Parameter Identification of the Jiles–Atherton Hysteresis Model Using Differential Evolution

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In this paper, parameters of the Jiles–Atherton (J-A) hysteresis model are identified using a stochastic search algorithm called differential evolution (DE). The J-A hysteresis model’s parameters are identified by DE in such a way, that best possible agreement is obtained between the measured and model calculated hysteresis loops. This agreement is furthermore increased by improving the J-A hysteresis model. The improvement is achieved by replacing a constant pinning parameter in the J-A hysteresis model with a variable one. Here, the variable pinning parameter is written as a function of a magnetic field. By DE identified parameters are used in the J-A hysteresis model, which is included in the dynamic model of a single-phase transformer. The effectiveness of the improved J-A hysteresis model and parameters identification approach is verified with experiments and simulations.

Index Terms—Jiles–Atherton model, magnetic hysteresis, optimization methods, parameters estimation.

I. INTRODUCTION

TRANSFORMERS and inductors are essential components over a wide variety of power system applications. When these devices are modeled and used in numerical calculations, accurate representation of the nonlinear behavior of the iron core is essential for achieving good agreement between calculated and measured variables. Univocal function is often used to represent nonlinear behavior of these devices. However, the hysteresis phenomenon is neglected in this case. A model, which takes the hysteresis phenomenon into account must be used to improve the accuracy.

Different models have been proposed to represent the hysteresis phenomenon. These models are basically grouped into two categories. In first category, mathematical models can be found [1], which ignore the physics of material behavior, and second category presents models based on physical considerations. In recent papers Preisach [2], [3] and Jiles–Atherton (J-A) [4], [5] hysteresis models from the second category have often been used. Since the J-A model is simpler for implementation and requires less computational effort, the J-A over the Preisach model was selected for this paper.

Regardless of model selection, the selected hysteresis model’s parameters must be correctly identified to achieve an accurate representation of the iron core’s nonlinear behavior. Since the J-A hysteresis model is based on physical assumptions of material behavior, it is natural to obtain parameters from the physical concepts of this behavior. This methodology was first proposed in [6], where parameters were obtained from measured hysteresis loops. Several similar methodologies followed including [7] and [8]. Other authors have proposed the use of optimization algorithms such as genetic algorithm (GA) [9], simulated annealing [10], and combinations of different optimization algorithms [11]. Among these algorithms, GA has been used more often than others to evaluate the parameters of a J-A hysteresis model.

After an initial population has been randomly generated, GA evolves thorough three main operators: selection; which simulates natural survival phenomena, crossover; which represents natural reproduction, and mutation; which introduces random modifications in genes. Due to the probabilistic transition rules employed by operators, GA is known for its robustness and applicability in multiobjective problems. However, in regard to practical problems, GA often has difficulties finding the global minimum due to premature convergence [9]. In order to overcome this disadvantage, differential evolution (DE) was introduced by Storn and Price as first described in [12]. DE, like GA, is a population based algorithm which uses similar operators: selection, crossover, and mutation. In literature, there are only a few studies where DE was used for parameters identification of different mathematical models. Since DE has proven to be fast and accurate in finding optimal solutions in these cases, it has been tested in this work in order to identify the parameters of the J-A hysteresis model. The goal of the optimization algorithm is to vary the parameters in such a way as to achieve best agreement between measured and J-A model calculated hysteresis loops.

Fitting accuracy depends on the selected optimization algorithm and, to a greater extent, on a detailed description of the J-A hysteresis model. An important component of this model is loss parameter or so-called pinning parameter, which is energy related to the domain walls’ motion in the presence of pinning sites. Detailed analysis of the measured hysteresis loops shows, that the pinning parameter depends on the amplitude of the magnetic field, or the magnetic field density. This was not taken into account with the original J-A hysteresis model [13], where the value of the pinning parameter was considered as constant.

This paper introduces a modified J-A hysteresis model, where the pinning parameter is considered to be magnetic field dependent. Variable pinning parameter enables better representation of the iron core’s nonlinear behavior and, therefore, better agreement between the measured and model calculated hysteresis loops. The better accuracy, offered by the variable pinning parameter can only be achieved if all parameters are identified using an optimization algorithm. In this paper, DE was used to identify the parameters of the J-A hysteresis model. By using
DE identified parameters, both the original and modified J-A hysteresis models were confirmed through comparing the calculated and measured hysteresis loops. The modified J-A hysteresis model was tested by the inclusion of this model into a dynamic model of a single-phase transformer. Very good agreement between currents is achieved through a comparison between measured and calculated currents. This signals a good reason for using DE for parameters identification, and the proposed modified J-A hysteresis model for numerical simulations.

II. DYNAMIC MODEL OF A SINGLE-PHASE TRANSFORMER AND JILES–ATHERTON HYSTERESIS MODEL

The dynamic model of a single-phase transformer is described with voltage equations (1)

\[
\begin{align*}
  u_1 &= i_1 R_1 + \frac{d\psi_1}{dt} = i_1 R_1 + L_{\sigma_1} \frac{di_1}{dt} + N_1 \frac{d\phi}{dt} \\
  u_2 &= i_2 R_2 + \frac{d\psi_2}{dt} = i_2 R_2 + L_{\sigma_2} \frac{di_2}{dt} + N_2 \frac{d\phi}{dt}
\end{align*}
\]

(1)

where \(u_1, u_2, i_1, i_2\) are the primary and secondary voltages and currents, \(R_1\) and \(R_2\) are the primary and secondary resistances, \(\psi_1\) and \(\psi_2\) are the primary and secondary flux linkages, \(L_{\sigma_1}\) and \(L_{\sigma_2}\) are the primary and secondary leakage inductances, \(N_1\) and \(N_2\) are the primary and secondary number of turns, and \(\phi\) is the magnetic flux. Magnetic flux \(\phi\) is magnetomotive force (MMF) dependent, which is defined as \(\theta = N_1 i_1 + N_2 i_2\). Partial derivative of the magnetic flux \(\phi\) is given by

\[
\frac{d\phi}{dt} = \frac{d\phi}{d\theta} \frac{d\theta}{dt} = \frac{d\phi}{d\theta} \left( N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt} \right).
\]

(2)

By inserting (2) in (1), the voltage equations of the transformer can be rewritten in the form

\[
\begin{align*}
  u_1 &= i_1 R_1 + L_{\sigma_1} \frac{di_1}{dt} + N_1 \frac{d\phi}{d\theta} \left( N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt} \right) \\
  u_2 &= i_2 R_2 + L_{\sigma_2} \frac{di_2}{dt} + N_2 \frac{d\phi}{d\theta} \left( N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt} \right).
\end{align*}
\]

(3)

In (3), the magnetically nonlinear behavior of the transformer is described by the term \(d\phi/d\theta\). Since the J-A hysteresis model describes the magnetically nonlinear behavior of the iron core, some simplifications must be made so that it is possible to include the hysteresis model in the dynamic model of a single-phase transformer. These simplifications consider using the average area of the transformer iron core \(A\) and the mean path length of the magnetic flux \(l\). By using these simplifications and knowing transformer geometry, a single-phase transformer model obtains the final form (4)

\[
\begin{align*}
  u_1 &= i_1 R_1 + L_{\sigma_1} \frac{di_1}{dt} + N_1 A \frac{dB}{dH} \left( \frac{N_1}{l} \frac{di_1}{dt} + \frac{N_2}{l} \frac{di_2}{dt} \right) \\
  u_2 &= i_2 R_2 + L_{\sigma_2} \frac{di_2}{dt} + N_2 A \frac{dB}{dH} \left( \frac{N_1}{l} \frac{di_1}{dt} + \frac{N_2}{l} \frac{di_2}{dt} \right).
\end{align*}
\]

(4)

In (4), term \(dB/dH\) can now be determined using the J-A hysteresis model.

A. Original Jiles–Atherton Hysteresis Model

The J-A hysteresis model is based on physical considerations of the material’s nonlinear behavior. It is developed on known comprehension of the domain walls’ bending, and on its motion. In the original J-A hysteresis model, the term \(dB/dH\) needed in the dynamic model of the transformer is calculated from term \(dM/dH\), which is given by differentiating the magnetization \(M\) with respect to applied magnetic field \(H\).

The original J-A hysteresis model with magnetic field \(H\) as an independent variable, delivered from [4] and [14] is given by

\[
\frac{dM}{dH} = \frac{(1 - c) dM_{\text{fer}}}{dH_c} + c \frac{dM_{\text{sat}}}{dH_c}
\]

(5)

\[
\frac{dM_{\text{sat}}}{dH_c} = \frac{M_s}{a} \left[ 1 - \coth^2 \left( \frac{H_c}{a} \right) + \left( \frac{a}{H_c} \right)^2 \right]
\]

(6)

\[
\frac{dM_{\text{fer}}}{dH_c} = \frac{\gamma (M_{\text{sat}} - M_{\text{fer}})}{k \delta}
\]

(7)

\[\delta = \begin{cases} +1; & \frac{dH_c}{dt} \geq 0, \\ -1; & \frac{dH_c}{dt} < 0. \end{cases}\]

\[\gamma = \begin{cases} 1; & (M_{\text{sat}} - M_{\text{fer}})dH_c \geq 0, \\ 0; & (M_{\text{sat}} - M_{\text{fer}})dH_c < 0. \end{cases}\]

(8)

\[
\frac{dB}{dH} = \mu_0 \left( 1 + \frac{dM}{dH} \right)
\]

(9)

where \(M_{\text{sat}}\) and \(M_{\text{fer}}\) are the anhysteretic and irreversible magnetizations, \(\mu_0\) is the permeability of vacuum, and \(dH_c\) is denoted as \(dH_c = dH + \alpha dM\). The parameters of the original J-A hysteresis model that must be identified are: the anhysteretic behavior \(\alpha\), the main field parameter \(\alpha\), the saturation magnetization \(M_s\), the parameter which is proportional to the hysteresis loop width and domain flexing constant \(c\), and the pinning parameter \(k\).

B. Modified Jiles–Atherton Hysteresis Model

The pinning parameter of the J-A hysteresis model reflects the impurities of the material, which constitutes the pinning places for the domain walls. At small values of the applied magnetic field \(H\), the predominant physical process of the material behavior is the domain walls’ motion. Detailed analysis of the measured hysteresis loops showed that the value of the pinning parameter is high at small values and low at high values of applied magnetic field \(H\). Based on this conclusion, the shape of the pinning parameter as a function of the applied magnetic field \(H\) was considered as a replacement for the constant value of the pinning parameter from the original J-A hysteresis model. Therefore, a Gaussian function was used in [9]. Gaussian function has no discontinuity around zero and is, therefore, appropriate for numerical simulations. The only drawback when using this function for pinning parameter is that it reaches zero for high negative and positive values of the applied magnetic field \(H\). Therefore, we use a shape of the pinning parameter as a sum of two terms. The first term is independent of the magnetic field and the second one is a Gaussian function of the applied magnetic field \(H\)

\[k = k_0 + k_1 e^{-\frac{H^2}{2\sigma^2}} = G(H),\]

(10)
In (10), \(k_0\) and \(k_1\) are the constant values and \(\sigma\) is the standard deviation of the Gaussian function. Modified J-A hysteresis model is given with (5)-(9) when (10) is inserted in (7). Parameters from the modified J-A hysteresis model must be identified separately from the original J-A hysteresis model’s parameters. Those parameters that must be identified are: the anhysteretic behavior \(\alpha\), the main field parameter \(\alpha_k\), the saturation magnetization \(M_s\), the parameter which is proportional to the hysteresis loop width and domain flexing constant \(c\), and three new parameters: the constants \(k_0\) and \(k_1\), and the standard deviation of the Gaussian function \(\sigma\).

III. USING DE FOR J-A HYSTERESIS MODEL

PARAMETERS IDENTIFICATION

The DE [15] used in this paper for J-A hysteresis model parameters identification is based on an optimization algorithm that varies the parameters and controls the root mean square error \(\varepsilon\), in such a way as to achieve the best agreement between measured \(B_{\text{meas}}\) and calculated magnetic flux densities from the model \(B_{\text{model}}\). The best agreement is achieved when the global minimum of used objective function (11) is found for the selected parameters

\[
\varepsilon = \sqrt{\frac{\sum_{i=1}^{N} (B_{\text{meas}} - B_{\text{model}})^2}{N}}, \quad (11)
\]

In (11), \(N\) denotes the number of points of measured magnetic flux density. The magnetic flux density and applied magnetic field of the transformer iron core were measured according to [16]. For this purpose, the transformer was supplied with sinusoidal voltage whilst the primary voltage and current were being measured. The hysteresis loop with magnetization curve was then determined from known geometry of the iron core and measured data.

Parameters of the original and modified J-A hysteresis model must be identified separately. Using DE, five parameters for the original J-A hysteresis model must be identified while seven are needed for the modified J-A model.

IV. RESULTS

A definition of the initial values of parameters is needed in order to identify the J-A hysteresis model’s parameters. Four parameters \(\alpha, \alpha_k, M_s\) and \(c\), that are common in both J-A hysteresis models are set to the same initial values in each J-A hysteresis model. Initial values of parameters \(k_0, k_1\) and \(\sigma\) must also be set, when corresponding J-A hysteresis model’s parameters are identified. Initial values of the parameters are then subjected to random variations in order to achieve a global minimum of the objective function. At global minimum, the parameters identified using DE for original and modified J-A hysteresis models are presented in Table I. In calculations for both models, the DE used 100 population members at the step size 0.5 and the crossover probability constant 0.7.

Use of identified parameters from Table I in the original and modified J-A hysteresis models (5)–(10) give calculated \(B-H\) hysteresis loops, which are shown in Fig. 1. The measured \(B-H\) hysteresis loop is also presented for comparison. Comparison of hysteresis loops shows that better agreement is achieved when parameter is considered to be a Gaussian function of applied magnetic field. Greater deviation is noticeable only in the region of the magnetizing curve. This is expected since anhysteretic magnetization does not fully represent the magnetization curve. The variable pinning parameter of the modified J-A hysteresis model as a Gaussian function of the applied magnetic field \(H\) is shown in Fig. 2.

A comparison between the measured and calculated currents of a single-phase transformer was performed in order to fully verify the proposed J-A hysteresis model and parameters identification by DE. For this purpose, modified J-A hysteresis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial value</th>
<th>Original J-A model</th>
<th>Modified J-A model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (A/m)</td>
<td>1000</td>
<td>226.25</td>
<td>142.76</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(1 \cdot 10^{-3})</td>
<td>(0.502 \cdot 10^{-3})</td>
<td>(0.345 \cdot 10^{-3})</td>
</tr>
<tr>
<td>(M_s) (A/m)</td>
<td>(2 \cdot 10^6)</td>
<td>(1.335 \cdot 10^6)</td>
<td>(1.190 \cdot 10^6)</td>
</tr>
<tr>
<td>(c)</td>
<td>1</td>
<td>0.724</td>
<td>0.698</td>
</tr>
<tr>
<td>(k) (A/m)</td>
<td>1000</td>
<td>300.05</td>
<td>/</td>
</tr>
<tr>
<td>(k_0) (A/m)</td>
<td>1000</td>
<td>/</td>
<td>152.08</td>
</tr>
<tr>
<td>(k_1) (A/m)</td>
<td>1000</td>
<td>/</td>
<td>288.84</td>
</tr>
<tr>
<td>(\sigma) (A/m)</td>
<td>100</td>
<td>/</td>
<td>53.66</td>
</tr>
<tr>
<td>Error (\varepsilon)</td>
<td>/</td>
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<td>0.02282</td>
</tr>
<tr>
<td>Iterations</td>
<td>/</td>
<td>926</td>
<td>1120</td>
</tr>
</tbody>
</table>

Fig. 1. Comparison of measured and calculated \(B-H\) hysteresis loops.

Fig. 2. Gaussian function of the pinning parameter.
model with identified parameters was included in the dynamic model of a single-phase transformer following (4)–(10). A single-phase transformer with data $R_1 = 0.49 \, \Omega$, $R_2 = 0.63 \, \Omega$, $L_{\sigma 1} = L_{\sigma 2} = 5.8 \times 10^{-3} \, \text{H}$, $N_1 = 345 \, \text{turns}$, $N_2 = 352 \, \text{turns}$, $A = 2650 \cdot 10^{-6} \, \text{m}^2$, $l = 0.6 \, \text{m}$ was excited with sinusoidal voltage and frequency 50 Hz, as shown in Fig. 3. The same figure shows calculated and measured currents in the case of no-load, steady-state operation.

V. CONCLUSIONS

This paper demonstrates the possible use of DE for identification of J-A hysteresis model parameters. The objective function is based on the root mean square difference between measured and calculated magnetic flux densities. The fitting accuracy was increased by considering the shape of the pinning parameter as a Gaussian function of applied magnetic field. This led to the proposed modified J-A hysteresis model. By considering a variable pinning parameter the number of unknown parameters of the J-A hysteresis model increased from five to seven. When parameters were identified using DE this caused only a slight increase in the number of iterations. Otherwise, DE very quickly found the true global minimum of the objective function, regardless of the initial parameter values. The modified J-A hysteresis model, and parameters identifications by DE were confirmed through a comparison of the measured and calculated currents, in the case of a single-phase transformer. Results show that both the model and the DE are appropriate for use in numerical simulations.

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