Nonlinear Elastic Constitutive Model of Soil-Structure Interfaces Under Relatively High Normal Stress

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Abstract: The shear characteristics of soil-structure interfaces with different roughness are studied systematically by using the DRS-1 high normal stress and residual shear apparatus. The experimental results indicate that, under a relatively high normal stress, normal stress and the coefficient of structural roughness are the most important factors affecting the mechanical interface characteristics. The relationship between shear stress and shear displacement of the soil-structure interface is a hyperbolic curve with high regression accuracy. Based on our experimental results, a nonlinear elastic constitutive model of the soil-structure interface under relatively high normal stress is established with a definite physical meaning for its parameters. The model can predict the strain hardening behavior of the soil during the shearing process. The results show an encouraging agreement with experimental data from direct shear tests.

Key words: high normal stress; standard medium sand; interface; strain hardening; constitutive model

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1 Introduction

The problems of soil-structure interaction are still widespread in such areas as civil engineering, hydraulic engineering, traffic engineering and mining engineering. At the soil-structure interface, besides the stress transfer between the soil and structure, discontinuous deformation behavior (i.e., partial opening, sliding, etc) also exists because the material on different sides of the interface have various forms of deformation, strength, stiffness and other mechanical properties. Therefore, under certain conditions, the mechanical responses of the interface are different from those of the soil and the structure. For interaction analyses, more attention should be paid to stress-strain relations and strength characteristics of the soil-structure interface.

Many studies have been carried out on the mechanical characteristics of the soil-structure interface and various soil-structure constitution models have been presented. For example, a hyperbolic shear stress-deformation model by Clough and Duncan and an elastic-perfectly plastic model by Chen, have been presented[1–2]. Boulon, et al established an elastoplasticity interface model[3]. Desa was the first to introduce the basic theories of damage mechanics into interface constitutive relations, which started a new way for constitutive relation research[4]. Yin proposed a rigid-plastic interface model[5]. Based on the sliding zone concept, Lu, et al established a nonlinear constitutive model, where the coupling of normal stiffness and shear stiffness on the interface were considered[6]. An, et al presented a three-dimensional elastic-viscoplastic model[7]. Based on the damage concept, a damage model of the interface between coarse, sandy soil and structure and an elastoplastic damage constitutive model of the interface between coarse grained soil and structure were established by Hu, et al respectively[8–9].

All of the afore mentioned studies were largely built...
up and developed under relatively low stress, but more and more investigations show that deep soils have special mechanical properties.

After systematic research on the shear strength characteristics of Fujian standard medium sand under high normal stress, Zhou, et al discovered that the mechanical characteristics of Fujian standard medium sand under high stress are different from those under low stress [10–11]. Cui, et al established the famous theory of additional stress in shaft linings by a thorough investigation of the interaction between deep soil bodies and shaft linings [12–17].

Based on the mechanical characteristics obtained in experiments of soil-structure interfaces with different roughness and under relatively high normal stress, a nonlinear elastic model of the soil-structure interface is presented.

2 Analysis of the Test Method and Results

To study the mechanical properties of interfaces with different roughness under high normal stress, a series of experiments were carried out with standard medium sand and steel plates on the DRS-1 high normal stress and residual shear test apparatus designed by the China University of Mining and Technology. Because concrete blocks have a poor repeatability, rough steel plates were used as an ideal interface in our experiments. In order to validate the rationality and practicability of substituting rough steel plates for concrete blocks, a series of direct shear tests of the interface between standard medium sand and rough concrete blocks were made under different normal stresses.

From our experiments, it is shown that, under each level of normal stress, the results obtained from the tests using concrete blocks are in good agreement with those from the test using steel plates with corresponding roughness. So we can use the test results obtained from rough steel plates instead of concrete blocks for analysis. This will not result in a large error.

Due to defects, irregularity, uncertainty and other factors resulting in various characteristics of structural surfaces, it is very difficult to describe precisely the characteristics of surfaces. Different levels of roughness of structural surfaces can be simulated by rotating the steel plate to make different angles with the shear direction. For more information about the method, see Reference [18].

The tests on interface properties between standard medium sand and steel plates with different roughness were carried out under four normal stresses (σ = 8, 10, 12 and 15 MPa). According to our experimental results, the following two conclusions can be drawn: 1) the shear stress vs. shear displacement curves are almost show the same trend under different normal stresses; 2) a good regression line of the shear stress on shear displacement for the interface between a certain rough steel plate and standard medium sand was obtained. The experimental and regression results are illustrated in Fig. 1.

![Fig. 1](image)

As seen in Fig. 1, the relationship between the shear stress and shear displacement of the soil-structure interface under relatively high normal stress is hyperbolic. The relationship can be expressed as:
where $\tau$ and $\sigma$ are the shear stress and normal stress, respectively, $\Delta s$ is the shear displacement, $d$ is the diameter of the specimen and $a$ and $b$ are experimental constants.

The experimental constants and the correlation coefficients can be obtained by regressing the experimental results. This is shown in Table 1.

![Table 1](image)

**Table 1** Hyperbola fitting parameters of experimental results of soil-structure interfaces under relatively high normal stress

Fig. 1 and Table 1 show that Eq.1 agrees well with our experimental results: all correlation coefficients are greater than 0.99.

3 Nonlinear Elastic Constitutive Model of the Soil-Structure Interface Under Relatively High Stress

Substituting $\gamma = \frac{\Delta s}{d}$ into Eq.1, we can obtain,

$$
\gamma = \frac{\Delta s}{d}
$$

Differentiating Eq.1 with $\gamma$ and denoting $\gamma \rightarrow 0$ (i.e. $\Delta s \rightarrow 0$), we obtain

$$
G_i = \frac{d\tau^i}{d\gamma} \bigg|_{\gamma \rightarrow 0} = \frac{1}{a}
$$

Thus it can be seen that, $a$ is related to the initial slope of our regressed $\tau/\sigma$ vs. $\Delta s/d$ curve (i.e. the so-called regressed initial slope), and the value of $a$ manifests the magnitude of the initial shear modulus.

If $\gamma \rightarrow \infty$ (i.e. $\Delta s \rightarrow \infty$), in the limit we finally can obtain from Eq.2

$$
\tau_{\text{ult}} = \left(\frac{\tau}{\sigma}\right)_{\text{ult}} = \lim_{\gamma \rightarrow \infty} \left(\frac{\gamma}{a + b\gamma}\right) = \frac{1}{b}
$$

From Eq.4 we know that $b$ is related to the the asymptotic value of the $\tau/\sigma$ vs. $\Delta s/d$ curve (i.e. the regressed initial shear modulus). The value of $b$ is the magnitude of the initial shear modulus.

Differentiating Eq.2 with respect to $\gamma$ we obtain the tangential value of the $\tau/\sigma$ vs. $\Delta s/d$ curve (i.e. the regressed tangent modulus), i.e.,

$$
G_i = \frac{d\tau^i}{d\gamma} = \frac{a}{(a + b\gamma)^2}
$$

Fig. 1 shows that, based on the direct shear tests of the soil-structure interface, the relationship of $\tau/\sigma$ and $\Delta s/d$ is a hyperbola without a significant shear stress peak. However, given the limitation of the direct shear apparatus itself, the soil-structure interface area becomes smaller and smaller with an increase in shear displacement. If the shear displacement is too large, the shear stress we measured is not the real interface shear stress. Therefore, it is impossible to make the $\Delta s$ infinite to evaluate the magnitude of $\tau_{\text{ult}}$ and in most cases the magnitude of $\tau^i_\text{ult}$ is confirmed according to the shear displacement. In our study, the magnitude of $\tau/\sigma$ corresponding to $\Delta s/d = 15\%$ was considered the regressed failure shear stress $\tau^i_{\text{ult}}$ and it can be seen in Fig. 1 that $\tau^i_{\text{ult}} < \tau^i_{\text{ult}}$.

If we denote $R_i$ as the failure ratio

$$
R_i = \frac{\tau^i}{\tau_{\text{ult}}}
$$

then given Eq.3, we can obtain

$$
a = \frac{1}{G_i}
$$

From Eq.4, we can obtain

$$
b = \frac{1}{\tau_{\text{ult}}}
$$

Substituting Eqs.6 to 8 into Eq.5 leads to

$$
G_i = \frac{1}{G_i} \left(1 - \frac{R_i}{\tau^i_{\text{ult}}}\right)^2
$$

It can be found from Eq.2 that

$$
\gamma = \frac{a\tau}{1 - b\tau}
$$

Substituting Eqs. 6 to 8 and 10 into Eq.9, then Eq.11 is obtained as follows

$$
G_i = G_i \left(1 - R_i \frac{\tau}{\tau^i_{\text{ult}}}\right)
$$

where $\tau = \frac{\tau}{\sigma}$, $\tau^i_{\text{ult}} = \tau^i_{\text{ult}}$. Eq.11 can be rewritten as

$$
G_i = G_i \left(1 - R_i \frac{\tau}{\tau^i_{\text{ult}}}\right)
$$

so that we obtain
\[
\frac{\tau}{\sigma} = G_I \left( 1 - R_f \frac{\tau}{\tau_f} \right) \cdot \frac{\Delta s}{d} \quad (13)
\]

This is the expression of the shear stress vs. shear displacement curve, i.e. the nonlinear elastic constitutive equation of the soil-structure interface.

From reference [18] it is known that \( \tau_f \) can be expressed as follows:

\[
\tau_f = \left( a_0 + a_1 D + a_2 D^2 + a_3 D^3 \right) \cdot \sigma + C
\]

Substituting this into Eq.13 we can obtain

\[
\frac{\tau}{\sigma} = G_I \left( 1 - R_f \left( a_0 + a_1 D + a_2 D^2 + a_3 D^3 \right) \cdot \sigma + C \right) \cdot \frac{\Delta s}{d} \quad (14)
\]

It is clear from Eq.14 that, for the nonlinear elastic constitutive equation of the soil-structure interface under relatively high normal stress, the main unknown parameters are: the regressed initial shear modulus, \( G_I \); the failure ratio, \( R_f \); the soil-structure interface cohesion, \( C \); the fractal dimension of structure surface roughness, \( D \); the experimental constants, \( a_0, a_1, a_2 \) and \( a_3 \). All of these parameters have a definite physical meaning, which can easily be obtained from experiments.

According to the regressed \( \tau / \sigma \) vs. \( \Delta s / d \) curve, we can obtain the regressed failure shear stress \( \tau_f \), the regressed limit shear stress \( \tau_{\text{ult}} \) and the failure ratio \( R_f \). The details are presented in Table 2.

<table>
<thead>
<tr>
<th>Roughness of Structure surface</th>
<th>( \tau_f )</th>
<th>( \tau_{\text{ult}} )</th>
<th>( R_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D=1.012 )</td>
<td>0.546225</td>
<td>0.589240</td>
<td>0.926998</td>
</tr>
<tr>
<td>( D=1.043 )</td>
<td>0.537697</td>
<td>0.604376</td>
<td>0.922765</td>
</tr>
<tr>
<td>( D=1.084 )</td>
<td>0.570345</td>
<td>0.622316</td>
<td>0.916487</td>
</tr>
<tr>
<td>( D=1.122 )</td>
<td>0.575139</td>
<td>0.634357</td>
<td>0.906649</td>
</tr>
<tr>
<td>( D=1.149 )</td>
<td>0.579413</td>
<td>0.634638</td>
<td>0.912981</td>
</tr>
<tr>
<td>( D=1.159 )</td>
<td>0.595626</td>
<td>0.661419</td>
<td>0.900527</td>
</tr>
</tbody>
</table>

Table 2 shows that the magnitude of \( R_f \) varies within the range of 0.900527–0.926998, which bears no relation to the roughness of the structural surface. So, in practical calculations, we assume that \( R_f = 0.9 \).

### 4 Nonlinear Elastic Model Validation

The experimental results with different normal stress and roughness are compared with the aforementioned nonlinear elastic model results, which are illustrated in Fig. 2.

![Fig. 2](image)

It is evident from Fig. 2 that the theoretical results coincide with the test results, which validate that our proposed nonlinear elastic model can predict the mechanical properties of soil-structure interfaces under relatively high normal stresses.

### 5 Conclusions

Under a relatively high normal stress, strain hardening characteristics occur on the soil-structure interface during the shearing process. The relationship between the regressed shear stress and shear displacement is a hyperbola.

Based on the experimental results of different rough steel plates and standard medium sand under a relatively high normal stress, a nonlinear elastic model of the soil-structure interface was established, which involved crucial factors (such as normal stress, structural roughness, etc). All of the model parameters have a definite physical meaning and can easily...
be obtained from tests. The afore mentioned nonlinear elastic model can present the strain hardening characteristics of soil-structure interfaces during shear processes.

References
