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Performance analyses of the IDEAL algorithm combined with the fuzzy control method for 3D incompressible fluid flow and heat transfer problems

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Abstract
IDEAL proposed by the present author is an efficient segregated algorithm for solving the incompressible fluid flow and heat transfer problems. However, its convergence rate is greatly influenced by the under-relaxation factor. The convergence rate under an optimum under-relaxation factor is dozens of times quicker than that under the most unfavorable under-relaxation factor. To lessen the influence, the IDEAL algorithm combined with a fuzzy control method, called IDEAL\textsuperscript{+}FC, is introduced to automatically regulate the values of the under-relaxation factor for accelerating the iteration convergence. Finally, it is demonstrated that IDEAL\textsuperscript{+}FC is superior to IDEAL in terms of convergence rate and robustness. The rapid convergence rate can be achieved by IDEAL\textsuperscript{+}FC even if the initial under-relaxation factor is the most unfavorable value.

1. Introduction

SIMPLE \cite{1} is the first pressure-correction algorithm for solving the fluid flow and heat transfer problems. There exist two major approximations in the SIMPLE algorithm. One is that the initial pressure and velocity fields are assumed independently. The other is that the velocity corrections of the neighboring grids are omitted. These two approximations have a considerable impact on the convergence rate and robustness of the algorithm \cite{2}. For overcoming the approximations mentioned above, many revised algorithms have been proposed after the invention of the SIMPLE algorithm, including SIMPLER \cite{3}, SIMPLEC \cite{4}, PISO \cite{5}, SIMPLET \cite{6}, SOAR \cite{7}, MSIMPLER \cite{8}, CSIMPLER \cite{9}, CLEAR \cite{10, 11}, and so on. Nevertheless, all of these algorithms only partially improve these two approximations in the SIMPLE algorithm.

In 2008, IDEAL was proposed by the present author \cite{12, 13}. The algorithm contains two inner iteration processes of pressure equation, which can almost completely eliminate the disadvantages of these two approximations in the SIMPLE algorithm. As a result, the convergence rate and stability of the solution process is greatly enhanced \cite{14}. So far, the IDEAL algorithm has been extended to the 3D staggered and collocated grid systems in orthogonal coordinates \cite{15, 16}; the 3D body-fitted grid system in nonorthogonal coordinates \cite{17}; and the unsteady two flows \cite{18} by the present author, the unstructured grid system by Ding and Sun \cite{19}, the multiscale multiphysicochemical processes by Luan et al. \cite{20} and Chen et al. \cite{21, 22}, and the weakly compressible flows by Lauriat et al. \cite{23–26}. Considering the high nonlinearity of fluid flow and heat transfer problems, the under-relaxation factor must be used to guarantee the iteration convergence. The value of the under-relaxation...
factor has a great impact on the convergence rate. The convergence rate under an optimum under-relaxation factor is dozens of times quicker than that under the most unfavorable under-relaxation factor. In this study, the lid-driven flow in a 3D inclined cavity is taken as an example to prove the importance of selecting the under-relaxation factor. Figure 1 shows the computation time of the IDEAL and SIMPLERM algorithms under different under-relaxation factors $\alpha$ and reveals that the IDEAL algorithm is more efficient and more robust than the SIMPLERM algorithm. The efficiency and robustness are embodied in the fact that (1) the shortest computation time of the IDEAL algorithm is only 32% of that of the SIMPLERM algorithm; (2) the convergence range of the IDEAL algorithm ($\alpha \leq 0.99$) is wider than that of the SIMPLERM algorithm ($\alpha \leq 0.8$). However,
the convergence properties of the IDEAL algorithm are highly dependent on the choice of the under-relaxation factor. The computation time of IDEAL under the optimum under-relaxation factor ($\alpha = 0.99$) and under the most unfavorable under-relaxation factor ($\alpha = 0.2$) is 172 s and 13354 s, respectively. That is to say, the convergence rate under the optimum under-relaxation factor increases 78 times when compared with the most unfavorable under-relaxation factor. Even in the range of $\alpha \leq 0.8$, the IDEAL algorithm takes more computation time than the SIMPLER algorithm. Therefore, it is very crucial to appropriately select the under-relaxation factor for the IDEAL algorithm.

The optimum under-relaxation factor varies with problems, and it cannot be known in advance. For overcoming this shortcoming, a fuzz control method could be used to automatically select the optimum under-relaxation factor. By now, some scholars have made success in using the fuzz control method to solve the fluid flow and heat transfer problems. Ryoo et al. [27] and Liu et al. [28] adopted a residual-based fuzzy control method to accelerate iteration convergence in CFD simulation. Later, Ryoo et al. [29] used an adaptive network-based fuzzy inference system (ANFIS) to control the convergence of a CFD algorithm. Recently, Dragojlovic et al. [30, 31] and Jain et al. [32, 33] developed a fuzzy control method based on a Fourier transform to achieve automatic convergence with minimum CPU time. All of the work mentioned above is conducted only for the SIMPLER algorithm and the 2D problems. Yet there is a very little information on the fuzzy control method used in the IDEAL algorithm and the 3D problems in the literature study. The extension of the dimensionality will cause to a significant increase in computational time. Therefore, it is significantly meaningful to apply the fuzzy control method to the acceleration of the solution process of the 3D problems.

Based on the analyses above, our study mainly focuses on combining the IDEAL algorithm with the fuzzy control method to accelerate the iteration convergence of the 3D problems. For convenience, the IDEAL algorithm combined the fuzzy control method is called IDEAL+FC for short.

In the following, the governing equations and their discretization forms are first described (Section 2). Then, the IDEAL algorithm is briefly reviewed (Section 3), and the fuzzy control method is introduced (Section 4). After that, the solving performances of SIMPLER, IDEAL, and IDEAL+FC are compared by two 3D problems (Section 5). Finally, some conclusions are drawn (Section 6).

**2. Governing equations and their discretization forms**

The incompressible laminar flow and heat transfer problems are considered in this study, and the numerical research is based on the 3D nonorthogonal curvilinear coordinate system (Figure 2). The corresponding governing equations in this coordinate system are written as the follows:

**Continuity equation:**

$$\frac{\partial}{\partial \xi} (\rho U) + \frac{\partial}{\partial \eta} (\rho V) + \frac{\partial}{\partial \zeta} (\rho W) = 0$$  

**Momentum equations:**

$$\frac{\partial}{\partial \xi} (\rho U^2) + \frac{\partial}{\partial \eta} (\rho U V) + \frac{\partial}{\partial \zeta} (\rho U W) = \frac{\partial}{\partial \xi} \left( \frac{\chi}{f} \eta \frac{\partial U}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \frac{\beta}{f} \eta \frac{\partial U}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( \frac{\gamma}{f} \eta \frac{\partial U}{\partial \zeta} \right)$$

$$- \chi_1 \frac{\partial P}{\partial \xi} - \beta_1 \frac{\partial P}{\partial \eta} - \gamma_1 \frac{\partial P}{\partial \zeta} + JS_u$$

$$\frac{\partial}{\partial \xi} (\rho U V) + \frac{\partial}{\partial \eta} (\rho V^2) + \frac{\partial}{\partial \zeta} (\rho V W) = \frac{\partial}{\partial \xi} \left( \frac{\chi}{f} \eta \frac{\partial V}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( \frac{\beta}{f} \eta \frac{\partial V}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( \frac{\gamma}{f} \eta \frac{\partial V}{\partial \zeta} \right)$$

$$- \chi_2 \frac{\partial P}{\partial \xi} - \beta_2 \frac{\partial P}{\partial \eta} - \gamma_2 \frac{\partial P}{\partial \zeta} + JS_v$$
\[
\frac{\partial}{\partial \xi} (\rho U) + \frac{\partial}{\partial \eta} (\rho V) + \frac{\partial}{\partial \zeta} (\rho W) = \frac{\partial}{\partial \xi} \left( \frac{\chi}{f} \frac{\partial w}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\beta}{f} \frac{\partial w}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( \frac{\gamma}{f} \frac{\partial w}{\partial \zeta} \right)
- \frac{\partial}{\partial \xi} \frac{\partial p}{\partial \xi} - \frac{\partial}{\partial \eta} \frac{\partial p}{\partial \eta} - \frac{\partial}{\partial \zeta} \frac{\partial p}{\partial \zeta} + JS'_w
\] (4)

Energy equation:
\[
\frac{\partial}{\partial \xi} (\rho UT) + \frac{\partial}{\partial \eta} (\rho VT) + \frac{\partial}{\partial \zeta} (\rho WT) = \frac{\partial}{\partial \xi} \left( \frac{\chi \lambda}{C_p} \frac{\partial T}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\beta \lambda}{C_p} \frac{\partial T}{\partial \eta} \right)
+ \frac{\partial}{\partial \zeta} \left( \frac{\gamma \lambda}{C_p} \frac{\partial T}{\partial \zeta} \right) + JS'_T
\] (5)

\(U, V, \) and \(W\) in Eqs. (1)–(5) are contravariant velocities and can be expressed as
\[
U = \chi_1 u + \chi_2 v + \chi_3 w
\] (6)
\[
V = \beta_1 u + \beta_2 v + \beta_3 w
\] (7)
\[
W = \gamma_1 u + \gamma_2 v + \gamma_3 w
\] (8)

The governing Eqs. (1)–(5) are discretized on the body-fitted collocated grid system (Figure 2) using the finite volume method (FVM) [34].

Discretized continuity equation:
\[
(\rho U)_{\xi} \Delta \eta \Delta \zeta - (\rho U)_{w} \Delta \eta \Delta \zeta + (\rho V)_{\eta} \Delta \zeta \Delta \zeta - (\rho V)_{\xi} \Delta \xi \Delta \zeta + (\rho W)_{\zeta} \Delta \xi \Delta \eta - (\rho W)_{b} \Delta \xi \Delta \eta = 0
\] (9)

Discretized momentum equations:
\[
\frac{d u^*}{d t} = \sum_{nb} a_{nb} u_{nb} + b^0 + (1 - \alpha_u) \frac{d}{\partial t} + P + \left( \frac{\chi}{f} \frac{\partial p}{\partial \xi} \right) \Delta \xi \Delta \eta \Delta \zeta
- \left( \frac{\beta}{f} \frac{\partial p}{\partial \eta} \right) \Delta \xi \Delta \eta \Delta \zeta
\] (10)
\[ \begin{align*}
\frac{d v}{dt} v_p &= \sum_{nb} a_{nb} v_{nb} + b_p + (1 - \alpha_v) \frac{d v}{dt} v_p^0 - \left( \chi_2 \frac{\partial p}{\partial \xi} \right)_p \Delta \xi \Delta \eta \Delta \zeta \\
&\quad - \left( \beta_2 \frac{\partial p}{\partial \eta} \right)_p \Delta \xi \Delta \eta \Delta \zeta - \left( \gamma_2 \frac{\partial p}{\partial \zeta} \right)_p \Delta \xi \Delta \eta \Delta \zeta \\
\frac{d w}{dt} w_p &= \sum_{nb} a_{nb} w_{nb} + b_w + (1 - \alpha_w) \frac{d w}{dt} w_p^0 - \left( \chi_3 \frac{\partial p}{\partial \xi} \right)_p \Delta \xi \Delta \eta \Delta \zeta \\
&\quad - \left( \beta_3 \frac{\partial p}{\partial \eta} \right)_p \Delta \xi \Delta \eta \Delta \zeta - \left( \gamma_3 \frac{\partial p}{\partial \zeta} \right)_p \Delta \xi \Delta \eta \Delta \zeta 
\end{align*} \]

where \( \alpha_v, \alpha_w, \) and \( \alpha_T \) are the under-relaxation factors. In Eqs. (1)–(13), the parameters \( \gamma, \chi_1, \chi_2, \chi_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3 \) are geometry factors. Their detailed expressions have been given in reference [17].

3. Brief review of IDEAL

The details of the implementation of IDEAL on 3D nonorthogonal curvilinear coordinates have been well documented in reference [17]. In this study, the solutions procedure of the IDEAL algorithm (Figure 3) is briefly reviewed as follows:

**Step-1**: Assume an initial velocity field.

**Step-2**: Calculate the coefficients and source terms of the discretized momentum equations based on the initial velocity field.

**Step-3**: Solve the pressure equation iteratively until the iteration time equals to the prespecified value of \( N_1 \). Once the first inner iteration process for solving pressure equation is over, the final temporary pressure is regarded as the initial pressure.

**Step-4**: Solve the discretized momentum equations based on the initial velocity and the initial pressure and obtain the intermediate velocity.

**Step-5**: Solve the pressure equation iteratively until the iteration time equals to the prespecified value of \( N_2 \). Once the second inner iteration process for solving pressure equation is finished, the final temporary velocity is taken as the final velocity of the current iteration level.

**Step-6**: Solve the discretized energy equation if necessary.

**Step-7**: Regard the final velocity of the current iteration level as the initial velocity of the next iteration level, then return to the Step-2 for the next iteration level, and repeat the iterative procedure until convergence is reached.

In the IDEAL algorithm, \( N_1 \) and \( N_2 \) could be adjusted to control the convergence rate and stability of the solution process.

4. Introduction to the fuzzy control method

The values of the under-relaxation factors \( \alpha_v, \alpha_w, \) and \( \alpha_T \) in Eqs. (10)–(13) are adjusted separately by the fuzzy control method. The adjustive measures are exactly the same for these four different under-relaxation factors. To simplify, the general form of the discretized Eqs. (10)–(13) is given as follows:

\[ \frac{d \phi}{dt} \phi_p = \sum_{nb} a_{nb} \phi_{nb} + b_p \]
where $\alpha$ refers to $\alpha_u$, $\alpha_v$, $\alpha_w$, and $\alpha_T$ and $\phi$ denotes $u$, $v$, $w$, and $T$. In the following, the fuzzy control method is introduced on the basis of Eq. (14).

### 4.1. Input variable and output variable

Generally, the iteration process tends to be divergent with the gradual increase of the residual. To prevent the divergence of the iteration process, the under-relaxation factor needs to be reduced. On the contrary, the under-relaxation factor should be appropriately increased to accelerate the iteration convergence as the residual decreases gradually. Based on the analyses above, the ratio of the residuals between two successive iteration levels is defined as the input variable of the fuzzy control, which is expressed as

$$e = \frac{d_n}{d_{n-1}}$$ (15)
where \(d_n\) is the maximum residual on the current iteration level \(n\).

\[
d_n = \text{MAX} \left\{ \left| \frac{d_p}{\alpha} \Phi_p - \left[ \sum_{nb} a_{nb} \Phi_{nb} + b_p \right] \right| \right\}_n
\]

(16)

\(d_{n-1}\) is the maximum residual on the previous iteration level \(n-1\).

\[
d_{n-1} = \text{MAX} \left\{ \left| \frac{d_p}{\alpha} \Phi_p - \left[ \sum_{nb} a_{nb} \Phi_{nb} + b_p \right] \right| \right\}_{n-1}
\]

(17)

The output variable of the fuzzy control is the change of the under-relaxation factor \(\Delta\alpha\).

### 4.2. Membership functions

A fuzzy set is characterized by its membership function. The value of the membership function could be set to any number between 0 and 1, representing the membership degree of the element in the fuzzy set.

The input variable \(e\) includes three fuzzy subsets: PS (positive small), PM (positive medium), and PB (positive big). Their corresponding membership functions are, respectively, \(\mu_e^{PS}\), \(\mu_e^{PM}\), and \(\mu_e^{PB}\), as shown in Figure 4a. The output variable \(\Delta\alpha\) also has three fuzzy subsets: PS (positive small), NS (negative small), and NB (negative big), which correspond, respectively, to the membership functions \(\mu_{\Delta\alpha}^{PS}\), \(\mu_{\Delta\alpha}^{NS}\), and \(\mu_{\Delta\alpha}^{NB}\) (Figure 4b).

### 4.3. Control rules

As shown in Table 1, there are three control rules. The first control rule can be described as “if \(e\) is PS, then \(\Delta\alpha\) is PS,” which indicates that \(d_n\) is less than \(d_{n-1}\). The iteration process tends to be convergent. To accelerate the iteration convergence, the under-relaxation factor should be increased slightly. The second control rule is that “if \(e\) is PM, then \(\Delta\alpha\) is NS,” which means that \(d_n\) is greater than \(d_{n-1}\). The under-relaxation factor should be decreased to prevent the divergence of the iteration process. The third control rule is that “if \(e\) is PB, then \(\Delta\alpha\) is NB.” In this case, \(d_n\) is much greater than \(d_{n-1}\), which results in prompt divergence of the iteration process. Therefore, the under-relaxation factor should be decreased greatly.

Based on the three control rules, we can obtain the fuzzy relation \(R\).

\[
R = (\mu_e^{PS} \times \mu_{\Delta\alpha}^{PS}) \cup (\mu_e^{PM} \times \mu_{\Delta\alpha}^{NS}) \cup (\mu_e^{PB} \times \mu_{\Delta\alpha}^{NB})
\]

where the symbol “\(\times\)” refers to direction production and the symbol “\(\cup\)” denotes dyadic production.

![Figure 4. Membership functions of input and output variables.](image-url)
4.4. Implementation strategy of the fuzzy control method

In this study, the single-input and single-output fuzzy control method is applied to automatically adjust the value of the under-relaxation factor. Its implementation strategy is introduced in detail as follows.

First, a given residual ratio $E$ can be obtained on one iteration level. $E$ is considered as a specific fuzzy set, namely fuzzy single point. Its corresponding membership function is expressed as $\mu_{e}^{input}$, in which the membership degree of any element is set as zero except that the membership degree of $E$ equals to one.

Then, the membership function of the output fuzzy set can be obtained by

$$m_{output}^{Da} = m_{input}^{e} \circ R$$

where the symbol “$\circ$” is the synthesizing operator.

After that, $m_{output}^{Da}$ is transformed into an exact value $Da$ by the process of defuzzification. Here is a center of gravity method used for defuzzification.

$$\Delta \chi^* = \frac{\int \Delta \chi m_{output}^{Da} d \Delta \chi}{\int m_{output}^{Da} d \Delta \chi}$$

Finally, the under-relaxation factor is updated by the following equation:

$$\alpha_{n+1} = \alpha_n + \Delta \chi^*$$

where $n$ and $n + 1$ denote the current iteration level and the next iteration level, respectively.

5. Numerical comparisons

Comprehensive comparisons are made among SIMPLERM, IDEAL, and IDEAL+FC by two 3D problems including laminar fluid flow through a twisted tape-inserted tube and lid-driven flow in a 3D inclined cavity. The numerical comparison conditions involve (1) a stability-guaranteed second-order different scheme (SGSD) [35] is adopted for discretizing the convection terms; (2) all of the algebraic equations, formed by discretizing governing equation, are solved by the alternative direction implicit method (ADI); (3) double precision digital is used to implement computation to reduce the truncated errors in our codes.

5.1. Problem 1: Laminar fluid flow through a twisted tape-inserted tube

The flow configuration of laminar fluid flow through a twisted tape-inserted tube is shown in Figure 5. $D$ is the tube outer diameter. $\delta$ denotes the thickness of the twisted tape. The twist ratio of the twisted tape is defined as

$$y = \frac{L}{D}$$

where $L$ refers to the axial length of the tape rotated 180°. In the study, the value of $y$ is set as 5.

Figure 6 shows the computation domain and the corresponding body-fitted grid system. The computation domain is the volume of the shadow region (Figure 5) rotated 360°. The no-slip velocity
The boundary condition is applied on the tube wall and the twisted tape. The tube inlet and outlet are considered as the periodic boundary condition.

The Reynolds number is defined as

$$Re = \frac{u_{in}D}{v}$$

where $u_{in}$ refers to the inlet average velocity. Here, calculations are conducted in the two cases of $Re = 100$ and $Re = 500$. The convergence criterion requires that the relative maximum mass and momentum residuals are all less than $10^{-6}$.

The friction factor of the twisted tape-inserted tube is defined as

$$f = \frac{(p_{in} - p_{out})D}{L\rho u_{in}^2}$$

Table 2 shows the predicted friction factors. The results computed by IDEAL+FC are excellently consistent with those calculated by SIMPLERM and IDEAL, verifying the reliability of the proposed IDEAL+FC method and the developed code. To obtain grid-independent solution, three grid systems of $185 \times 20 \times 20$, $276 \times 30 \times 30$, and $370 \times 40 \times 40$ are considered. As shown in Table 2, the maximum deviation among different grid systems is only $0.5\%$, proving that the computation results are grid-independent solutions. Thus, the grid of $276 \times 30 \times 30$ is used to make the later simulation.

Figure 7 shows the computation time of SIMPLERM, IDEAL, and IDEAL+FC under different initial under-relaxation factors ($\alpha^0$), which refer to 0.2, 0.4, 0.6, 0.8, 0.9, 0.94, and 1, respectively. For SIMPLERM and IDEAL, $\alpha^0$ remains unchanged in the iteration process. However, for IDEAL+FC, $\alpha^0$ is only an initial value and its value in the iteration process will be constantly adjusted to achieve rapid convergence.

As shown in this figure, IDEAL is more efficient and more robust than SIMPLERM. In the cases of $Re = 100$ and $Re = 500$, the shortest computation time of IDEAL is only 55 and 42\% of that of SIMPLERM, respectively; the convergence ranges of IDEAL ($\alpha^0 \leq 0.94$) far surpass those of SIMPLERM ($\alpha^0 \leq 0.6$). However, the consumed computation time of IDEAL is almost the same as that of SIMPLERM under the small initial under-relaxation factors ($\alpha^0 \leq 0.6$). For IDEAL, the ratios of the longest computation time and the shortest computation time are up to 6.1–8.6. For example,
the longest computation time is 6919 s under the most unfavorable initial under-relaxation factor ($\alpha^0 = 0.2$) and the shortest computation time is 805 s under the optimum initial under-relaxation factor ($\alpha^0 = 0.94$) in the case of $Re = 500$. To overcome this shortcoming, IDEAL+FC is proposed to automatically select the optimum under-relaxation factor.

The ratios of the longest computation time and the shortest computation time are only 1.04–1.2 for IDEAL+FC, far superior to IDEAL. It is indicated that the fast convergence rate can be achieved by IDEAL+FC even the initial under-relaxation factor is the most unfavorable value. The ratios of the consumed computation time between IDEAL+FC and IDEAL are 0.14–0.16 under the most unfavorable initial under-relaxation factor ($\alpha^0 = 0.2$), meaning that the convergence rate of IDEAL+FC...
improves about 6–7 times when compared with IDEAL. The ratios of the consumed computation time between IDEAL+FC and IDEAL are 0.96–1.07 under the optimum initial under-relaxation factor ($\alpha^0 = 0.94$). The convergence rate of IDEAL+FC is almost the same as that of IDEAL in this situation. In the aspect of robustness, IDEAL+FC is also superior to IDEAL. IDEAL+FC can obtain the converged solutions in a wider range ($\alpha^0 \leq 0.99$).

For the purpose of studying the internal mechanism of accelerating iteration convergence in IDEAL+FC, we give the variation curves of the adjusted under-relaxation factor $\alpha^a$ versus outer iteration number under three different $\alpha^0$ (Figure 8). The dotted line in Figure 8 represents a value close to the optimum under-relaxation factor. With the advance of iteration, $\alpha^a$ gradually gets closer to the optimum under-relaxation factor and then fluctuates upper and lower around the optimum under-relaxation factor. Thus, the fuzzy control regulation mechanism is mainly embodied in two aspects: (1) when $\alpha^a$ is less than the optimum under-relaxation factor, increasing the value of under-relaxation factor can accelerate iteration convergence; (2) when $\alpha^a$ is larger than the optimum under-relaxation factor, decreasing the value of under-relaxation factor can prevent iterative divergence.

5.2. Problem 2: Lid-driven flow in a 3D inclined cavity

Figure 9 shows the flow configuration of lid-driven flow in a 3D inclined cavity. In the figure, the values of $L$, $H$, and $W$ are all equal to one and $\theta$ denotes the inclination angle. Calculations are conducted in the three cases of $\theta = 90$, 45, and 5°. The corresponding Reynolds number is defined as

$$Re = \frac{u_{lid}H}{v}$$

(25)

where $Re$ is set as 500. The convergence criterion requires that the relative maximum mass and momentum residuals are all less than $10^{-7}$.

Figure 10 shows the corresponding body-fitted grid systems, which are generated by the transfinite interpolation method [36]. As shown in this figure, the grid skewness increases with the decrease of $\theta$.

The predicted $u$-velocity profiles along the central line “CL” are presented in Figure 11. The specific location of CL is shown in Figure 9. Results calculated by SIMPLERM, IDEAL, and IDEAL+FC agree very well with each other, further verifying the reliability of the proposed IDEAL+FC method. From this figure, it is also found that the calculation deviation among three different grid systems
Figure 9. Schematic diagram of the 3D inclined cavity.

Figure 10. Body-fitted grid systems of the 3D inclined cavities.

Figure 11. Comparison of $u$-velocity profiles along the central line "CL" at $\theta = 5^\circ$. 
almost tends to zero, proving that the computation results are grid-independent solutions. Thus, the grid of $50 \times 50 \times 50$ is used to make the later simulation.

Figure 12 shows the solving performances of SIMPLERM, IDEAL, and IDEAL+FC in the three different cases of $\theta = 90, 45$, and $5^\circ$. As shown in this figure, the superiority of IDEAL to SIMPLERM is more obvious with the decrease of $\theta$, i.e., with the increase of the grid skewness. The shortest computation time of IDEAL is, respectively, reduced about two, three, and eight times as compared with SIMPLERM in the three different cases of $\theta = 90, 45$, and $5^\circ$. However, IDEAL consumes more computation time than SIMPLERM under the small initial under-relaxation factors. The ratios of the longest computation time and the shortest computation time are up to 59.5–87.7 for IDEAL, far beyond the ratios for IDEAL+FC (1.8–4.0). It reveals that IDEAL+FC could always get solutions with high convergence rate under different initial under-relaxation factors. Meanwhile, IDEAL+FC has the same robustness as IDEAL. Both of them can obtain the converged solution in the range of $\alpha^0 \leq 0.99$. All above analyses prove that IDEAL+FC is superior to IDEAL.

6. Conclusion

In the study, the method of IDEAL+FC is introduced to automatically adjust the value of the under-relaxation factor for accelerating the iteration convergence. Finally, advantages of IDEAL+FC are evaluated by comparison with SIMPLERM and IDEAL through two 3D problems. Main conclusions are as follows:

1. IDEAL is more efficient and more robust than SIMPLERM. However, IDEAL consumes more computation time than SIMPLERM under the small initial under-relaxation factors. The ratios
of the longest computation time and the shortest computation time are up to 6.1–87.7 for IDEAL.

2. IDEAL+FC is superior to IDEAL in the aspects of convergence rate and robustness. The ratios of the longest computation time and the shortest computation time are only 1.04–4 for IDEAL+FC, far superior to IDEAL. It is indicated that the fast convergence rate can be achieved by IDEAL+FC even the initial under-relaxation factor is the most unfavorable value. Meanwhile, IDEAL+FC can obtain the converged solutions in a wider range when compared with IDEAL.

3. The convergence rate of IDEAL+FC increases about 6–36 times when compared with IDEAL, when the initial under-relaxation factor is the most unfavorable value.

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