} \) to \( p_{jT} \), where \( p_{jU} \geq p_{jT} \) from Fig. 3. Thus, firm \( i \)'s realized demand for the consumer of type \( \theta \) is \( D_i^R(p_i^{TU}, p_j^{UT}; \theta) \), which is smaller than the expected demand \( D_i^E(p_i^{TU}, p_j^{UU}; \theta) \). The strength of strategic interaction, i.e., the difference of the two demands, is as follows (Eq. (B.1));

\[
D_i^E(p_i^{TU}, p_j^{UU}; \theta) - D_i^R(p_i^{TU}, p_j^{UT}; \theta) = \frac{\gamma}{1-\gamma^2}(p_{jU} - p_{jT}^2). \tag{B.1}
\]

Thus, the degree of strategic interaction effect increases as \( \gamma \) increases since \((\partial(D_i^E(\theta) - D_i^R(\theta)))/(\partial(\gamma)) = (4\gamma(45 + 15\gamma + \gamma^3 - \gamma^4))/(3 - \gamma^2)(1 + \gamma)^2(15 - \gamma^2)^2 > 0\), implying that the negative effects of strategic interaction become greater with the increase of \( \gamma \).

References


