Target Location and Height Estimation via Multipath Signal and 2D Array for Sky-Wave Over-the-Horizon Radar

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In sky-wave over-the-horizon radar (OTHR), it is quite difficult to handle the issues of target location (ground range) and height (altitude) estimation due to their joint effect on group range. This work addresses the joint estimation of target location and height by the means of multipath propagation of OTHR signals and structure of a two-dimensional (2D) array. Usually, the multipath signal results from ionosphere propagation in OTHR and can be produced in multi-input–multi-output (MIMO) radar by transmitting signals of various carrier frequencies. A 2D array provides the potential of elevation resolution, which is related to ground and slant ranges. By the multi-quasi-parabolic (MQP) ionospheric model, we formulate the multipath propagation and signal model for the OTHR with a target at location \( r \) and height \( h \). Moreover, the joint maximum-likelihood estimates (MLEs) of \( r \) and \( h \) are derived, and the joint Fisher information matrix (FIM) is calculated. With the so-obtained FIM, the estimability is analyzed; that is, \( r \) and \( h \) are estimable if and only if either a multipath signal or 2D array is available. The joint Cramér-Rao bound (CRB) is computed and discussed for accuracy improvement. Additionally, the estimability is also extended to the joint estimation of target location, height, and velocity.

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beacon [16], and improvement of the Doppler spectrum of sea echoes [17]. In this paper, we will study the 2D array’s effects on target location and height estimation performance.

Second, the multipath effect is beneficial for target localization. Compared with micro-multipath in OTHR, the multipath effect is more widely used for target localization in ordinary radar (non-OTHR), especially in multi-input–multi-output (MIMO) radar. Multiple time delays produced by multistation radars can be estimated for locating a target [18]. Non-collocated MIMO radar places the transmitters and receivers widely separated and so produces spatial diversity gain on target detection and estimation performance [19–22]. Godrich and He analyzed the antenna placement for the accuracy of target localization and velocity respectively [23, 24]. Unlike ordinary radar with antennas widely separated, an OTHR system is usually bistatic. In OTHR, the multipath effect may be produced by an ionosphere propagation characteristic, which permits a single signal to travel through different ionosphere layers to illuminate the same area. Generally, there are one to four paths for one signal. In conventional OTHR, some multipath association tracking algorithms have been developed without considering target altitude [25–29].

Herein, we consider making use of the multipath effect for estimating target location and height in signal processing in fast time. Ray tracing is employed based on a widely used sophisticated ionosphere model—the multi-quasi-parabolic (MQP) model [3, 15, 27]. The MQP model combines quasi-parabolic layers and joint layers together and provides a profile of ionosphere electron density, from which the electron magnetic waves’ propagation can be deduced. In addition, the multipath effect depends on the ionosphere state and range of interest. When the multipath effect is not produced by a single signal, OTHR can employ the MIMO technique and transmit multiple signals. Each signal bears a unique carrier frequency to guarantee and accentuate the multipath effect, as electron magnetic waves at different carrier frequencies can reach the same area by various paths. It is worth noting that our proposed MIMO-OTHR is different from the OTHR in [30–32], in which multiple transmit signals share one carrier frequency, so that the correlations of various signals and paths can be utilized, as the multipath situation is similar to that of conventional OTHR. Our work is also different from the MIMO technique in [33], where height estimation depends on Doppler signatures generated by micro-multipath effects in slow time.

This paper analyzes the estimability and estimation of target location and height in fast time for sky-wave OTHR, considering the effects of multipath propagation and a 2D array. Traditional azimuth is omitted in this analysis for simplification. Estimability denotes the feasibility of effective estimation, i.e., whether target location and height are estimable in a certain situation. We also analyze the joint Cramér-Rao bound (CRB) and discuss the approaches on accuracy improvement for an OTHR that wishes to estimate target altitude effectively.

The remainder of the paper is organized as follows. In section II, we develop a multipath propagation and receive signal model when OTHR illuminates a target at a certain altitude. The joint maximum-likelihood estimate (MLE) and Fisher information matrix (FIM) of the target location and height are derived in section III. Based on the FIM, the estimability is analyzed in section IV. We discuss sufficient conditions for effective estimation and also give some instructive advice on radar system construction and operation for accuracy improvement. Section V presents simulation results. Section VI extends the study to the target velocity problem. Finally, conclusions are drawn in section VII.

Notation: Throughout this paper, we use bold uppercase letters to denote matrices, and bold lowercase letters to signify vectors. Superscripts \( \{ \cdot \}^\text{H}, \{ \cdot \}^\dagger, \{ \cdot \}^\text{T} \) denote the complex conjugate transpose, conjugate, and transpose of a matrix or vector, respectively, and \( \text{diag} \{ \cdot \} \) denotes a diagonal matrix with its diagonal given by the vector \( \cdot \). We use \( \mathbb{E}\{ \cdot \} \) for expectation with respect to all the random variables within the bracket, and \( | \cdot | \) as the modulus for a complex number.

II. MULTIPATH PROPAGATION AND SIGNAL MODEL

Bistatic OTHR system places transmit antennas and receive antennas separately in two 2D arrays. Owing to the large propagation distances compared with the separation between transmit and receive arrays, OTHR is considered as a monostatic system in research. The MIMO technique employs \( M \) transmit signals totally. Suppose that there are \( M \) rows in the transmit array, and each row emits one signal. Therefore, there is beamforming in azimuth, and omnidirectional emitting in elevation. We ignore the azimuth analysis and focus on the 2D elevation plane to offer insight into ionosphere propagation.

Consider monostatic OTHR with \( M \) transmit antennas and \( N \) receive antennas in the 2D elevation plane (vertical to the horizon), as depicted in Fig. 1, where adjacent receive antennas have distance \( d \) spacing in a uniform linear array. The \( m \)-th transmit antenna, for \( m = 1, \ldots, M \), emits a narrowband waveform given by \( \sqrt{E_m} s_m(t) \), where \( E_m \) is the transmitted energy, and \( \int_{-\infty}^{+\infty} |s_m(t)|^2 dt = 1 \), at carrier frequency \( f_m \). The carrier frequency increment between any two transmitted signals is assumed large enough, so that the spectrums are non-overlapping to keep signals mutually orthogonal. A target located at ground range \( r \) and height \( h \) is considered. For the convenience of analysis, the influence of velocity is not considered here.

The ionosphere is modeled as MQP layers for multipath deducing and ray tracing [3, 15]. Each layer is parameterized by its critical frequency \( f_{cr} \), layer height \( z_i \), and layer thickness \( z_h \), for \( i = 0, \ldots, I \), where \( I \) denotes the number of layers in the MQP model. These parameters are assumed to be achieved by the acquisition system of OTHR [1, 14], and they remain constant during the
applies to the joining layers, and the positive sign applies to the

\[ \beta \]

ranges corresponding to elevation angles found in [3]. From altitude 0 to the horizon. With the effect of Earth’s magnetic field starting from the ground, passes through the ionosphere, and reaches the ground region. Under the MQP model, a one-way ray path is evaluated under the propagation equations [3]

\[ G_{\mu}(\beta) = 2 \int_{z_0}^{z_\beta} \frac{zdz}{z^2 \mu_2(z) - z_0^2 \cos^2 \beta}, \]

(1)

in each path, and the ground range satisfies

\[ g_\mu(\beta) = 2z_0^2 \cos \beta \int_{z_0}^{z_\beta} \frac{dz}{z^2 \mu_2(z) - z_0^2 \cos^2 \beta}, \]

(2)

where \( z_0 \) and \( z_\beta \) denote the radius of Earth and the apogee height of the ray path (away from Earth’s core), respectively; \( G_{\mu}(\beta) \) and \( g_\mu(\beta) \) denote the slant range and ground range, respectively; and

\[ \mu_\mu^2(z) = 1 - \frac{f_0^2}{f_0^2} \left[ \pm \frac{z - z_i}{z_h} \left( \frac{z_i - z_h}{z} \right)^2 \right]; \]

(3)

For the positive/negative sign in (3), the negative sign applies to the joining layers, and the positive sign applies to others. The detailed procedure of layering the ionosphere and computing the apogee height \( z_\beta \) can be found in [3]. From altitude 0 to \( h \), the ground and slant ranges corresponding to elevation \( \beta \) are

\[ G_{\delta}(\beta, h) = \int_{z_0}^{z_0+h} \frac{zdz}{\sqrt{z^2 \mu^2(z) - z_0^2 \cos^2 \beta}}, \]

(4)

and

\[ g_\delta(\beta, h) = z_0^2 \cos \beta \int_{z_0}^{z_0+h} \frac{dz}{z \sqrt{z^2 \mu^2(z) - z_0^2 \cos^2 \beta}}, \]

(5)

respectively.

By the ray-tracing functions (1–5), we can calculate the multipath along which the transmitted waveform \( s_m(t) \) reaches the target at \( (r, h) \) and returns to the receive array. Suppose the number of multipaths for \( s_m(t) \) is \( L_m \). The time delay corresponding to the \( l \)th path of \( s_m(t) \), for \( l = 1, \ldots, L_m \), is

\[ \tau_{ml} = \left( R_{ml}^F + R_{ml}^B \right)/c, \]

(6)

where \( c \) denotes the speed of light, and \( R_{ml}^F \) and \( R_{ml}^B \) denote the slant range of the forward propagation (the transmitted electromagnetic wave travels through the ionosphere and reaches the target) and the backward propagation (the electromagnetic wave reflected by the target travels through the ionosphere and reaches the receive array) corresponding to the \( l \)th path of the \( m \)th transmitted signal. The elevation angles corresponding to \( R_{ml}^F \) and \( R_{ml}^B \), are denoted by \( \beta_{ml}^F \) and \( \beta_{ml}^B \), respectively. Then, the slant range and height \( h \) satisfy the following propagation equations [3]

\[ R_{ml}^F = G_{\mu}(\beta_{ml}^F) - G_{\mu}(\beta_{ml}^F, h), \]

(7)

\[ R_{ml}^B = G_{\mu}(\beta_{ml}^B) - G_{\mu}(\beta_{ml}^B, h). \]

(8)

Target location \( r \) and height \( h \) satisfy

\[ r = g_\mu(\beta_{ml}^F) - g_\mu(\beta_{ml}^B, h), \]

(9)

\[ = g_\mu(\beta_{ml}^F) - g_\mu(\beta_{ml}^B, h). \]

(10)

By now, the relationship between the time delays and target state \( (r, h) \) under the multipath effect is established by (1–10). Herein we briefly explain their usage. Given the ionosphere parameters, transmitting signals, and target state \( (r, h) \), (9) becomes a function of \( \beta_{ml}^F \) and (10) becomes one of \( \beta_{ml}^B \). The two functions are solved for possible solutions of \( \beta_{ml}^F \) and \( \beta_{ml}^B \) (there may be no solution, one root, or multiple roots). Then, practical solutions are determined, and \( L_m \) is evaluated under the constraints of geography, atmosphere, and radar system. Substitute the practical \( \beta_{ml}^F \) and \( \beta_{ml}^B \) in (7) and (8) for \( R_{ml}^F \) and \( R_{ml}^B \). Finally, \( \tau_{ml} \) is obtained by (6). Since the aforementioned path parameters vary for carrier frequencies, multipath diversity is produced as long as OTHR transmits two or more signals accessible to the target. The number of paths for all signals is \( L = \sum_{m=1}^M L_m \). Hence, \( L \) is determined by \( L_m \) directly, constrained by \( M \) indirectly.

Considering the target reflection coefficients, an assumption is made as follows.

ASSUMPTION 1 The reflection coefficient corresponding to the \( l \)th path of the \( m \)th signal is a zero-mean complex Gaussian random variable \( \xi_m \sim \mathcal{CN}(0, 2 \sigma_m^2) \), and it remains constant during the observation. For \( m \neq m' \) or \( l \neq l' \), \( \xi_m \) and \( \xi_{m'} \) are independent.

The background noise level varies with the spectrum in the high-frequency band [1, 14]. When OTHR employs a unique carrier frequency for each signal, the noise levels at the receivers can be different for various signals.
ASSUMPTION 2. The noise due to the mth signal at the nth antenna is a temporally white, zero-mean complex Gaussian random process $u_{nm}(t) \sim \mathcal{CN}(0, \sigma_{um}^2)$, where $\sigma_{um}^2$ is a constant. The noise is independent for different receivers or signals, $\mathbb{E}[u_{nm}(t)u_{n'm'}(t)] = 0$, if $n \neq n'$ or $m \neq m'$.

Under Assumptions 1 and 2, the received signal by the nth receiver due to the mth signal is given by

$$y_{nm}(t) = \sum_{l=1}^{L_m} \xi_{ml} \sqrt{E_m} s_m(t - \tau_{ml}) e^{j(\omega_{ml})} + u_{nm}(t), \tag{11}$$

where

$$\varphi_{ml} = 2\pi d \cos \frac{\beta_{ml}^B}{c} f_m/c \tag{12}$$

denotes the phase interval due to the receive antenna spacing.

Let $\tilde{y}_{nm}(t)$ denote the observed data by the nth antenna due to the mth signal. List the observed data in a column vector

$$\tilde{y}(t) = [\tilde{y}_1^T(t), ..., \tilde{y}_m^T(t), ..., \tilde{y}_M^T(t)]^T, \tag{13}$$

where

$$\tilde{y}_n(t) = [\tilde{y}_{1n}(t), ..., \tilde{y}_{nm}(t), ..., \tilde{y}_{Nm}(t)]^T. \tag{14}$$

We intend to estimate target state $(r, h)$ from the observed data $\tilde{y}(t)$ and investigate the accuracy if estimable. These are the main works of the following two sections.

III. JOINT MLE AND FIM

It is known from estimation theory that vector estimation performance is closely related to the joint FIM of unknown vector elements [34]. If the FIM is invertible, the minimum variances of the estimates are given by the diagonal elements of the inverse matrix of the FIM—the so-called CRB matrix. If the FIM is noninvertible, the joint estimate is ineffective. In this section, we derive the joint FIM of $(r, h)$ and then analyze the estimation performance in the following section.

To derive the joint FIM, we calculate the likelihood function $\Lambda(\tilde{y}(t) | r, h)$ necessarily. Due to the mutual independence of the noise in Assumption 2 and reflection coefficients in Assumption 1, the log-likelihood function is calculated as

$$\ln \Lambda(\tilde{y}(t) | r, h) = Cons + \sum_{m=1}^{M} \sum_{l=1}^{L_m} 2\sigma_{nm}^2 E_m \int_{-\infty}^{+\infty} \tilde{y}_{nm}(t) s_m^*(t - \tau_{ml}) e^{-j(\varphi_{ml})} dt - \int_{-\infty}^{+\infty} f |S_m(f)|^2 df. \tag{23}$$

From the log-likelihood function in (15), the MLE of target location and height is given by

$$\hat{r}, \hat{h} = \arg \max_{r, h} \sum_{m=1}^{M} \sum_{l=1}^{L_m} 2\sigma_{nm}^2 E_m \int_{-\infty}^{+\infty} \tilde{y}_{nm}(t) s_m^*(t - \tau_{ml}) e^{-j(\varphi_{ml})} dt - \int_{-\infty}^{+\infty} f |S_m(f)|^2 df, \tag{16}$$

It can be seen that the coherent processing approach is employed in processing the received signals by $N$ receive antennas due to the same path, while the noncoherent processing approach is employed in processing the signals due to different paths.

Based on the log-likelihood function in (15), the joint FIM of $(r, h)$ is given as

$$\mathbf{F}(r, h) = \sum_{m=1}^{M} \sum_{l=1}^{L_m} \gamma_{ml} \left[ a_{ml}^2 b_{ml}^2 + w_{ml}^2 \psi_{ml} \right]^2, \tag{17}$$

where

$$a_{ml} = \frac{\partial \tau_{ml}}{\partial r} = \frac{1}{c} \left( \frac{\partial R^E_{ml}}{\partial r} + \frac{\partial R^B_{ml}}{\partial r} \right), \tag{18}$$

$$b_{ml} = \frac{\partial \tau_{ml}}{\partial h} = \frac{1}{c} \left( \frac{\partial R^E_{ml}}{\partial h} + \frac{\partial R^B_{ml}}{\partial h} \right), \tag{19}$$

$$w_{ml} = \frac{\partial \rho_{ml}^B}{\partial r}, \tag{20}$$

and

$$\gamma_{ml} = \frac{32\pi^2 N^2 E_m^2 \sigma_{nm}^4}{\sigma_{um}^2 (\sigma_{nm}^2 + 2N E_m \sigma_{ml}^2)}. \tag{22}$$

$$e_{ml} = \int_{-\infty}^{+\infty} \left[ S_m(f) \right]^2 df - \int_{-\infty}^{+\infty} f \left[ S_m(f) \right]^2 df, \tag{23}$$

$$\psi_{ml} = \frac{N^2 - 1}{12} \left( \frac{d \sin \rho_{ml}^B}{\lambda_m} \right)^2. \tag{24}$$

Herein, $S_m(f)$ is the Fourier transform of $s_m(t)$, and $\lambda_m = c/f_m$ denotes the wavelength of the mth signal. (See Appendix A for derivation.)

Path parameters $\partial R^E_{ml} / \partial r, \partial R^B_{ml} / \partial h, \partial \rho_{ml}^B / \partial r$, and $\partial \rho_{ml}^B / \partial h$ in (18–21) can be derived from (1–10), though the expressions are too complicated to present here. Actually, they can be easily evaluated by numerical
methods. Generally $\partial R^\text{F(B)}_n/\partial r$ and $\partial R^\text{F(B)}_n/\partial h$ are positive, while $\partial b^\text{F(B)}_m/\partial r$ and $\partial b^\text{F(B)}_m/\partial h$ are negative. Hence $a_m$ and $b_m$ are positive, and $w_m$ and $v_m$ are negative.

Next, we briefly interpret the meanings of the variables in (22–24). The variable $\gamma_m$ shows the influences of signal-to-noise ratio (SNR) $2E_{\text{am}}/\sigma^2_{\text{am}}$ and receive antenna number $N$. Related to range estimation accuracy, $\varepsilon_m$ is a measure of signal bandwidth, coinciding with the traditional radar theory [34]. The novel variable $\psi_m$ is produced by the receive array in the elevation dimension. It denotes the effect of elevation resolution on range estimation, due to the essential relationship between the elevation angle and the slant/ground range in OTHR.

IV. ESTIMABILITY OF LOCATION AND HEIGHT

The MLE in (16) and the joint FIM $F(r, h)$ in (17) both depend on propagation paths and the receive array. In the following, we analyze the rank of $F(r, h)$ for various paths and antennas, as the indication of feasibility of target location and height estimation. If estimable, the CRBs are presented, and the accuracy is discussed.

The deduction in Appendix B examines the rank of $F(r, h)$ and concludes

$$\text{rank}[F(r, h)] = \min(LN, 2).$$

(25)

Hereby conclusions of estimability are drawn as follows:

1) For multiple paths or antennas $LN > 1$, the joint FIM $F(r, h)$ is full rank and invertible. Target location and height $(r, h)$ are estimable.

2) For single path and single antenna $LN = 1$, the joint FIM $F(r, h)$ is a singular matrix. Target location and height $(r, h)$ are inestimable.

This reveals that target location and height estimability or inestimability depends on the number of paths and antennas in OTHR. In the following, we investigate the estimability, accuracy, and inestimability in detail.

A. Estimable for $LN > 1$

For multiple paths or antennas $LN > 1$, target location and height can be estimated by the MLE in (16). The lower bounds for jointly estimating target location and height are given by

$$\sigma^2_{\text{min}}(\hat{r}) = \frac{\sum_{l=1}^L \gamma_l (b_l^2 \varepsilon_l + \psi^2_l)}{\text{det}[F(r, h)]},$$

(26)

$$\sigma^2_{\text{min}}(\hat{h}) = \frac{\sum_{l=1}^L \gamma_l (a_l^2 \varepsilon_l + w_l^2 \psi_l)}{\text{det}[F(r, h)]},$$

(27)

where

$$\text{det}[F(r, h)] = \frac{1}{2} \sum_{l=1}^L \sum_{\ell=1}^L \gamma_l \gamma_{\ell}$$

$$\times \left[ \varepsilon_l \varepsilon_{\ell} (a_l b_{\ell} - a_{\ell} b_l)^2 + \psi_l \psi_{\ell} (w_l v_{\ell} - w_{\ell} v_l)^2 \right. +$$

$$\left. \psi_l \psi_{\ell} (w_l v_{\ell} - w_{\ell} v_l)^2 + \varepsilon_l \varepsilon_{\ell} (w_l v_{\ell} - w_{\ell} v_l)^2 \right].$$

(28)

and $\ell$ denotes the serial number of the $(m, l)$ path in the multipath sequence, $\ell = \sum_{m'=1}^m L_{m'-1} + l$, for $L_0 = 0$.

There are two sufficient conditions for jointly estimating target location and height. One is multiple paths, and the other is multiple antennas. Multipath in OTHR provides the diversity gain for target localization, like that in a multistatic radar system and non-collocated MIMO radar. For a single path, multiple antennas are necessary for joint estimation. For $L = 1$, $F(r, h)$ is invertible only when $N > 1$ makes $\psi_m > 0$ in (24). In principle, it is the elevation resolution brought by the antennas in the elevation dimension that provides the ability of decoupling target location and height from the single echo.

REMARK There is no clear definition about the threshold of the differences between paths for claiming “two paths are different and the multipath effect is produced.” In the formula of $F(r, h)$ in (17), $(a_m, b_m, w_m, v_m)$ are the path parameters, which change continuously and accordingly as the path varies. There are situations when the parameters for two paths are not equal but very close. In the MQP model, a tiny difference between two carrier frequencies leads to slightly different parameters $(a_m, b_m, w_m, v_m)$. Then, $F(r, h)$ is numerically invertible; however, it gives rise to CRBs that are too huge to be practical. Such situations can be regarded as the “transition region” between multipath and single path. In the transition region, the CRB exists in theory, but it is unrealistic in practice.

Generally, a reasonable multipath for effective estimation is produced under either of the following conditions. 1) At least two paths are through different layers of the ionosphere. 2) If through the same layer, carrier frequencies are different enough to make far different path parameters. We will further simulate this in section V.

B. Discussion on Accuracy Improvement

Next, we investigate the approaches that OTHR can take to improve the estimation for $LN > 1$. The joint location and height CRBs shed light on the accuracy. As the CRB formulas in (26) and (27) are complicated, we turn back to the joint FIM in (17). Before the discussion, a useful lemma is introduced.

**Lemma 1** Given a $2 \times 2$ symmetric positive definite (SPD) matrix

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{12} & F_{22} \end{bmatrix},$$

(29)

and a $2 \times 2$ symmetric nonnegative definite (SND) matrix

$$F' = \begin{bmatrix} F'_{11} & F'_{12} \\ F'_{12} & F'_{22} \end{bmatrix},$$

(30)

for which the diagonal elements are positive $F'_{11} > 0$, $F'_{22} > 0$, the diagonal elements of the inverse matrix $F^{-1}$ are not less than the corresponding diagonal
elements of $(F + F')^{-1}$, i.e.,
\[ [F^{-1}]_{11} \geq [(F + F')^{-1}]_{11}, \]  
(31)
\[ [F^{-1}]_{22} \geq [(F + F')^{-1}]_{22}. \]  
(32)
Equation (31) holds if and only if $\text{det}(F') = 0$, and
\[ F_{22}F'_{12} = F'_{22}F_{12}. \]  
(33)
Equation (32) holds if and only if $\text{det}(F') = 0$, and
\[ F_{11}F'_{12} = F'_{11}F_{12}. \]  
(34)

**Proof:** See Appendix C.

**Remark:** For the FIM in (17), the matrix inside the summation is SND (SPD for $N > 1$). An increase of any one variable $L$, $\varepsilon_{ml}$, $\gamma_{ml}$, or $\psi_{ml}$ produces an equivalent effect of adding an SND matrix with positive diagonal elements to the original FIM. By Lemma 1, the diagonal elements of the inverse matrix, i.e., the joint CRBs, are decreased (the equation conditions are not taken into consideration due to their rather low probability in practice).

Next, we discuss the approaches for accuracy improvement, roughly classified in three aspects.

1) Traditional factors of a radar system, including transmit energy $E_m$, signal bandwidth $\varepsilon_{ml}$, antenna number $N$, and interval $d$. Due to $\gamma_{ml}$ being an increasing function of $E_m$ and $N$ in (22), and $\psi_{ml}$ being an increasing function of $N$ and $d$ in (24), it is concluded that increasing $E_m$, $N$, or $d$ decreases the CRBs. Hence, traditional approaches are still effective for better estimation of target location and height in OTHR, such as increasing transmit energy $E_m$ and bandwidth $\varepsilon_{ml}$. The accuracy can also be improved by enlarging the array aperture $d \times N$ with higher elevation resolution. As the interval $d$ merely affects array aperture, the number $N$ also contributes to noncoherent SNR.

2) Multipath and MIMO technique. Each sub-FIM of path $(m, l)$ is SND (SPD for $N > 1$), so that the CRBs are decreased by growing $L$, owing to Lemma 1. There are two ways to improve multipath $L$. The first is to increase the transmitting signal number $M$ by the MIMO technique. Signals of various carrier frequencies are used to explore the potential of multipath propagation through the ionosphere. The second is to increase the employed path number $L_m$ instead of abandoning multipath signals, at a price of more complicated signal processing compared to that of single path in conventional OTHR.

3) Carrier frequency selection. Carrier frequency $f_m$ determines not only the path number $L_m$, but also path parameters $(a_{ml}, b_{ml}, w_{ml}, v_{ml})$, which are closely related to the CRBs. In addition, the echo average amplitude $\sigma_{ml}$ and noise covariance $\sigma_{um}$ in each path depend on $f_m$ too. As the carrier frequency influences the CRBs through multiple factors that are related to the real-time environment state, it is hard to analyze the optimal carrier frequency theoretically. Herein, we propose to select carrier frequency via a numeric method for accuracy improvement. First, the CRBs of various carrier frequencies are calculated by (26) and (27), based on the real-time ionosphere state, surveillance data, and interested area. Then, we select the optimal carrier frequency of the least CRBs, which is supposed to achieve the best accuracy. As the signal number in OTHR is not large, a numeric method will be efficient. The calculation and selection can be included in the process of the frequency management system (FMS) during OTHR operation.

C. Inestimable for $LN = 1$

For single path and single antenna $LN = 1$, $F(r, h)$ is nonlinear, and the MLE in (16) does not work. It denotes the disability of decoupling $r$ and $h$ from the time delay of a single path without any elevation information. This is the situation that conventional OTHR usually faces.

However, in signal processing, we can take a substitutive method instead of the MLE in (16). Suppose that by some prior information, the “predicted” target height is $h_0$ (e.g., $h_0 = 0$ at sea surface, which may be the target’s real height, not for sure). Then, the substitutive location estimate is modified as
\[ \hat{r}_0 = \arg \max_r \frac{2 \sigma_{11}^2 E_1 \cdot \int \tilde{y}_{11}(t) s_1^1(t - \tau_{11}) dt |^2}{\sigma_{a1}^2 (\sigma_{a1}^2 + 2 E_1 \sigma_{11}^2)} \bigg|_{h = h_0} \]
(35)
that is, the target location is estimated based on the assumed height $h_0$. The location estimate $\hat{r}_0$ in (35) and the predicted height $h_0$ constitute an estimated state $(\hat{r}_0, h_0)$, which is expected to achieve the same time delay as the real state $(r, h)$ basically. Specially, if the real height coincides with the “predicted height” $h_0$, then $\hat{r}_0$ is efficient. The corresponding CRB is easy to derive, though it is not represented here. Otherwise, $\hat{r}_0$ is biased for $h \neq h_0$.

It is not surprising to see a supposed target height in OTHR. For existing phased-array OTHR with a 1D array, conventional signal processing methods estimate target location despite the height factor, or alternately speaking, under zero height by default. This simplistic consideration works in practice and leads to no degradation of target detection performance. Here, our analysis provides theoretical support for the reasonability of the zero-height assumption in conventional OTHR.

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1 Average amplitude $\sigma_{ml}$ can refer to the backscattering ionogram from the backscatter soundings. A conventional 1D array may valuate $\sigma_{ml}$ approximately, while a 2D array can provide an accurate estimate of $\sigma_{ml}$ since its receive beamforming suppresses other multimode signals [14]. Noise covariance $\sigma_{um}$ can be read by spectral surveillance of the occupancy of the channels and the level of noise.
V. SIMULATION AND PERFORMANCE ANALYSIS

This section demonstrates the joint MLE of the target state and investigates the CRBs by numeric simulations. Based on the set MQP parameters, the root mean-square-error (RMSE) of the MLE is simulated, compared to the root CRB (RCRB). Then, we illustrate the dependence of the RCRB on the receive array and multipath, as well as selection of carrier frequencies.

A. The MQP Model and MLE Performance

First of all, ionosphere parameters are set as follows: the number of layers \( I = 3 \), where \( i = 1 \) denotes the layer \( E \), and \( f_{E1} = 3.7 \) MHz, \( z_1 = z_0 + 115 \) km, and \( z_{h1} = 15 \) km; \( i = 3 \) denotes the layer \( F2 \), and \( f_{E3} = 12.7 \) MHz, \( z_3 = z_0 + 310 \) km, and \( z_{h3} = 100 \) km; \( i = 2 \) denotes the joint layer between the layers \( E \) and \( F2 \); and \( z_0 = 6743 \) km for the beginning of the free space region.

Transmitting waveforms are the most widely used linear frequency modulated continuous waveform (LFMCW) modulated on various carrier frequencies

\[
s_m(t) = \frac{1}{\sqrt{P T}} \sum_{p=0}^{P-1} \text{rect} \left( \frac{t - p T}{T} \right) s_0(t - p T) \exp(j 2 \pi f_m t), \tag{36}
\]

where \( s_0(t) = \exp(j \pi B_T^2 T) \) for \( 0 < t < T \), \( \text{rect}(\cdot) \) denotes a rectangular window, and \( \text{rect}(t) = 1 \) for \( t \in (0, 1) \) and otherwise \( \text{rect}(t) = 0 \). In addition, \( B \), \( T \), and \( P \) denote the bandwidth, period length, and number, respectively, set as \( B = 20 \) kHz, \( T = 0.02 \) s, and \( P = 5 \) in the following simulations. A target is located at ground range \( r = 1500 \) km and height \( h = 20 \) km. The variances of reflection coefficients in multipath are assumed to be the same, \( 2 \sigma_{ml}^2 = 1 \), and the transmit energy of each waveform is \( E_m = 1 \). Noise covariance is set uniformly \( \sigma_{im}^2 = \sigma_{m}^2 \) for various carrier frequencies. We vary \( \sigma_{m}^2 \) to set the SNR in each receiver.

The relationship between the slant/ground range and elevation angle for a target with an altitude is calculated by the MQP model functions (1–10), via numeric methods, e.g., the secant method used here. Fig. 2 depicts the slant/ground range corresponding to an elevation of \( 0^\circ \sim 35^\circ \) for a target with an altitude of \( 0 \) km (sea surface) or 20 km, at carrier frequency 20 MHz or 25 MHz. It can be seen that there is one forward/backward propagation for 25 MHz to reach a ground range of 1500 km at elevation 7.6° for 0 km and 6.1° for 20 km (the rising part is ignored). Contrarily, there are two forward/backward propagations for 20 MHz, at elevations of \( 7^\circ \) and \( 31.6^\circ \) for 0 km, and \( 5.5^\circ \) and \( 31.4^\circ \) for 20 km. Thus, 25 MHz produces a single path \( L = 1 \), and 20 MHz produces a multipath \( L = 4 \).

The performance of the location and height MLE in (16) versus SNR is investigated by employing the waveform in (36) as a transmit signal, at carrier frequency 20 MHz or 25 MHz, and \( M = 1 \). The RMSE of 1000 Monte Carlo simulations is drawn in Fig. 3, compared with the RCRB, for \( N = 200 \), and \( d = 10 \) m. We can see that the RMSE is close to the RCRB for \( \text{SNR} \geq 0 \) dB. In the low SNR region, the location RMSE is much greater than the location RCRB, and the height RMSE is limited by the height search range \( 0 \sim 40 \) km. The advantages of multipath are shown in two points. First, it can be seen that the location/height RCRB of 20 MHz is less than the corresponding RCRB of 25 MHz at the same SNR. Second, the RMSE approximates the RCRB at \( \text{SNR} = 10 \) dB of 20 MHz, i.e., lower than \( \text{SNR} = 0 \) dB of 25 MHz. It is worth noting that the CRBs seem relatively large, because the radar system parameters are evaluated relatively small, as limited by our simulation platform. For instance, the practical coherent integral time may exceed 5 s, and bandwidth may be over 100 kHz.

B. Investigation on Receive Array

In the following, we simulate the effects of a receive array on the location and height RCRB for multipath or single path. The effect of antenna number \( N \) is investigated in Fig. 4, for \( \text{SNR} = 0 \) dB, \( d = 10 \) m, and \( M = 1 \), at carrier
Fig. 4. Root CRB of target location and height versus number of receive antennas $N$, by carrier frequency 20 MHz or 25 MHz, for $M = 1$, $d = 10$ m, and SNR = 0 dB.

Fig. 5. Root CRB of target location and height versus adjacent receive antennas interval $d$, by carrier frequency 20 MHz or 25 MHz, for $M = 1$, $N = 40$, and SNR = 0 dB.

Fig. 6. Root CRB of target location and height versus carrier frequency $f_1$ employing one signal $s_1(t)$ in (36) for $M = 1$, $N = 200$, $d = 10$ m, and SNR = 0 dB.

frequency 20 MHz or 25 MHz. We can see that the RCRB decreases as $N$ grows for both 20 MHz and 25 MHz, at different speeds. When $N$ is small, the RCRB of 20 MHz is much lower than the RCRB of 25 MHz. As $N$ grows, the speeds of RCRB decline are about $1/\sqrt{N}$ for 20 MHz, and $1/(2\sqrt{N})$ for 25 MHz, due to the different significance of $\psi_{ml}$ in (24) for the location and height CRB in multipath and single path. The RCRB of 25 MHz catches up with that of 25 MHz as $N$ grows in Fig. 4. However, it is found that the two speeds exchange with each other when $N$ exceeds a threshold $2^{12}$, a value so huge and unrealistic that the simulations are not shown here.

The influence of antenna interval $d$ on target location and height RCRB is depicted in Fig. 5, for SNR = 0 dB, $N = 40$, $M = 1$, by the waveform in (36) at carrier frequency 20 MHz or 25 MHz. It can be seen that the RCRB is almost not affected by $d$ for multipath at 20 MHz, while it deceases obviously for single path at 25 MHz. This shows the different significance of $\psi_{ml}$ to the CRB for multipath and singe path. The RCRB is much lower in 20 MHz than 25 MHz at small $d$, and it catches up as $d$ grows, similar to $N$ in Fig. 4.

The simulations demonstrate that the receive array plays different roles in estimation accuracy for multipath and single path. In the case of multipath, greater array aperture (with fixed $N$) produces little gain on estimation accuracy. However, for single path, enlarging $d$ is helpful to improve the accuracy. In both cases, increasing the number of antennas is an effective way to improve accuracy.

C. Investigation on Carrier Frequency and Multipath

The joint CRB also depends on path parameters that are determined by the ionosphere and signal carrier frequencies. Next, we investigate various situations of carrier frequencies and paths, for single or multiple antennas.

In the case of multiple antennas, we set $N = 200$ and $d = 10$ m, so that target location and height are estimable even for single path. OTHR transmits only one waveform $s_1(t)$ as formulated in (36), under the same ionosphere parameters as set in section VA. The location and height RCRBs are depicted in Fig. 6 as $f_1$ varies from 5 to 28 MHz, for SNR = 0 dB. We can see that the RCRB behaves differently in three stages. In the lower-frequency region (about 5–14 MHz), both the location and height RCRBs decrease slowly as $f_1$ grows. In the higher-frequency region (about 22.5–28 MHz) for growing $f_1$, the height RCRB decreases still, while the location RCRB nearly remains unchanged. There is only one path in both regions. The middle-frequency region, where multipath effect is produced, produces the lowest RCRB for the whole band. Note that there are some singular points where path parameters hop, such as the two minimums at 17.5 MHz and 21.5 MHz (18 MHz is regular with single path). The reason is that the partial derivative $\partial \tau_{ml}/\partial h$ is extremely large in some areas (which may be due to bugs in MQP model; the minimums need further demonstration by real data). Herein, if OTHR
Fig. 7. Joint CRB by employing two signals $s_1(t)$ and $s_2(t)$ in (36) at carrier frequency combination $(f_1, f_2)$, for $M = 2$, $N = 1$, and SNR = 0 dB: (a) location RCRB, (b) height RCRB.

needs to illuminate a target at location 1500 km and height 20 km by only one signal and expects better accuracy, the optimal carrier frequency is within 14.5–17 MHz.

In the case of a single antenna $N = 1$, OTHR can estimate target location and height jointly only when $L > 1$, as analyzed before. To guarantee the multipath effect, OTHR transmits two signals at carrier frequencies $f_1$ and $f_2$ varying independently in 5–28 MHz. Other parameters are set the same as the previous subsection. The location and height RCRBs versus combinations of $[f_1, f_2]$ are depicted in Fig. 7, for SNR = 0 dB. It is obvious that the location and height RCRBs depend on the carrier frequency combination significantly.

In detail, there are three different regions. The first is the transition region, where the RCRB does not exist or is huge and unpractical, mainly when $f_1$ and $f_2$ belong to the same band 5–14.5 MHz, 17.7–18.4 MHz, and 22.1–28 MHz. When $f_1 \approx f_2$ produces one path each, the path parameters of one signal are the same or close to the other path, leading to singular FIM. The second region achieves the best RCRB and suboptimal RCRB when both signals or only one signal produces multipath, respectively. In the last region, where two carrier frequencies differ greatly, and each signal produces single path, the RCRB retains in an available level, larger than that of multipath by about 14%–28%. It is worth noting that the minimum height RCRB appears in 17.3–17.7 MHz and 21.5–21.9 MHz in Fig. 7b, corresponding to the two minimums at 17.5 MHz and 21.5 MHz in Fig. 6.

D. Comments on Practical Application

The simulation results demonstrate that OTHR can effectively estimate target location and height jointly, as long as it satisfies either of two sufficient conditions. It may employ multiple antennas in the elevation dimension, or, alternatively, it may transmit one signal with multiple paths or multiple signals with quite different carrier frequencies. The last method shows the significance of the MIMO technique to produce the multipath effect.

OTHR can draw lessons from the analysis and investigations herein. First, sufficient conditions for effective estimation provide necessary guides for the OTHR system and array construction. Researchers can evaluate the probability of the multipath effect for a single signal in routine operation and then decide whether a 2D array or the MIMO technique is necessary. Take the simulated situation for example. If the multipath is satisfied in regular missions (such as location 1500 km and frequency 20 MHz), then OTHR may not need to employ a 2D array or the MIMO technique. Otherwise, OTHR has to decide whether to build a 2D array or update the radar system to use the MIMO technique. The joint CRBs in (26) and (27) reflect the estimation accuracy, giving guidance on the details of the 2D array, MIMO technique, transmit power and bandwidth, etc.

Second, accuracy analysis is instructive for radar operation. Based on the joint CRBs, OTHR can choose to operate in an appropriate mode for better accuracy. After the ionosphere parameters and interested area are obtained in real time, OTHR needs to decide the carrier frequencies of transmit signals, which are crucial for propagation paths. Ray paths are predicted by (1–10). The location CRB $\sigma_{\min}^2(\hat{r})$ and height CRB $\sigma_{\min}^2(\hat{h})$ are calculated by (26) and (27) for various carrier frequencies. Then, optimal carrier frequencies are determined according to OTHR’s overall consideration of $\sigma_{\min}^2(\hat{r})$ and $\sigma_{\min}^2(\hat{h})$. Fig. 7 can be considered as a simple example for $M = 2$, assuming that both signal amplitude and noise covariance are uniform for various carrier frequencies.

VI. EXTENSION TO VELOCITY (SPEED AND HEADING)

In the previous sections, we have studied the target location and height estimation. Here, we extend the study...
to velocity, still in the 2D elevation plane. The velocity information includes two aspects: speed and heading.

A. Signal Model and Joint FIM

Consider a target at location \( r \) and height \( h \) that is moving with a constant speed \( v \) and heading direction angle \( \alpha \). The relationships of target state and incident and reflection paths are depicted in Fig. 8.

The time delay corresponding to the \( l \)th path of \( s_m(t) \) is given by (6), and the Doppler shift is given by

\[
f_{d,ml} = f_m v \left[ \cos(\alpha - \beta_{ml}^E) + \cos(\alpha - \beta_{ml}^B) \right] / c. \tag{37}\]

Under Assumptions 1–2, the received signal by the \( n \)th receiver due to the \( m \)th antenna moving with a constant speed \( v \) given by (6), and the Doppler shift is given by

\[
y_{nm}^V(t) = \sum_{l=1}^{L_n} \xi_{ml} \sqrt{E_m} s_m(t - \tau_{ml}) e^{j2\pi f_{d,ml} t} e^{j(\alpha - \beta_{ml})} + u_{nm}(t). \tag{38}\]

There are four unknown parameters to be estimated, i.e., target location, height, speed, and heading, listed in a vector

\[
\theta = [r, h, v, \alpha]. \tag{39}\]

By derivations similar to section III, the joint MLE of \( \theta \) is given by

\[
\hat{\theta} = \arg \max_{\theta} \sum_{m=1}^{M} \sum_{l=1}^{L_n} \frac{2\sigma_m^2}{\sigma_m^2 + 2NE_m \sigma_{ml}^2} \left( \int_{-\infty}^{\infty} y_{nm}^V(t) s_m^* (t - \tau_{ml}) e^{-j2\pi f_{d,ml} t} e^{-j(\alpha - \beta_{ml})} dt \right)^2,
\tag{40}\]

\[
F^V(\theta) = \sum_{m=1}^{M} \sum_{l=1}^{L_n} [F_m^V(\theta)]_{ij} = \begin{bmatrix}
[F_m^V(\theta)]_{11} & [F_m^V(\theta)]_{12} & \sigma_m^2 & \sigma_m^2 \\
[F_m^V(\theta)]_{12} & [F_m^V(\theta)]_{22} & \sigma_m^2 & \sigma_m^2
\end{bmatrix}
\times
\begin{bmatrix}
p_{ml}(\rho_m + \rho_{ml} + k_m \eta_m + \rho_{ml} \eta_m) & q_{ml}(\rho_m + \rho_{ml} + k_m \eta_m + \rho_{ml} \eta_m) \\
p_{ml}(\rho_m + \rho_{ml} + k_m \eta_m + \rho_{ml} \eta_m) & q_{ml}(\rho_m + \rho_{ml} + k_m \eta_m + \rho_{ml} \eta_m)
\end{bmatrix},
\tag{41}\]

\[
[F_m^V(\theta)]_{11} = a_{ml}^2 \epsilon_{ml} + 2a_{ml} \epsilon_{ml} \rho_{ml} + a_{ml}^2 \eta_{ml} + w_{ml}^2 \eta_{ml},
\tag{42}\]

\[
[F_m^V(\theta)]_{12} = a_{ml}(b_{ml} \epsilon_{ml} + k_m \rho_{ml} + b_{ml} \eta_{ml}) + a_{ml} \epsilon_{ml} (b_{ml} \rho_{ml} + k_m \eta_{ml}) + w_{ml} v_{ml} \psi_{ml},
\tag{43}\]

and \( \Im \{ \cdot \} \) denotes the imaginary operator.
B. Investigation on Estimability

Detailed analysis on the rank of $F^V(\theta)$ is not presented here for brevity. The conclusions on estimability are drawn as follows.

1) For multipath $L > 1$, the FIM $F^V(\theta)$ is full rank, $[F^V(\theta)] = 4$. Target location $r$, height $h$, speed $v$, and heading $\alpha$ are estimable by the MLE in (40), and the CRBs are given by the diagonal elements of the inverse matrix of $F^V(\theta)$.

2) For single path $L = 1$, the FIM $F^V(\theta)$ is singular. Thus, target parameters $(r, h, v, \alpha)$ are inestimable. Specially, rank $[F^V(\theta)] = 3$ for $N > 1$, rank $[F^V(\theta)] = 2$ for $N = 1$.

These conclusions indicate that multipath is essential for the estimability of target speed and heading. When OTHR illuminates a target from various directions by multiple paths, spatial diversity is brought so that target speed and heading are estimable. Otherwise, for the case of single path, the echo carries the information of a Doppler shift due to the radial velocity as the projection of target speed and heading $(v, \alpha)$. However, the real speed and heading cannot be resolved from the echo, even when elevation information is provided. Elevation information is unhelpful for estimating target speed and heading in single path (though useful for estimating target location and height). The joint MLE in (40) fails for single path, as a series of combinations of speed and heading evaluation can maximize the log-likelihood function. Meanwhile, location and height estimating still works, as analyzed in section IVA.

In the worst case for a single path and single antenna $LN = 1$, all the target parameters are inestimable. Instead, we can take a substitutive method, similar to section IVC. Two parameters are needed to be supposed by prior information. For example, consider the supposed height $h_0$ and heading $\alpha_0$, the parameters left to be estimated are $(r, v)$. The substitutive MLE is modified as

$$
(\hat{r}, \hat{v}) = \arg \max_{r, v} \int_{-\infty}^{\infty} \left. \mathcal{L}^V(t) \mathcal{L}^F(t - \tau_{11}) e^{-j2\pi f_s t} dt \right|_{h = h_0, \alpha = \alpha_0},
$$

(51)

The estimate $(\hat{r}, \hat{v})$ in (51) and supposed $(h_0, \alpha_0)$ form the estimated state $(\hat{r}_0, h_0, \hat{v}_0, \alpha_0)$, which is expected to achieve the same time delay and Doppler shift as the real state $(r, h, v, \alpha)$ basically.

The consideration of assuming height and heading makes sense in practical operation. For example, for a vessel in the sea, the height and heading angle are both zero. A plane has its frequent cruising altitude and generally flies parallel to the horizon, except in maneuvering stages.

C. Simulation on the MLE

Numeric simulations are employed to demonstrate the joint estimation of target location, height, speed, and heading under the multipath effect. The ionosphere parameters are set the same as in section VA, and the transmit waveform is given in (36) at carrier frequency 20 MHz, $M = 1$, and $L = 4$. Target parameters are set $(r, h, v, \alpha) = (1500 \text{ km}, 20 \text{ km}, 100 \text{ m/s}, 10^\circ)$. Receive data are simulated by the signal model in (38) for various SNR values. The MLE in (40) is employed for jointly estimating target state. The RMSE and RCRB for the four parameters are depicted in Fig. 9, for $N = 1$. It can be seen that the RMSE is close to the RCRB for SNR $\geq 12.5 \text{ dB}$. In the low SNR region, the height and heading RMSE values are lower than the RCRB, because of the limited search range. It is demonstrated that OTHR is able to estimate target location, height, speed, and heading jointly under the multipath effect, and it does not require multiple antennas necessarily.

VII. CONCLUSION

This paper has addressed the joint estimation of target location and height in OTHR by employing the diversity of multipath signal and structure of a 2D array. By determining the joint FIM, it is revealed that an efficient estimation can be obtained provided that the diversity of multipath signals or the structure of a 2D array can be employed. The multipath effect may be produced by transmitting a signal that experiences multipath propagation. However, such a favorite carrier frequency may not exist. As an alternative, the MIMO technique can be employed to produce the multipath effect by transmitting multiple signals at the same instant. We also analyze the performance improvement based on the joint CRBs. Several points of advice are proposed for the OTHR construction and operation.
To derive the joint FIM $F(\theta)$, we define an intermediate parameter vector
\[
\theta = (\tau_{11}, \ldots, \tau_{MLu}, \beta_{11}^B, \ldots, \beta_{MLu}^B).
\] (52)

Observe that location $r$ and height $h$ are the unique indexes of target state, while the time delays $\tau_{ml}$ and elevation angles $\beta_{ml}^B$ in $\theta$ are regarded as the target's projections onto multiple paths. Using the chain rule, the joint FIM is given by
\[
F(r, h) = \nabla \theta \cdot F(\theta) \cdot [\nabla \theta]^T,
\] (53)
where
\[
\nabla \theta = \begin{bmatrix}
\frac{\partial \tau_{11}}{\partial r} & \ldots & \frac{\partial \tau_{MLu}}{\partial r} & \frac{\partial \beta_{11}^B}{\partial h} & \ldots & \frac{\partial \beta_{MLu}^B}{\partial h}
\end{bmatrix},
\] (54)
which represents the differentials of $\theta$ with respect to $(r, h)$, for which the elements are computed in (18–21).

By the signal model in (11) under Assumptions 1–2, $F(\theta)$ is derived after lengthy calculations, partitioned as
\[
F(\theta) = \begin{bmatrix}
F_{rr} & \Omega_{rp} \\
\Omega_{rp}^T & F_{\beta \beta}
\end{bmatrix},
\] (55)
where $\Omega_{rp}$ denotes an $L \times L$ null matrix, and
\[
F_{rr} = \text{diag}[\gamma_{11} \psi_{11}, \ldots, \gamma_{MLu} \psi_{MLu}],
\] (56)
\[
F_{\beta \beta} = \text{diag}[\gamma_{11} \psi_{11}, \ldots, \gamma_{MLu} \psi_{MLu}],
\] (57)
with elements listed in (22–24). Plug formulas (54–57) and (18–24) into (53), then $F(r, h)$ is obtained by (17).

### APPENDIX A. DERIVATION OF THE FIM IN (17)

The rank of the FIM $F(r, h)$ is analyzed based on the formula (53). The elements of $\nabla \theta$ represent the relationship of the ground/slant ranges to elevation angles in various paths. Basically, for an arbitrary path $(m, l)$, vectors $[\frac{\partial \tau_{ml}}{\partial r}, \frac{\partial \beta_{ml}^B}{\partial h}]$ are not linearly dependent. Thus, $\nabla \theta$ is full rank, i.e., rank $\nabla \theta$. In the following, we discuss the rank of $F(r, h)$ in two cases, according to the number of receive antennas.

In the case of multiple receive antennas $N > 1$, for $\psi_{ml} > 0$ in (24), $F(\theta)$ in (55) is full rank, $[F(\theta)] = 2L$. Since the product of full rank matrices is a full rank matrix, it is inferred that $F(r, h)$ is full rank, i.e., rank $F(r, h) = 2L$.

In the case of a single receive antenna $N = 1$, due to $\psi_{ml} = 0$ in (24), the formula of $F(r, h)$ in (53) can be further simplified as
\[
F(r, h) = (\nabla \theta)_{L \times L} \cdot F_{rr} \cdot (\nabla \theta)^T_{L \times L},
\] (58)
where $F_{rr}$ is full rank, rank $[F_{rr}] = L$. For various paths, the differentials of time delays to $r$ or $h$ in $[\nabla \theta]_{L \times L}$ are linearly independent generally, hereby $\text{rank}([\nabla \theta]_{L \times L}) = \text{min}(L, 2)$. For multipath $L > 1$, both $[\nabla \theta]_{L \times L}$ and $F_{rr}$ are full rank, so that $F(r, h)$ is full rank. For single path $L = 1$, rank $[F_{rr}] = 1$, rank $([\nabla \theta]_{L \times L}) = 1$, and so rank $F(r, h) = 1$.

Summarizing the various situations of paths and antennas, it is concluded that:
\[
\text{rank } [F(r, h)] = \min(LN, 2).
\] (59)
This implies that $F(r, h)$ is full rank as long as there is multipath or multiple antennas alternately.

### APPENDIX C. PROOF OF LEMMA 1

**Proof** The sum matrix is represented as
\[
F'' = F + F' = \begin{bmatrix}
F_{11} + F'_{11} & F_{12} + F'_{12} \\
F_{12} + F'_{12} & F_{22} + F''_{22}
\end{bmatrix}.
\] (60)

Since the sum of an SPD matrix and an SND matrix is an SPD matrix, $F''$ is SPD and invertible. Then, we compare the diagonal elements of inverse matrices $F^{-1}$ and $F''^{-1}$. Without loss of generality, the differences of the $(1,1)$th elements are computed
\[
D_{11} = [F^{-1}]_{11} - [F''^{-1}]_{11}
\] (61)
\[
= \frac{F_{22} \cdot \det(F'') - (F_{22} + F''_{22}) \cdot \det(F)}{\det(F) \cdot \det(F''')}
\] (62)
We are interested in the sign of $D_{11}$. Due to the property of an SPD matrix, we have $\det(F) > 0$, and $\det(F'') > 0$, so the denominator of $D_{11}$ is positive. By $F_{22} > 0$ and $F'_{22} > 0$, the numerator of $D_{11}$ is rewritten as
\[
F_{22} \cdot \det(F') - (F_{22} + F''_{22}) \cdot \det(F')
\]
\[
= \frac{1}{F_{22}}(F_{22}^2 F_{11}^2 - 2F_{22} F_{12} F_{12}^2 + F_{22}^2 + F_{12}^2) + F_{22} \cdot \det(F')
\]
\[
\geq \frac{1}{F_{22}}(F_{22}^2 F_{11}^2 - 2F_{22} F_{12} F_{12}^2 + F_{22}^2 F_{12}^2)
\] (63)
\[
= \frac{1}{F_{22}}(F_{22}^2 F_{12}^2 - F_{22}^2 F_{12}^2)
\] (64)
where (63) holds for $\det(F') = F_{22} F_{11}^2 - F_{12}^2 = 0$, and (64) holds for $F_{22} F_{12}^2 - F_{22}^2 F_{12}^2 = 0$.

Since the denominator of $D_{11}$ is positive and the numerator is nonnegative, we have $D_{11} \geq 0$; i.e., $[F^{-1}]_{11} \geq [F''^{-1}]_{11}$, where equality holds if and only if $\det(F') = 0$, and
\[
F_{22} F_{12} = F'_{22} F_{12}.
\] (65)

Similarly, the difference of the $(2,2)$th elements of $F^{-1}$ and $F''^{-1}$ is computed, with $D_{22} = [F^{-1}]_{22} - [F''^{-1}]_{22} \geq 0$, i.e., $[F^{-1}]_{22} \geq ([F + F']^{-1})_{22}$, where the equality holds if and only if $\det(F') = 0$, and
\[
F_{11} F_{12} = F'_{11} F_{12}.
\] (66)
Hereby the proof is concluded.
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