EMD interval thresholding denoising based on similarity measure to select relevant modes

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\textbf{Article info}

\textbf{Abstract}

This paper introduces a novel EMD interval thresholding (EMD-IT) denoising, where relevant modes are selected using a $l_2$-norm measure between the probability density function (pdf) of the input and that of each mode, thresholds are estimated by the characteristics of fractional Gaussian noise (fGn) through EMD. To solve the problem of more relevant modes included when the signal is corrupted by fGn with the $H$ increase, a modified $l_2$-norm method was given. The computational complexity of EMD-IT denoising is also analyzed. And the time complexity of it is equal to that of EMD. Numerical simulation and real data test were carried out to evaluate the effectiveness of the proposed method. Other traditional denoisings, such as correlation-based EMD partial reconstruction (EMD-PR), EMD direct thresholding (EMD-DT) and NeighCoeff-db4 wavelet denoising are investigated to provide a comparison with the proposed one. Simulation and test results show its superior performance over other traditional denoisings in whole.

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\section{Introduction}

Empirical mode decomposition (EMD), first introduced by Huang et al. in [1], has been widely used to analyze the non-stationary and non-linear signal processes by adaptively decomposing any signal into oscillatory components called intrinsic mode functions (IMFs), where wavelet thresholding has been the dominant techniques for many years. The fundamental reasoning of wavelet thresholding is that all coefficients lower than a threshold are set to zero, according to the fact that the energy of a signal and noise spread among wavelet coefficients in wavelet domain. A main drawback of this approach is that the basis functions are predefined, leading to mismatch varying nature of signals [5]. In contrast to wavelet thresholding, EMD expresses the signal as an expansion of basis functions that are derived directly from the signal itself [6,7]. The decomposition is based on the sequential extraction of energy associated with various intrinsic time scales of the signal starting from finer temporal scales to coarser ones. As a powerful adaptive decomposition tool, EMD is well suited to estimate the noise or frequency in measurement domains, apart from the specific applications such as biomedical, watermarking, and audio processing.

Recently, the statistical characteristics of white Gaussian noise and fractional Gaussian noise (fGn) through EMD have been revealed in [8–12]. According to these characteristics, each mode can be classified based on its energy density spread function or power spectral density (PSD) criteria. Consequently, many EMD-based denoisings are provided to remove noises from observed data. In [13,14], Boudraa et al. proposed a signal denoising scheme with each pre-filtered IMFs to estimate the signal. However, this study is limited to signals corrupted by white Gaussian random noise. Boudraa et al. have later proved that EMD filtering based on partial reconstruction of relevant modes performs in an adaptive
EMD can adaptively break down any signal \( x(t) \) into a number \( L \) of IMFs, termed \( h^{(i)}(t) \) (1 \( \leq i \leq L \)). Those basic IMFs are obtained through a sifting process according to the following steps [1,5,24] shown in Fig. 1.

The extracted modes are nearly orthogonal to each other, which form a complete set because accumulating all modes with the residual can restore the decomposed signal. The signal can be expressed as follows:

\[
x(t) = \sum_{i=1}^{L} h^{(i)}(t) + r_L(t)
\]  

Fig. 2(a) depicts as an example the EMD of a Bumps signal with length 2048. It is contaminated by white Gaussian noise, where the SNR is fixed to 5 dB. EMD results in nine IMFs and the last residual shown in Fig. 2(b) and (c).

### 3. Criterion of selecting relevant modes

Consider a noiseless signal \( y(t) \) contaminated by an additive noise \( n(t) \)

\[
x(t) = y(t) + n(t)
\]  

The denoising is to find an estimate \( \hat{x}(t) \) of the observed signal \( y(t) \). For EMD-based denoising, one of the important steps is to discriminate between relevant and irrelevant modes. EMD denoising based on partial reconstruction, called EMD-PR, is given by

\[
\hat{x}(t) = \sum_{i=k_m}^{L} h^{(i)}(t) + r_L(t)
\]  

The \( k_m \) can be determined by an estimation of correlation coefficient between the original data and decomposition modes. The estimated \( \hat{x}(t) \) can be rewritten as

\[
\hat{x}_m(t) = x(t) - \sum_{i=1}^{m} h^{(i)}(t)
\]

---

**Fig. 1.** Pictorial representation of empirical mode decomposition.
where \( m = k_{ih} - 1 \). Calculate the correlation coefficient between \( x(t) \) and \( \hat{x}_m(t) \) as follows:

\[
\rho(m) = \sum_{t=1}^{N} x(t)\hat{x}_m(t)/\left( \sqrt{\sum_{t=1}^{N} x^2(t)} \sqrt{\sum_{t=1}^{N} \hat{x}_m^2(t)} \right)
\]

in which \( N \) denotes the length of data. The \( k_{ih} \) is determined when the \( \rho(m) \) starts smaller than some constant \( C \). Usually, \( C \) belongs to \([0.75, 0.85]\). In this paper, the \( C \) is set to 0.80. The \( k_{ih} \) is given by Eq. (6)

\[ k_{ih} = \arg \text{last} \{\rho(m) \geq 0.8\} + 1 \tag{6} \]

where “last” stands for the last value in series \( \rho(m) \) bigger than 0.8. However, this method is sensitive to some noisy signals with different SNR. To analyze this question, we carried out 50 trials on the above noisy signal to get the statistical value of \( k_{ih} \) using Eq. (6). Fig. 3 shows the \( k_{ih} \) increases with the SNR increase.

A more robust filtering scheme is presented in [22,23] on the basis of similarity measure between the pdf of the input and that of each mode, because the pdf denoting the distribution shape of data can reflect the difference in two signals. To identify the relevant modes of signal, the signal is first decomposed into several IMFs followed by an estimation of their pdfs using the kernel density estimator. Generally, similarity measure can be classified into two main categories, the information-theoretic measures and the metrics-based measures. And the best results are given by geometric similarity measure, especially \( l_2 \)-norm. It is usually used to calculate the distance between two points in a plane or in a space. For two pdfs, named \( P \) and \( Q \), it can be seen as two sets of points. The \( l_2 \)-norm is defined by

\[
\|P - Q\|_2 = \left( \int_{-\infty}^{+\infty} (P(z) - Q(z))^2 dz \right)^{1/2}
\]

From Eq. (7), we can see the \( l_2 \)-norm is sensitive to the variation of the distance of two corresponding points. The similarity measure, \( L \), between the pdf of \( x(t) \) and that of \( h^{(i)}(t) \) is defined as follows:

\[ L(i) = \text{dist} \{\text{pdf}(x(t)), \text{pdf}(h^{(i)}(t))\} \tag{8} \]

The “dist” stands for the distance between two pdfs measured by \( l_2 \)-norm. The first selected mode is the one when the distance starts decreasing after the first local maximum. The \( k_{ih} \) is identified by

\[ k_{ih} = \arg \max_{1 \leq i \leq L} \{L(i)\} + 1 \tag{9} \]

Take the noisy signal shown in Fig. 2 as an example. Fig. 4(c) shows the curve of correlation coefficient with respect to IMFs and gives the specific location of \( k_{ih} \). The relevant modes only include IMFs 6–9. Similarly, the curve of pdf measured by \( l_2 \)-norm is shown in Fig. 4(d). The number 4 is the first local maximum. Thus, the IMFs 5–9 are identified as relevant modes. Fig. 4(a) and (b) gives the corresponding value calculated by these two methods.

It is seen that the number of relevant modes determined by correlation coefficient is different from that by pdf. In some cases, two numbers are close. However, when the SNR is low, or the noise is \( \text{Gn} \), the two ways exhibit their own disadvantages. For example, the SNR of the Bumps signals with white Gaussian noise is fixed to 2 dB,
the relevant modes include IMFs 3–9 using correlation coefficient shown in Fig. 3. In contrast, the \( l_2 \)-norm is not sensitive to the SNR, but to the form of noise in choosing the relevant modes. We carried out 50 trials on the above noisy signal to get the statistical value of the first local maximum in \( l_2 \)-norm with \( H \) varying from 0.2 to 0.9, where the SNR is fixed to \(-2 \) dB and \( 5 \) dB respectively. The results are shown in Fig. 5.

From Fig. 5, we can see that the first local maximum shifts towards the smaller number with the \( H \) increase. That is, more relevant modes are included when the noise is positive long-range dependence (\( 0.5 < H < 1 \)). To resolve
where $h(t)$ is the threshold of the $i$th IMF.

A good choice of the $k_{th}$ is $L-2$. In other words, the last two IMFs do not get thresholded.

4. EMD-based denoising

4.1. IMF thresholding-based denoising

EMD-based denoising reported in literature can be classified into two main categories, partial reconstruction based on relevant modes, whole reconstruction with filtered modes. In some cases, especially when the SNR is high, useful signal is also decomposed into low-order modes which are considered as discarded irrelevant modes. To enhance the performance of the conventional EMD-PR, researchers developed a series of novel EMD-based denoising methods inspired by wavelet thresholding in [13–14,18–20]. When a direct wavelet thresholding is applied into the EMD, it generates an EMD-DT. The denoised signal could be expressed as

$$\hat{x}(t) = \sum_{i=1}^{M_1} \hat{h}^{(i)}(t) + \sum_{i=M_2+1}^{L} h^{(i)}(t) + r(t)$$  \hspace{1cm} (11)

where

$$\hat{h}^{(i)}(t) =\begin{cases} h^{(i)}(t) & |h^{(i)}(t)| > T_i \\ 0 & |h^{(i)}(t)| \leq T_i \end{cases}$$  \hspace{1cm} (12)

for hard thresholding and

$$\hat{h}^{(i)}(t) =\begin{cases} \text{sgn}(h^{(i)}(t))(h^{(i)}(t) - T_i) & |h^{(i)}(t)| > T_i \\ 0 & |h^{(i)}(t)| \leq T_i \end{cases}$$  \hspace{1cm} (13)

for soft thresholding. Here, two parameters are introduced. $M_1$ and $M_2$ denote the low-order and high-order of IMFs, respectively. $T_i$ is the threshold of the $i$th IMF. A good choice of $M_1, M_2$ is also given in [19,20]. However, EMD-DT can result in disadvantageous consequence for the continuity of the denoised signal. When the signal between the two adjacent zero-crossings within IMFs is defined as a mode cell, and treated the mode cell as the basic analyzable object, a novel EMD-IT is developed based on the mode cell filter which can reduce the discontinuity induced by EMD-DT. Considering two adjacent zero-crossings $z_{j}^{(i)} = [z_{j}^{(i)}, z_{j+1}^{(i)}]$ in the $i$th IMF, the denoising translates to

$$\hat{h}^{(i)}(z_{j}^{(i)}) =\begin{cases} h^{(i)}(z_{j}^{(i)}) & |h^{(i)}(z_{j}^{(i)})| > T_i \\ 0 & |h^{(i)}(z_{j}^{(i)})| \leq T_i \end{cases}$$  \hspace{1cm} (14)

for hard thresholding and

$$\hat{h}^{(i)}(z_{j}^{(i)}) =\begin{cases} \text{sgn}(h^{(i)}(z_{j}^{(i)}))(h^{(i)}(z_{j}^{(i)}) - T_i) & |h^{(i)}(z_{j}^{(i)})| > T_i \\ 0 & |h^{(i)}(z_{j}^{(i)})| \leq T_i \end{cases}$$  \hspace{1cm} (15)

for soft thresholding. Where $h^{(i)}(z_{j}^{(i)})$ is the single extremum of the corresponding zero-crossing interval, $\hat{h}^{(i)}(z_{j}^{(i)})$ represents all samples from $z_{j}^{(i)}$ to $z_{j+1}^{(i)}$.

To show the difference between the direct and the interval EMD thresholding, the fifth IMF of the signal shown in Fig. 2 is used as an example in Fig. 6. To further illustrate the effectiveness of EMD-IT in reducing the discontinuity of denoised signal, EMD-DT and EMD-IT based on hard

![Fig. 5. The variation of $k_{th}$ with the $H$ change. (a) The variation of $k_{th}$ with the SNR = -2 dB, (b) the variation of $k_{th}$ with the SNR = 5 dB.](image-url)

![Fig. 6. Results of direct thresholding and interval thresholding. (a) Direct thresholding, (b) interval thresholding.](image-url)
thresholding are respectively applied on the ECG signal corrupted by white Gaussian noise ($H = 0.5$, SNR = 5 dB) shown in Fig. 7.

Fig. 8 shows the results when EMD-based denoisings above-mentioned are imposed on the noisy signal shown in Fig. 2. As a comparison, NeighCoeff-db4 in [3] is provided because of the well-developed characteristics of fGn through wavelet. The red line is the noisy signal, and the black one is the denoised signal. Clearly, Fig. 8(d) contains more discontinuities in the denoised signal. The top-left numbers are the SNR values after denoising. And the highest SNR is given in Fig. 8(e) by EMD-IT.

4.2. Principle of selecting threshold

With respect to the threshold selection, the universal threshold $T = \sigma \sqrt{2 \ln N}$ is a popular candidate [25]. In this paper, the standard deviation of the noise is estimated using a robust estimator on the first IMF.

$$\hat{\sigma}_i = \frac{\text{median}(|h^{(1)}(t)|; t = 1, \cdots N)}{0.6745}$$

For the white Gaussian noise, the characteristic of decomposition through EMD is that the power spectra of the other IMFs, apart from the first noise-only IMF, exhibit self-similar characteristics. The energy of IMFs, $E_i$, decreases linearly in a semilog diagram according to

$$E_i = \frac{E_1}{\beta \rho^{i-1}} \quad i = 2, 3, \cdots L$$

where $\beta$ and $\rho$ are parameters that have been estimated by a large number of independent noise realizations and their IMFs. Flandrin et al. [26] have proposed the values 0.719 and 2.01 for these two parameters, respectively. According to the relationship between $E$ and $\sigma$, the $T_i$ can translate to

$$T_i = \hat{\sigma}_i \sqrt{2 \ln N} = \sqrt{E_i} \times 2 \ln N \quad i = 1, 2, 3, \cdots L$$

For the fGn, its statistical characteristics are determined merely by its second-order structure, which depends on a parameter $H$, known as Hurst parameter ($0 < H < 1$). When the fGn is decomposed by EMD, the variance
relation among all IMFs can be derived from
\[ V'(i) = \rho_H^{(2H-2)i-1} V(i) \quad i > i \geq 2 \] (19)
where
\[ \rho_H \approx 2.01 + 0.2(H-0.5) + 0.12(H-0.5)^2 \] (20)
Actually, \( \rho_H \approx 2 \). Robust estimator is also used to estimate the standard deviation \( \sigma(1) \) and \( \sigma(2) \) from the first two IMFs. So the Eq. (19) can be rewritten as
\[ V(i) = \rho_H^{(2H-2)i-2} V(2) = \rho_H^{(2H-2i-2)} \delta(2)^2 i > 2 \] (21)
Thus, the universal threshold \( T_i \) is
\[ T_i = \begin{cases} \delta_i \sqrt{2 \ln N} & \text{for } i = 1, 2 \\ \sqrt{V(i) \times 2 \ln N} & \text{for } i = 3, 4 \ldots L \end{cases} \] (22)
To further illustrate the difference of selection threshold based on white Gaussian noise or fGn, we yield independent noises with different parameter \( H \), and obtain the threshold by Eq. (18) and Eq. (22), respectively. The decrease curves of thresholds are shown in Fig. 9. We observe that the threshold calculated by Eq. (22) decays slower than that by Eq. (18) when the \( H \) is close to 1. So, if ignoring the form of noise, each threshold is selected on

<table>
<thead>
<tr>
<th>( H )</th>
<th>Methods</th>
<th>( k_n ) (SNR = -2 dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>correlation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>pdf</td>
<td>6</td>
</tr>
<tr>
<td>0.5</td>
<td>correlation</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>pdf</td>
<td>5</td>
</tr>
<tr>
<td>0.8</td>
<td>correlation</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>unmodified pdf</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>modified pdf</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 10. Results of EMD-based denoising. (a) EMD-PR(cor), (b) NeighCoeff-db4, (c) EMD-PR(pdf), (d) EMD-DT(pdf), (e) EMD-IT(pdf).
Fig. 11. SNR after EMD-based denoising on the noisy Bumps. (a) Mean of SNR after EMD-based denoising, (b) variance of SNR after EMD-based denoising.

Fig. 12. The diagram of EMD-based denoising methods.
the assumption that the noise is white Gaussian noise, more signal energy will discard in the denoised signal when \(0 < H < 0.5\), more noise energy will remain in the denoised signal when \(0.5 < H < 1\).

In addition, we have found that the \(k_{th}\) using \(l_2\)-norm becomes smaller with the \(H\) increase in Fig. 5, which means more relevant modes are included as described in Eq. (11). On the contrary, we expect that the number of relevant modes is less with the \(H\) close to 1 shown in Fig. 9. Thus, the consideration about the modified \(l_2\)-norm method is necessary, and it also meets the characteristics of \(f\)-Gn decomposition through EMD, where the noise energy decays decreases in different degrees. That is, of \(f\)-Gn decomposition through EMD, where the noise energy decays decreases in different degrees. That is, the closer to 1 thefiltering methods decreases in different degrees. That is,

Eq. (6) for correlation coefficient and Eq. (10) for \(l_2\)-norm. Table 2 gives the formula about the selection of threshold.

<table>
<thead>
<tr>
<th>Function</th>
<th>(M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMD-PR(cor)</td>
<td>(\left(13 + n_m\right) + \frac{n_m}{\text{correlation}} + \frac{n_m}{\text{reconstruction}}) float</td>
</tr>
<tr>
<td>EMD-PR(pdf)</td>
<td>(\left(13 + n_m\right) + n_m + \frac{n_m}{\text{reconstruction}}) float</td>
</tr>
<tr>
<td>EMD-DT(pdf)</td>
<td>(\left(13 + n_m\right) + n_m + \frac{n_m}{\text{reconstruction}}) float</td>
</tr>
<tr>
<td>EMD-IT(pdf)</td>
<td>(\left(13 + n_m\right) + n_m + \frac{n_m}{\text{reconstruction}}) float</td>
</tr>
</tbody>
</table>

Concluded that EMD-IT(pdf) has the highest SNR among these methods. But the best variance of denoising is given by NeighCoeff_db4 as seen in Fig. 11(b). The variance of EMD-DT(pdf) is smaller than that of EMD-IT(pdf) because the thresholding is applied to each sample not the zero-crossing interval which includes a set of samples.

The framework of EMD-based denoising methods is shown in Fig. 12, where the selection of relevant modes is according to Eq. (6) for correlation coefficient and Eq. (10) for \(l_2\)-norm. Eq. (22) gives the formula about the selection of threshold.

### 4.3. Computational complexity of EMD-based denoising

The computational complexity of an algorithm itself should be concerned except for the performance of it. In this section, we analyze EMD-based algorithm's time and space complexity. The arithmetic operations involved include addition (ADD), multiplication (MUL), division (DIV) and comparison (COMP). The CPU time for ADD, MUL, DIV and COMP depends on the hardware used. Here, we assume all operations require the same number of flops to simplify the analysis, which means the cost time is same for all operations. In [27], Wang et al. analyzed and provided the time and space complexity of EMD. On the basis of this work, we provide the time and space complexity of EMD-based denoising.

An input with length of \(n\) through EMD is extracted \(n_m\) IMFs where \(n_m=\text{log}_2 n\). The space complexity for it is \(M=(13 + n_m)\cdot n\) float. When \(n_m=\text{log}_2 n\), \(M=(13 + \text{log}_2 n)\cdot n\) float. The total storage for EMD-based denoising is obtained in Table 2, where the \(\text{log}_2 n\) stands for the max value of \(n_m\) IMFs. For EMD-PR, EMD-DT and EMD-IT, the incremental

\[n\] | EMD-PR (cor) | EMD-PR (pdf) | EMD-DT (pdf) | EMD-IT (pdf) | NeighCoeff_db4 |
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2^2</td>
<td>0.4516</td>
<td>0.6162</td>
<td>0.1663</td>
<td>0.1659</td>
<td>0.0024</td>
</tr>
<tr>
<td>2^3</td>
<td>0.4483</td>
<td>0.1735</td>
<td>0.1761</td>
<td>0.1764</td>
<td>0.0025</td>
</tr>
<tr>
<td>2^4</td>
<td>0.2299</td>
<td>0.2609</td>
<td>0.2654</td>
<td>0.2663</td>
<td>0.0026</td>
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<td>2^5</td>
<td>0.4550</td>
<td>0.5004</td>
<td>0.5013</td>
<td>0.5038</td>
<td>0.0029</td>
</tr>
<tr>
<td>2^6</td>
<td>0.7257</td>
<td>0.7947</td>
<td>0.8135</td>
<td>0.8111</td>
<td>0.0037</td>
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<td>2^7</td>
<td>1.7466</td>
<td>1.8730</td>
<td>1.8954</td>
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<td>6.2277</td>
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</tr>
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<td>2^9</td>
<td>10.5107</td>
<td>10.7312</td>
<td>10.9427</td>
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<td>2^11</td>
<td>100.6059</td>
<td>101.0530</td>
<td>102.0845</td>
<td>102.3095</td>
<td>0.0641</td>
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<td>269.9729</td>
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<td>272.2481</td>
<td>0.1360</td>
</tr>
</tbody>
</table>

### Table 3

The time complexity of EMD-based denoising.

<table>
<thead>
<tr>
<th>Function</th>
<th>(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMD-PR(cor)</td>
<td>(41\text{NS}_{n_m} + \frac{\left(3\text{MUL}\right) n_m + \text{DIV} n_m + \left(1\text{ADD}\right) n_m}{\text{correlation}} + \frac{\left(1\text{ADD}\right) n_m}{\text{reconstruction}})</td>
</tr>
<tr>
<td>EMD-PR(pdf)</td>
<td>(41\text{NS}_{n_m} + n_m + 100 n_m + \frac{\left(1\text{ADD}\right) n_m + \left(1\text{MUL}\right) n_m + \left(1\text{ADD}\right) n_m}{\text{reconstruction}})</td>
</tr>
<tr>
<td>EMD-DT(pdf)</td>
<td>(41\text{NS}_{n_m} + n_m + 100 n_m + \frac{\left(1\text{ADD}\right) n_m + \left(1\text{MUL}\right) n_m + \left(1\text{ADD}\right) n_m}{\text{reconstruction}})</td>
</tr>
<tr>
<td>EMD-IT(pdf)</td>
<td>(41\text{NS}_{n_m} + n_m + 100 n_m + \frac{\left(1\text{ADD}\right) n_m + \left(1\text{MUL}\right) n_m + \left(1\text{ADD}\right) n_m}{\text{reconstruction}})</td>
</tr>
</tbody>
</table>

Table 4

The execution time of EMD-based denoising for Bumps signal (s).
Fig. 13. Three noisy signals ($H=0.5$, SNR=8 dB).

Fig. 14. Results of EMD-based denoising for Blocks signal. (a) EMD-PR(cor), (b) NeighCoeff-db4, (c) EMD-PR(pdf), (d) EMD-DT(pdf), (e) EMD-IT(pdf).

Fig. 15. Comparison of EMD-DT and EMD-IT for Blocks signal.
Fig. 16. Results of EMD-based denoising for three signals. (a) Results of EMD-based denoising for Blocks signal. (b) Results of EMD-based denoising for Heavysine signal. (c) Results of EMD-based denoising for Doppler signal.
space complexity includes \( n \) float for storage the denoised signal and several float for storage computing intermediate values, such as \( n_m \) for correlation and \( 100 n_m \) for pdf, where the density estimate is evaluated at 100 points.

The time complexity of EMD for extracting \( n_m \) IMFs is \( T = 41 N S n m n m \), where \( N S \) stands for the sifting iteration and \( N S = O(10)^{+1} \). Substitute log_2 \( n \) for \( n_m \), \( T \leq 41 N S n m n m = O(n \log_2 n) \), the time complexity of EMD is \( O(n \log_2 n) \). In addition, the time complexity for calculating each pdf is \( O(n) \). So, the time complexity about EMD-based denoising is listed in Table 3.

In Table 3, for EMD-PR(cor, pdf), the reconstruction signal may be \( n_m \) IMFs, \( 0 \leq n_m \leq n_m \). For EMD-DT(pdf) and EMD-IT(pdf), the number of IMFs thresholded may be \( n_m \leq n_m \) and \( n_m = n_m \). When we assume that \( n_m = \log_2 n \), \( n_m = n_m \) and \( n_m = n_m \), the time complexity of EMD-based denoising is \( O(n \log_2 n) \), which is equal to that of EMD.

We have performed a series of experiments to verify our analysis of the time complexity of EMD-based denoising. The test signal is Bumps with the length \( n \) ranging from \( 2^8 \) to \( 2^{18} \), where the \( H = 0.5 \) and \( SNR = 5 \) dB. The computer’s configuration is informed as follows: Intel(R) Core(TM)2 @2.80 GHz CPU and 2.00 GB RAM memory running windows XP. The EMD’s parameters are also informed as follows: \( N S = 100 \), \( n_m = 15 \) and the stopping threshold \( \epsilon = 0.05 \).

The execution time is shown in Table 4. As analyzed the smaller time is the discrete wavelet transform (DWT) technique. The time complexity of proposed method is \( O(n \log_2 n) \), which is equal to the EMD. How to reduce the complexity of these algorithms, especially for EMD, will be concerned in future work.

5. Results and discussions

5.1. Experiment 1

To verify the effectiveness of this novel denoising method, the noisy signals considered in this section include simulation and real test signals.

We consider a length of 2048, "Blocks", "Heavysine" and "Doppler" signal, which are generated by the function of whose noise in Matlab. We have adopted short and long range dependence noises with different SNR to assess the performance of the EMD-based filtering. As an example, these signal contaminated by white Gaussian noise is shown in Fig. 13, where the SNR = 8 dB. The code of EMD in [28] can serve as reference implementation.

Take the Blocks signal as an example. We carried out 50 trials for different \( H \) level to verify the effectiveness of this proposed method. The results are displayed in the form of histogram in Fig. 14. The figure indicates that the presented approach performs is better than the other methods, especially when the \( H = 0.2 \).

The comparison of EMD-DT and EMD-IT is seen in Fig. 15. An amplified figure pointed by an arrow clearly shows the discontinuity induced by EMD-DT.

The results of EMD-based denoising on the noisy signal: Blocks, Heavysine, Doppler with SNR varying from –8 to 10 dB are displayed in Fig. 16. Among these methods, the EMD-IT(pdf) gives the best results as expected under different SNR and \( H \). We also observe that EMD is well suited for filtering signals with jumps(Blocks) or oscillating signals(Heavysine and Doppler). And the performance of each denoising decreases with the \( H \) close to 1. We define the \( SNR_{out} \) is the SNR of output signal and \( SNR_{in} \) is the SNR.

---

**Table 5**

<table>
<thead>
<tr>
<th>Methods</th>
<th>( H = 0.8, SNR = -2 ) dB</th>
<th>( H = 0.8, SNR = 8 ) dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR</td>
<td>RMSE</td>
</tr>
<tr>
<td>Blocks signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMD PR(cor)</td>
<td>2.71</td>
<td>1.45</td>
</tr>
<tr>
<td>EMD PR(pdf)</td>
<td>2.71</td>
<td>1.45</td>
</tr>
<tr>
<td>EMD DT(pdf)</td>
<td>3.56</td>
<td>1.31</td>
</tr>
<tr>
<td>EMD IT(pdf)</td>
<td>4.21</td>
<td>1.02</td>
</tr>
<tr>
<td>NeighCoeff-db4</td>
<td>-0.61</td>
<td>2.21</td>
</tr>
<tr>
<td>Heavysine signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMD PR(cor)</td>
<td>1.02</td>
<td>2.64</td>
</tr>
<tr>
<td>EMD PR(pdf)</td>
<td>5.29</td>
<td>1.62</td>
</tr>
<tr>
<td>EMD DT(pdf)</td>
<td>5.39</td>
<td>1.59</td>
</tr>
<tr>
<td>EMD IT(pdf)</td>
<td>5.33</td>
<td>1.60</td>
</tr>
<tr>
<td>NeighCoeff-db4</td>
<td>-0.60</td>
<td>3.18</td>
</tr>
<tr>
<td>Doppler signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMD PR(cor)</td>
<td>0.81</td>
<td>0.26</td>
</tr>
<tr>
<td>EMD PR(pdf)</td>
<td>5.41</td>
<td>0.16</td>
</tr>
<tr>
<td>EMD DT(pdf)</td>
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</tr>
<tr>
<td>NeighCoeff-db4</td>
<td>-0.64</td>
<td>0.31</td>
</tr>
</tbody>
</table>

---

Fig. 17. FOG original signal and the selection of \( k_{th} \) (a) FOG original signal, (b) the selection of \( k_{th} \) using correlation and pdf.
of input signal. Take the Fig. 15(a) as an example, SNR \text{out} is up to 9 dB by EMD-IT(pdf) when \( H = 0.2 \) and SNR \text{in} = −8 dB. But it is down to 5 dB with \( H = 0.5 \) and SNR \text{in} = −8 dB. Finally, it is only 0.8 dB when \( H = 0.8 \), SNR \text{in} = −8 dB.

The root mean square error (RMSE) and SNR are employed to compare the performance of denoising methods. The RMSE is defined as follows:

\[
RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (x(t) - \hat{x}(t))^2}
\]  

(26)

In Table 5, we only record the experiment results when \( H = 0.8 \), SNR = −2 dB and \( H = 0.8 \), SNR = 8 dB. These results show the proposed method provides the lowest RMSE and the highest SNR. The performance of EMD-IT(pdf) is verified through this relatively large \( H \).

5.2. Experiment 2

As a kind of inertial measurement instrument based on optical Sagnac effect, Fiber optic gyro (FOG) has been widely used in Inertial Navigation System (INS) due to its advantages such as small size, low cost, light weight, no moving parts, long lifespan, large dynamic range, short startup time, etc. Due to the characteristics of FOG work, structural features and their working environment, the performance of FOGs is vulnerable to interference of various factors such as temperature, vibration, and pressure. Especially, the bias drift is an important factor in affecting the FOG working precision, which is weak non-linear, non-stationary, slowly time-varying, sensitive to environmental fluctuations. Because of the bias drift existence, the performance of FOG greatly restricts the navigation precision of INS. How to effectively reduce this kind of drift and noise is of crucial importance for improving the INS precision [29–31]. This paper tries to filter the random drift of FOG using the proposed method.

Under the normal temperature (25 °C), we make experiment testing for FOG drift signal. The sample period is 100 ms, and the total number of samples \( N = 145,570 \). To avoid the variation of temperature in startup time, this paper selects the samples from 4097 to 36,864 to analyze the filtering methods. The FOG signal shown in Fig. 16(a) is broken down into 14 modes through EMD, where the \( H \) of \( \text{Fbn} \) is 0.78 estimated by the wfbmestefunct in Matlab. The \( k_{th} \) is marked by an arrow around the points from which starts the partial reconstruction shown in Fig. 17(b).

The \( k_{th} \) is 3 calculated by the correlation coefficient, which is equal to the value by the unmodified pdf method. So, the performance of filtering based on EMD-PR(cor) and EMD-PR(pdf) is the same, as shown in Fig. 18. The modified EMD-IT(pdf) with \( k_{th} = 12 \) is not very smooth like the EMD-DT(pdf), because the \( \tilde{h}^\circ(t) \) \((i = 1,2,...,12)\) contains filtered signal which is usually discontinuity.

Fig. 19 shows an enlarged figure to see the distinction between EMD-DT(pdf) and EMD-IT(pdf), where the signal
discontinuity induced by EMD-DT(pdf) can be effectively reduced by EMD-IT(pdf).

The Allan variance analysis of the noisy signal and denoised signal of the FOG using EMD-based denoisings are plotted in Fig. 20. Compare the Allan variance curve before and after filtering, we can see the curves after five filtering methods decline in some degree, which mean the random error after filtering decrease. And the best results are given by EMD-IT(pdf) method.

6. Conclusion

In this paper, a novel modified EMD-IT method combined with similarity measure between the pdf of original signal and that of IMFs is proposed, where the $l_2$-norm is used to select relevant modes. But the first local maximum obtained from $l_2$-norm is sensitive to the form of noise. Thus, according to the value of $H$, the selection of relevant modes is modified. In addition, the threshold is estimated on the basis of the characteristics of fGn through EMD. The computational complexity of EMD-based denoisings presented in the paper has been analyzed. And the order of the time complexity of them is equal to that of EMD. To illustrate the effectiveness of this proposed method, we carried out numerical simulation with different SNR under each $H$ level. This improved method exhibits an enhanced performance of denoising compared with EMD-PR based on correlation coefficient, EMD-PR based on the pdfs, EMD-DT based on pdfs and NeighCoeff-db4. Finally, this method is applied to the random drift denoising for FOG. From the Allan variance curve, we can see the random error has been greatly decreased by the EMD-IT based on pdf.

Acknowledgments

The project is supported by the National Natural Science Foundation of China (61340044) and the Fundamental Research Funds for the Central Universities (YWF-10-01-B30). The authors acknowledge Beijing Aerospace Times Optical-Electronics Technology Co., Ltd. for providing the FOG data.

References


