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A new interval optimization method considering tolerance design

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This study considers the design variable uncertainty in the actual manufacturing process for a product or structure and proposes a new interval optimization method based on tolerance design, which can provide not only an optimal design but also the allowable maximal manufacturing errors that the design can bear. The design variables’ manufacturing errors are depicted using the interval method, and an interval optimization model for the structure is constructed. A dimensionless design tolerance index is defined to describe the overall uncertainty of all design variables, and by combining the nominal objective function, a deterministic two-objective optimization model is built. The possibility degree of interval is used to represent the reliability of the constraints under uncertainty, through which the model is transformed to a deterministic optimization problem. Three numerical examples are investigated to verify the effectiveness of the present method.

Keywords: interval optimization; non-probabilistic uncertainty; tolerance design; possibility degree of interval

1. Introduction

In traditional uncertain optimization, the probability model is generally used to address parametric uncertainties. Therefore, a great number of experimental samples are necessary to obtain accurate probability distributions for the uncertain parameters. For many real-life engineering problems, owing to considerations of the measurement conditions and cost, there is usually insufficient sample information, leading to a bottleneck in traditional stochastic optimization. Interval optimization is a new type of uncertain optimization method that requires only the parameters’ lower and upper bounds, instead of their probability distributions. This method provides an effective analysis tool for uncertain optimization design in complex engineering problems that lack sample information. Currently, a series of achievements exist in the field of interval optimization. Tanaka, Okuda, and Asai (1973) proposed a linear interval programming method with uncertain coefficients in the objective function. Chanas and Kuchta (1996a, 1996b) transformed the linear interval optimization to a conventional optimization problem based on an order relation of interval. Tong (1994) presented a linear interval programming model for problems involving interval coefficients in both the objective function and the constraints. Sengupta, Pal,

For the above interval optimizations, all uncertain parameters are depicted using intervals, and it is necessary to specify a deterministic interval for each imprecise parameter before the optimization so that the optimized design can satisfy the design requirements under the given interval uncertainty. Therefore, for these methods, the parameters’ intervals must be provided beforehand based on the actual situation, and furthermore the interval variables and the design variables are generally not the same. For real-life engineering applications, another important type of interval optimization problem also exists, that is, the intervals of the uncertain variables cannot be identified beforehand during the design phase; furthermore, the uncertain variables and the design variables constitute the same parameters. For example, in actual practice, the design variables usually include (but are not limited to) the structure sizes and the material characteristics. These parameters will be associated with uncertainty due to errors generated from the manufacture and measurement. In addition, owing to the complexity and diversity of the manufacturing and measurement conditions, their uncertain intervals cannot be determined beforehand under many circumstances. In this case, the parameters’ errors at the design stage should be considered so that the optimal design will be achieved, and the product’s performance under a relatively large uncertainty interval will be ensured, since a larger allowable interval means better processability of the parameter and hence a reduced manufacturing cost. Few references can be found regarding studies of this type of interval optimization method. The present study proposes a new interval optimization method based on tolerance design, which can provide not only the optimal design variables but also their optimal design tolerances. In this method, the optimality of the design objective, the processability of the design variables and the reliability of the constraints are comprehensively considered.

The rest of the article is organized as follows. Section 2 describes the interval optimization modelling; Section 3 provides a solution method for the interval optimization model; Section 4 analyses three numerical examples; and finally, Section 5 gives the overall conclusions. It should be pointed out that besides the present method another essential approach in the field of uncertain optimization, evidence-based design optimization (EBDO) (Mourelatos and Zhou 2006; Salehghaffari 2013; Salehghaffari et al. 2013), also introduced the concept of interval. However, these two methods are quite different. First, in interval optimization only the lower and upper bounds are required for each uncertain parameter, whereas in EBDO an evidence variable is used to deal with each uncertain parameter; this generally consists of several intervals called focal elements, each of which is given a probability assignment. Thus, to a certain extent, an evidence variable is more like a discrete probability variable. Secondly, in interval optimization the bounds of the objective function and constraints caused by the interval variables are generally required when finding the optimal solution, whereas in EBDO it is usually necessary to calculate the probability bounds of the constraints. These two methods also have some similarities.
When each evidence variable contains only one focal element, EBDO will degenerate into an interval optimization. Thus, in this sense, interval optimization can be regarded as a special case of EBDO.

2. Interval optimization modelling

A general optimization design problem can be expressed as:

$$\min_{\mathbf{X}} f(\mathbf{X})$$

s.t.

$$g_j(\mathbf{X}) \leq b_j, \quad j = 1, 2, \ldots, l$$

$$\mathbf{X}_l \leq \mathbf{X} \leq \mathbf{X}_u$$

(1)

where $\mathbf{X}$ is an $n$-dimensional design vector, with $\mathbf{X}_u$ and $\mathbf{X}_l$ representing its upper and lower design ranges; $f$ and $g$ represent the objective function and the constraint function, respectively; $l$ is the number of constraints; and $b_j$ represents the maximum allowed value for the $j$th constraint.

By optimizing Equation (1), an optimal design $\mathbf{X}_d$ can be obtained. In actual structures, the design variables $\mathbf{X}$ are usually the structural sizes, material characteristics, loads, etc. There usually exists uncertainty in these variables, due to factors such as manufacture and measurement errors. In many cases, a minor deviation of $\mathbf{X}_d$ in the manufacturing process can lead to a large fluctuation in the objective function or constraints, which therefore causes a low performance or even failure of the structure. The uncertainty of $\mathbf{X}_d$ can be reduced significantly by improving the manufacturability to improve the structure performance, but this procedure may also significantly increase the manufacturing cost. To solve the above-mentioned problem, the interval analysis method (Moore 1979; Ben-Haim and Elishakoff 1990; Qiu and Wang 2005; Gao, Song, and Tin-Loi 2010; Luo et al. 2009; Zhou, Jiang, and Han 2006) will be introduced in this study to measure the uncertainty of the design variables and thereby construct a corresponding interval optimization model, through which the present method aims to guarantee the comprehensive performance of the product or structure in terms of the design objective, constraint reliability and the processability of the design variables.

When errors from manufacture and measurement are considered, the actual values of $\mathbf{X}$ would belong to an interval vector $\mathbf{X}^I$:

$$X^I_i = [X^L_i, X^R_i] = \{X_i | X^L_i \leq X_i \leq X^R_i, X_i \in R\}, \quad i = 1, 2, \ldots, n$$

(2)

where the superscripts $I$, $L$ and $R$ represent the interval, the interval’s lower bound and the interval’s upper bound, respectively. When $X^L_i = X^R_i$, the interval $X^I_i$ degrades to a real number $X_i$. $\mathbf{X}^I$ can be represented in the format below:

$$X^I_i = [X^c_i, X^w_i] = \{X_i | X^c_i - X^w_i \leq X_i \leq X^c_i + X^w_i\}, \quad i = 1, 2, \ldots, n$$

(3)

where the superscripts $c$ and $w$ represent the interval’s midpoint and radius, respectively:

$$X^c_i = \frac{X^L_i + X^R_i}{2}, \quad X^w_i = \frac{X^R_i - X^L_i}{2}, \quad i = 1, 2, \ldots, n$$

(4)

For an actual product design, $X^c_i$ can be considered as the nominal design of $X_i$, whereas $X^w_i$ can be considered as the design tolerance of $X_i$. To match the engineering design convention,
\( X^I \) can also be expressed in the symmetrical tolerance format, as follows:

\[
X^I_i = X^c_i \pm X^w_i, \quad i = 1, 2, \ldots, n
\]

(5)

Using the interval to describe the uncertainty of the design variables, Equation (1) can be converted to the following interval optimization problem:

\[
\min_{X^I} f(X^I)
\]

s.t.

\[
g_j(X^I) \leq b_j, \quad j = 1, 2, \ldots, l
\]

\[
X_l \leq X^I \leq X_u
\]

\[
X^I_i = [X^L_i, X^R_i], \quad i = 1, 2, \ldots, n
\]

(6)

Compared with the deterministic optimization in Equation (1), the design variables for the interval optimization no longer form a real number vector \( X \) but an interval vector \( X^I \). By solving Equation (6), the optimal intervals for the design variables can be obtained to ensure the overall performance of the problem under an uncertain environment.

Because the interval of each design variable can be uniquely determined by its nominal design and tolerance, Equation (6) can also be expressed in an equivalent format as below:

\[
\min_{(X^c, X^w)} f((X^c, X^w))
\]

s.t.

\[
g_j((X^c, X^w)) \leq b_j, \quad j = 1, 2, \ldots, l
\]

\[
X_l \leq (X^c, X^w) \leq X_u
\]

(7)

For an interval optimization problem, the number of optimization variables becomes \( 2n \), not \( n \) as in the original optimization problem. The optimization variables in Equation (6) consist of \( n \) lower bounds and \( n \) upper bounds of \( X \), while those in Equation (7) consist of \( n \) nominal design variables and \( n \) tolerances of \( X \). It should be pointed out that in Equation (7) the present method assumes that all the design variables have a symmetrical tolerance. However, in practical applications some product features (holes, part thickness, etc.) may have different upper and lower tolerances, and in this case it is feasible to solve the interval optimization directly by means of Equation (6); that is, to find the optimal lower and upper bounds for each interval design variable. Therefore, just as in Equation (7), the number of optimization variables for asymmetrical tolerance problems is still \( 2n \).

3. Interval optimization solution

This section proposes an interval optimization method that converts the uncertain optimization to a regular deterministic optimization problem for solution. During the conversion process, the performance requirements for the objective function, the design variables’ manufacturability and the constraint reliability are all considered.
3.1. Creation of the deterministic objective function

In real-life engineering problems, under the premise of ensuring the performance of a product or structure, the smaller the design variable’s interval radius (i.e. the design tolerance), the higher the required manufacturing precision; in addition, the manufacturing cost would increase, and vice versa. Therefore, tolerance is a critical design indicator, as it reflects the manufacturability of the product or structure. To evaluate the level of the design variables’ tolerances, a design tolerance index \( W \) is defined in this article:

\[
W = \sqrt[n]{\prod_{i=1}^{n} \frac{X_i^w}{\psi_i}} \tag{8}
\]

where \( \psi_i \) is the normalization factor, which can be selected as \( \psi_i = |X_i^c| \). From Equation (8), \( W \) is a dimensionless parameter that reflects the size of the tolerances for all design variables, and the larger the \( W \), the larger the overall tolerance. Equation (8) will make sense only when the normalization factor \( \psi_i = |X_i^c| \) is not equal to zero. In actual structures or products, the design variables usually have positive values, and thus the above condition generally can be satisfied. The other forms of definition can also be used for the design tolerance index, provided that they can reflect the overall uncertainty of the interval variables. For example, for convenience of analysis, all the tolerances of the interval variables are given the same weights in Equation (8), while in a new definition of the design tolerance index they could be assigned different weights according to the importance of each design variable.

In addition, the nominal objective function \( f(X^c) \) can be used to depict the objective function’s average performance under the condition of interval uncertainty. By combining the above design tolerance index and the nominal objective function, a deterministic multi-objective optimization problem can be constructed as follows:

\[
\min_{X^c, X^w} [f(X^c), -W] = \begin{bmatrix} f(X^c), -\sqrt[n]{\prod_{i=1}^{n} \frac{X_i^w}{\psi_i}} \end{bmatrix} \tag{9}
\]

Through the first objective function, the average performance of the original objective function under uncertainty can be optimized; through the second objective function, the allowable tolerances of the design variables can be maximized and hence the manufacturing cost can be minimized.

3.2. Creation of the deterministic constraints

The possibility degree of interval can be used to quantitatively describe the degree of one interval being larger or better than another interval; in interval optimization, the possibility degree is usually used for constraint processing. The authors’ previous work (Jiang, Han, and Li 2012) proposed a reliability-based possibility degree of interval (RPDI), which could perform effective comparisons for various intervals under separated or overlapping states. For two intervals \( A^l \) and \( B^l \), the RPDI has the following form:

\[
p_r(A^l \leq B^l) = \frac{B^R - A^L}{2A^W + 2B^W} \tag{10}
\]

where \( p_r \) represents the possibility of interval \( A^l \) being smaller than interval \( B^l \). In addition, the \( p_r(A^l \leq B^l) \) has the following characteristics:

(1) \( -\infty \leq p_r(A^l \leq B^l) \leq +\infty. \)
(2) If \( A^R \leq B^L \), then \( p_r(A^I \leq B^I) \geq 1 \).
(3) If \( A^L \geq B^R \), then \( p_r(A^I \leq B^I) \leq 0 \).
(4) If \( p_r(A^I \leq B^I) = q \), then \( p_r(B^I \leq A^I) = 1 - q \), where \( q \in [-\infty, \infty] \).
(5) \( p_r(A^I \leq B^I) = \frac{1}{2} \), only when \( A^L + A^R = B^L + B^R \).
(6) For three intervals, \( A^I, B^I \) and \( C^I \), if \( p_r(A^I \leq B^I) \geq q \) and \( p_r(B^I \leq C^I) \geq q \), then \( p_r(A^I \leq C^I) \geq q \), where \( q \in [-\infty, \infty] \).

When \( B^I \) degrades to a real number \( B \), the RPDI still applies and has the following format:

\[
p_r(A^I \leq B) = \frac{B - A^L}{2A^w} \tag{11}
\]

In Equation (7), towards any \( X^I \), the value of the constraint \( g_j((X^c, X^w)) \) will belong to an interval \( g^I_j \):

\[
g^I_j = [g^L_j, g^R_j] = \left[ \min_{X_c \in (X^c, X^c)} g_j(X), \max_{X_c \in (X^c, X^c)} g_j(X) \right]. \quad j = 1, 2, \ldots, l \tag{12}
\]

Then, based on the RPDI model in Equation (11), the interval constraints in Equation (7) can be converted to the following deterministic constraints:

\[
p_r(g_j((X^c, X^w)) \leq b_j) = \frac{b_j - g^L_j}{2g^w_j} \geq \lambda_j, \quad j = 1, 2, \ldots, l \tag{13}
\]

where \( \lambda_j \) denotes the RPDI level for the \( j \)th interval constraint. A larger \( \lambda_j \) represents a stricter reliability requirement for the \( j \)th interval constraint, which therefore makes the feasible region for \( X^c \) and \( X^w \) in Equation (13) smaller. The RPDI level should be given beforehand based on the actual problem’s reliability requirement, and the constraints can be given different RPDI levels.

### 3.3. Creation of the deterministic optimization problem

After the above treatments, the interval optimization model in Equation (7) can eventually be converted into a regular deterministic optimization problem:

\[
\min_{X_c, X_w} \left[ f(X^c), -W \right] = \left[ f(X^c), -\sum_{i=1}^{n} X^w_i \prod_{i=1}^{n} \psi_i \right]
\]
s.t.

\[
p_r(g_j((X^c, X^w)) \leq b_j) = \frac{b_j - g^L_j}{2g^w_j} \geq \lambda_j, \quad j = 1, 2, \ldots, l
\]

\[
X_l \leq X^c - X^w \leq X^c + X^w \leq X_u
\tag{14}
\]

The above formula is a nesting optimization problem; the external layer is used for optimizing the design vectors \( X^c \) and \( X^w \), and the internal layer is used for the solution on the constraint interval \( [g^L_j, g^R_j], j = 1, 2, \ldots, l \). This study adopts the improved non-dominated sorting genetic algorithm (NSGA-II) (Deb et al. 2000) for external layer multi-objective optimization and adopts sequential quadratic programming (Nocedal and Wright 1999) to perform internal layer optimization. By solving the above optimization, an optimal nominal design vector \( X^c \) is given to the engineering staff, and the allowable maximum tolerances for the design variables can also be provided.
In addition, for many engineering problems, the constraint boundaries \( b_j, j = 1, 2, \ldots, n \) may also exhibit some level of uncertainty; therefore, the interval vector \( b_I^j = (b_c^j, b_w^j), j = 1, 2, \ldots, n \) can be used to depict the uncertainty. For example, it is required that the maximum stress of a structure does not exceed its yield strength, while the yield strength cannot be given an accurate value owing to the dispersion characteristic of the material, and only the interval to which it belongs can be known. In this case, the yield strength can thus also be treated as an interval. Correspondingly, the interval optimization Equation (7) would have the following form:

\[
\begin{align*}
\min_{\langle X^c, X^w \rangle} & \quad f(\langle X^c, X^w \rangle) \\
\text{s.t.} & \quad g_j(\langle X^c, X^w \rangle) \leq (b_c^j, b_w^j), \quad j = 1, 2, \ldots, l \\
& \quad X_l \leq \langle X^c, X^w \rangle \leq X_u 
\end{align*}
\]

It can also be converted to a deterministic optimization problem similar to Equation (14) using the above proposed method, except that here Equation (10) should be used to compute the constraint possibilities instead of Equation (11). For all the following numerical examples in this article, only the first case, whose constraint limits \( b_j, j = 1, 2, \ldots, n \) are deterministic values, has been considered.

4. Numerical analysis and discussion

4.1. Analytic function problem

Considering the following optimization problem with two design variables:

\[
\begin{align*}
\min_X & \quad f(X_1, X_2) = 2X_1 + 21X_2 - X_1X_2 + 100 \\
\text{s.t.} & \quad g_1(X_1, X_2) = 3(X_1 - 15)^2 + (X_2 - 20)^2 \leq 220 \\
& \quad g_2(X_1, X_2) = X_1X_2 + 12X_2 \leq 430 \\
& \quad 10 \leq X_1 \leq 25, 5 \leq X_2 \leq 15 
\end{align*}
\]

(16) After considering the manufacturing errors of the design variables \( X_1 \) and \( X_2 \), the above formula can be converted to an interval optimization problem, and by using Equation (14), it can eventually be converted to a deterministic multi-objective optimization:

\[
\begin{align*}
\min_{\langle X^c, X^w \rangle} & \quad \left[ 2X_1^c + 21X_2^c - X_1^cX_2^c + 100, -\sqrt{\frac{X_1^w}{|X_1^c|} \cdot \frac{X_2^w}{|X_2^c|}} \right] \\
\text{s.t.} & \quad p_r(3((X_1^c, X_1^w) - 15)^2 + ((X_2^c, X_2^w) - 20)^2 \leq 220) \geq \lambda_1 \\
& \quad p_r((X_1^c, X_1^w)(X_2^c, X_2^w) + 12(X_2^c, X_2^w) \leq 430) \geq \lambda_2 \\
& \quad 10 \leq X_1^c - X_1^w \leq X_1^c + X_1^w \leq 25, 5 \leq X_2^c - X_2^w \leq X_2^c + X_2^w \leq 15 
\end{align*}
\]

(17) where the optimization variables are \( X_1^c, X_1^w, X_2^w \) for the above problem.
In the interval optimization process, the same RPDI levels are given for two interval constraints. Concurrently, to analyse the impacts of the different constraint possibility levels on the optimization results, four different RPDI levels of 0.9, 1.0, 1.1 and 1.2 are given to perform the interval optimization. Figure 1 displays the Pareto optimal solutions under the four conditions. Tables 1–4 list the partial Pareto optimal solutions for the different RPDI levels. In each table, for the six groups of solutions the weight applied to the nominal objective function decreases in order, while the one applied to the design tolerance index increases. First, from the results in the tables, it is known that, unlike the regular deterministic optimization, the optimal solution of each design variable derived by the proposed interval optimization is no longer a deterministic value; on the contrary, it is an interval. Using this interval, not only can the optimized nominal design...
be provided, but also the allowable maximum tolerance for the design variable can be offered; therefore, the uncertainty of the design variables in the manufacturing process can be taken into account. Secondly, the present method indicates that under any one RPDI level, towards different weight values, the two objective functions at the optimal design variables exhibit a contradiction. As the weight value for the design tolerance index $W$ continues to grow, the optimal design tolerance index displays an ascending trend, leading to improved manufacturability of the design variables and reduced manufacturing cost, but the optimal nominal objective function gradually becomes worse. As shown in Table 2, when $\lambda_1 = \lambda_2 = 1.0$, the allowable design tolerances of $X_1$ and $X_2$ for the first group of optimal solution are $\pm 0.26$ and $\pm 0.43$, respectively; whereas for the sixth group, the allowable design tolerances are $\pm 1.53$ and $\pm 1.45$, respectively, which are approximately six and three times the former group, and hence the manufacturability significantly improves. However, the nominal objective function of the sixth group is 154, an increase of approximately 15% compared with 134 in the first group, leading to a worse average objective performance. For this reason, in real-life applications, it is indispensable to balance the relation between the above two indicators of the manufacturability and the nominal design objective based on actual situations and hence provide reasonable weights for them in the optimization process. If the manufacturing cost is the biggest concern for an actual problem, a larger weight should be given to the design tolerance index; by contrast, the nominal objective function should be given a larger weight in a case where the objective performance is of much more significance for the problem.

In additional, Figure 1 shows that the RPDI level $\lambda$ has an obvious impact on the interval optimization results. As $\lambda$ increases, the Pareto frontier gradually deviates from the origin of the objective function space; specifically, the optimal design tolerance index and nominal objective function simultaneously grow larger. This trend arises because $\lambda$ actually represents the reliability of the interval constraints. A larger $\lambda$ implies a smaller feasible region for the constraints in Equation (14), leading to a loss of both the design tolerance index and the nominal objective function. Therefore, the computational results indicate that in the present interval optimization model, not only are the optimality of the objective function and the manufacturability of the design variables considered, but the reliability of the constraints subjected to the manufacture and measurement errors is also taken into account.

### Table 3. Partial Pareto optimal solutions when $\lambda = 1.1$ (analytic function problem).

<table>
<thead>
<tr>
<th>No.</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$g_1$</th>
<th>$g^r_1$</th>
<th>$g_2$</th>
<th>$g^r_2$</th>
<th>$f^c$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.89 ± 0.25</td>
<td>12.27 ± 0.23</td>
<td>[189,216]</td>
<td>1.14</td>
<td>[405,427]</td>
<td>1.16</td>
<td>133</td>
<td>0.0146</td>
</tr>
<tr>
<td>2</td>
<td>21.39 ± 0.52</td>
<td>12.21 ± 0.32</td>
<td>[159,209]</td>
<td>1.22</td>
<td>[391,422]</td>
<td>1.14</td>
<td>138</td>
<td>0.0255</td>
</tr>
<tr>
<td>3</td>
<td>20.75 ± 0.67</td>
<td>11.60 ± 1.01</td>
<td>[132,212]</td>
<td>1.10</td>
<td>[340,421]</td>
<td>1.10</td>
<td>144</td>
<td>0.0529</td>
</tr>
<tr>
<td>4</td>
<td>20.31 ± 0.88</td>
<td>11.52 ± 1.11</td>
<td>[113,207]</td>
<td>1.14</td>
<td>[327,419]</td>
<td>1.12</td>
<td>149</td>
<td>0.0646</td>
</tr>
<tr>
<td>6</td>
<td>19.23 ± 1.58</td>
<td>11.08 ± 1.34</td>
<td>[78,206]</td>
<td>1.11</td>
<td>[289,408]</td>
<td>1.19</td>
<td>158</td>
<td>0.0997</td>
</tr>
</tbody>
</table>

### Table 4. Partial Pareto optimal solutions when $\lambda = 1.2$ (analytic function problem).

<table>
<thead>
<tr>
<th>No.</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$g_1$</th>
<th>$g^r_1$</th>
<th>$g^r_2$</th>
<th>$f^c$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.58 ± 0.17</td>
<td>11.70 ± 0.48</td>
<td>[184,214]</td>
<td>1.21</td>
<td>[375,411]</td>
<td>1.52</td>
<td>136</td>
</tr>
<tr>
<td>2</td>
<td>21.10 ± 0.47</td>
<td>11.83 ± 0.47</td>
<td>[154,204]</td>
<td>1.31</td>
<td>[371,413]</td>
<td>1.41</td>
<td>141</td>
</tr>
<tr>
<td>3</td>
<td>20.73 ± 0.70</td>
<td>11.65 ± 0.68</td>
<td>[135,205]</td>
<td>1.20</td>
<td>[352,412]</td>
<td>1.29</td>
<td>145</td>
</tr>
<tr>
<td>4</td>
<td>20.14 ± 0.78</td>
<td>11.20 ± 1.06</td>
<td>[117,203]</td>
<td>1.20</td>
<td>[318,404]</td>
<td>1.31</td>
<td>150</td>
</tr>
<tr>
<td>5</td>
<td>19.67 ± 0.96</td>
<td>11.11 ± 1.27</td>
<td>[99,199]</td>
<td>1.22</td>
<td>[302,404]</td>
<td>1.26</td>
<td>154</td>
</tr>
</tbody>
</table>
4.2. Cantilever beam

A cantilever beam as shown in Figure 2 is considered, which is modified from Du (2008). \( L = 100 \text{ cm} \) denotes the length of the beam, and \( b \) and \( h \) denote two cross-sectional sizes. The beam is subjected to a horizontal load \( P_x = 50 \text{ kN} \) and a vertical load \( P_y = 25 \text{ kN} \). The maximum stress at the fixed end of the cantilever beam can be analytically obtained:

\[
\sigma = \frac{6P_xL}{b^2h} + \frac{6P_yL}{bh^2} \quad (18)
\]

The structure needs to be optimized with the design variables \( b \) and \( h \) to have the minimum volume \( V \) under the premise of satisfying an allowable stress \( \sigma_s = 250 \text{ MPa} \). Thus, the following optimization problem can be established:

\[
\min V(b, h)
\]

s.t.

\[
\sigma(b, h) = \frac{6P_xL}{b^2h} + \frac{6P_yL}{bh^2} \leq \sigma_s
\]

\[
5 \text{ cm} \leq b \leq 20 \text{ cm}, \quad 5 \text{ cm} \leq h \leq 20 \text{ cm} \quad (19)
\]

By considering the uncertainty of the design variables \( b \) and \( h \) during the actual manufacturing process, the above problem can be solved by the proposed interval optimization method. As in the first numerical example, the RPDI levels of four different constraints in the range of 0.9–1.2 are investigated, and their corresponding Pareto optimal solutions are obtained in Figure 3.

Figure 2. Cantilever beam structure (Du 2008).

Figure 3. Pareto optimal solutions under different reliability-based possibility degree of interval (RPDI) levels (cantilever beam structure).
The results reveal a similar phenomenon to that observed in numerical example 1: under any one RPDI level, towards different weight values, the nominal objective function and design tolerance index at the optimal design variables have opposite trends; that is, as the design tolerance index increases, the value of the nominal objective function decreases. As the RPDI level increases, giving the stress constraint a stricter reliability requirement, the optimal nominal objective function and design tolerances increase simultaneously. Table 5 lists the partial Pareto solutions for RPDI level $\lambda = 1$. The results indicate that under this RPDI level, the change interval of the stress constraint caused by $b'$ and $h'$ is completely within the range of the allowable stress $\sigma_s = 250$ MPa, which means that the structure has a high reliability. In this numerical example, the contradiction between the nominal objective function and the design tolerance index is less obvious than in numerical example 1.

### 4.3. Automobile side-impact safety analysis

Automobile side impact is a main factor leading to traffic deaths, ranking second after front impact. During the process of automobile side impact, the main factors affecting passenger safety are the intrusion and intrusion velocity of the sidewall structure (Zhang and Su 2008). The sides of the automobile are where the rigidity and strength are the weakest. During a side impact, the side door, the B pillar and the sidewall structure play the major load-bearing roles. Figure 4 presents a side-impact problem for one type of automobile, which is analysed by the finite element method (FEM) with 720,383 shell elements. The moving deformable barrier includes 148,040 elements, and its initial collision velocity is 62 km/h. During the collision process, the
B pillar’s maximum intrusion Intr is an important safety evaluation parameter and must be controlled below an allowable value. As a main load-bearing component, here the B pillar needs to be optimized on its inner plate thickness \( t_1 \) and external plate thickness \( t_2 \), so that the mass \( m \) is minimal and at the same time the B pillar’s maximum intrusion can satisfy the requirement. Therefore, the following optimization problem can be created:

\[
\min m(t_1, t_2)
\]

s.t.

\[
\text{Intr}(t_1, t_2) \leq \text{Intr}_a
\]

\[
1.0 \text{ mm} \leq t_1 \leq 2.0 \text{ mm}, 1.0 \text{ mm} \leq t_2 \leq 2.0 \text{ mm}
\]  

(20)

where \( \text{Intr}_a = 350 \text{ mm} \) denotes the allowable value for the B pillar’s maximum intrusion. By considering the manufacture errors of the internal and external plates’ thicknesses, the proposed interval optimization method is used to solve the above problem. To improve the optimization efficiency, the Latin hypercube design method (Stephen, Bhaskar, and Keane 2003) is adopted to select 10 samples of the design variables. After performing the FEM analysis for these samples, a quadratic polynomial approximation is created for the maximum intrusion Intr:

\[
\text{Intr}(t_1, t_2) = 436.54 - 41.20t_1 - 11.56t_2 - 35.92t_1t_2 + 23.08t_2^2 + 7.99t_1^2
\]  

(21)

To test the accuracy of the approximate model, four points in the design space are selected to compare their results from the FEM and approximate model. As shown in Table 6, the maximum relative error of the approximate model from the FEM result is only 2%, which means that the created approximate model in Equation (21) has a high predictive accuracy for the maximum intrusion.

The interval optimization is then conducted based on this approximate model. The RPDI level of the constraint during the optimization process is set at \( \lambda = 1 \), and the Pareto optimal solutions are displayed in Figure 5. It can be found that the optimal mass of the internal and external plates and their tolerances display opposite changing trends. During the actual design process, a proper design should be selected based on the engineering requirement. For example, if the desired minimum mass \( m^c \) of the B pillar is set to 7.0 kg, the corresponding interval optimization results are as shown in Table 7. Under this condition, the optimal design thicknesses for the internal and external plates are 1.68 mm and 1.72 mm, respectively, and their allowable maximum manufacturing tolerances can be 0.05 mm and 0.09 mm, respectively. Under this design, the changing interval of the maximum intrusion of the B pillar is \( \text{Intr}^d = [315.0 \text{ mm}, 350.0 \text{ mm}] \), which completely satisfies the assigned design requirement \( \text{Intr}_a = 350 \text{ mm} \).

<table>
<thead>
<tr>
<th>( t_1 ) (mm)</th>
<th>( t_2 ) (mm)</th>
<th>FEM (mm)</th>
<th>Approximate model (mm)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>1.2</td>
<td>360.4</td>
<td>357.3</td>
<td>0.9</td>
</tr>
<tr>
<td>1.3</td>
<td>2.0</td>
<td>338.4</td>
<td>337.4</td>
<td>0.3</td>
</tr>
<tr>
<td>1.0</td>
<td>1.3</td>
<td>362.9</td>
<td>370.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1.6</td>
<td>1.5</td>
<td>342.5</td>
<td>344.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: FEM = finite element method.
5. Conclusion

This study proposes a new interval optimization method based on tolerance design by comprehensively considering the optimality of the objective function, the manufacturability of the design variables and the reliability of the constraints. By optimizing a design tolerance index, the optimal design variables can be made to endure a maximum manufacturing error under the premise of satisfying the product performance, thereby leading to better processability and lower manufacturing cost. Using the possibility degree of interval to handle uncertain constraints, the reliability of the optimization problem under uncertainty can be ensured. In practical applications, it is necessary to make a good trade-off among the nominal objective function, tolerances of the design variables and reliability of the constraints, and thereby provide engineering staff with a reasonable interval optimization result. By combining the present method with the ellipsoidal model (Ben-Haim and Elishakoff 1990; Luo, Kang, and Alex 2009; Kang and Luo 2010; Jiang et al. 2011), new interval optimization methods may be developed in future, which could take into account the correlations between the interval variables.

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References


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