AMBIGUITY SUPPRESSION IN SARs USING ADAPTIVE ARRAY TECHNIQUES

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Abstract
A novel technique is proposed for overcoming the fundamental tradeoff between azimuthal resolution and swath width in a Synthetic Aperture Radar (SAR). The technique uses an elevation-plane adaptive array to steer nulls at the range-ambiguous responses, allowing a considerably-widened swath.

Firstly the geometry of the system is considered, deriving expressions for the angular extent and rate of change of angle of the ambiguous responses, as a function of system parameters and required degree of suppression. This allows the degree of complexity of the adaptive array and the update rate of the adaptive algorithm to be determined. Next, various types of adaptive algorithm are considered, and it is shown that either a variant of the Howells-Applebaum algorithm implemented on board the satellite, or the Direct Matrix Inversion algorithm, implemented off-line as part of the ground segment processing, represent feasible options.

Keywords: SAR; Ambiguities; Adaptive Arrays

1. INTRODUCTION

It is well known that Synthetic Aperture Radars (SARs) suffer a fundamental limit on the swath width achievable for a given azimuth resolution. This limit arises because of the presence of range or azimuth ambiguities - that is, radar echoes that cannot unambiguously be identified with a given transmitted pulse.

The range and azimuth ambiguities are related via the Pulse Repetition Frequency (PRF) of the radar. Referring to the geometry of Figure 1, to ensure that the echoes are unambiguous in range:

\[ \frac{c}{2 \text{PRF}} > \Delta R \quad \ldots (1) \]

or for a given swath width W:

\[ \text{PRF} < \frac{c}{2W \sin \varphi} \quad \ldots (2) \]

The azimuth ambiguities result from aliased echoes in the undersampled Doppler bandwidth. For a broadside-looking SAR the Doppler bandwidth extends from \(-v L_{a}\) to \(+v L_{a}\), where \(v\) is the platform velocity and \(L_{a}\) is the antenna dimension in azimuth. To properly sample this bandwidth, the Nyquist sampling theorem tells us that:

\[ \text{PRF} > \frac{2v}{L_{a}} \quad \ldots (3) \]

Since the maximum azimuth resolution \(\Delta_{ax}\) of the SAR is just \(L_{a}/2\), equations (2) and (3) together impose a limit on the achievable azimuth resolution and swath width:

\[ \frac{v}{\text{PRF}} < \frac{c}{2W \sin \varphi} \quad \ldots (4) \]

In practice, even when this condition is fulfilled ambiguities will still be present, because of the finite sidelobe levels of the antenna patterns in both azimuth and elevation; this will set a constraint on the maximum allowable sidelobe levels.

Various methods have been proposed to attempt to overcome this limitation. Switched or steered antenna beams may be used in azimuth or elevation [1], or multiple receive antennas displaced along-track [2].

Here we consider an alternative approach. The SAR is operated with a PRF appropriate to the desired azimuth resolution, but with a broader elevation-plane beamwidth than implied by equation (2), so that echoes corresponding to \(n (>1)\) transmit pulses are received simultaneously. The SAR antenna on receive is an adaptive array, which forms \(n\) separate receive patterns in elevation, each with nulls in the directions of \(n-1\) ambiguities, and a beam in the direction of the appropriate wanted echo.

We begin by deriving expressions for the angular extent and rate of change of direction of the ambiguities. This sets a specification for the adaptive array and adaptive algorithm, following which several adaptive algorithms are considered and the tradeoffs between them described.

2. RANGE AMBIGUITIES

Each echo has a significant angular extent, determined by the projection of the (uncompressed) pulse length on the surface. Also, the direction of arrival of each echo is constantly changing as the footprint corresponding to each transmitted pulse moves outwards.

Under the assumption of a flat Earth and zero surface roughness, the across-track extent \(\Delta r_{\text{ground}}\) of the uncompressed pulse-limited footprint (Figure 1) is given by:

\[ \Delta r_{\text{ground}} = \frac{ct}{2 \sin \varphi} \quad \ldots (5) \]

where \(c\) is the uncompressed pulse length. From this, the projected extent seen from the satellite is just \(ct/(2 \tan \varphi)\), which subtends an angle at the satellite of:
Inserting typical values of $\varphi = 45^\circ$, $h = 800$ km and $t = 20\mu s$ gives a value of $\Delta R_{\text{ground}}$ of the order of 4.2 km, and shows that the angular extent of the echo will be of the order of 0.15'.

This result is interesting, because it indicates the extent to which non-adaptive (i.e. open-loop) null steering might be expected to suppress the ambiguities. Allowing also for a contribution of (say) 0.5° due to platform mispointing, the total uncertainty in ambiguity angular position may in the worst case be as great as 0.8°, and in view of the null pattern shapes discussed below, we can conclude that adaptive operation will be necessary, though the open-loop null positions would provide a good 'first guess' for any iterative algorithm to converge from.

The rate of change of direction of each echo is evaluated in a similar manner, and is given by:

$$\frac{d\varphi}{dt} = \frac{c \cos^2 \varphi}{2h \sin \varphi} \text{(radians/sec)}$$

(9)

This, and the echo angular extent derived from equation (6) are tabulated below as a function of angle $\varphi$:

<table>
<thead>
<tr>
<th>$\varphi$ (degrees)</th>
<th>$\varphi = 30^\circ$</th>
<th>$\varphi = 45^\circ$</th>
<th>$\varphi = 60^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>281 rad/\mu s = 1.67/100 $\mu s$</td>
<td>0.3°</td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>132 rad/\mu s = 0.73/100 $\mu s$</td>
<td>0.15°</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>54 rad/\mu s = 0.31/100 $\mu s$</td>
<td>0.06°</td>
<td></td>
</tr>
</tbody>
</table>

This shows that in both respects the worst case occurs closest to nadir, where the angular extent of the echo is greatest and the rate of change of echo direction is most rapid (which is just what would be expected).

We now consider these values in the context of the null shape and the degree of suppression required. Assuming a simple single null pattern, as shown in Figure 3a, it is immediately evident that even in the static case, because of the finite angular extent of the ambiguous echo, we do not obtain perfect suppression. The degree of suppression may be expressed as:

$$10 \log_{10} \left( \frac{S(\varphi) G(\varphi) d\varphi}{S(\varphi) d\varphi} \right) \text{ (dB)}$$

(9)

Approximating the single null $G(\varphi)$ by a $\sin^2$ function and adopting the worst-case figure of 0.3° for the angular extent of the ambiguity, this can be evaluated, yielding a suppression of -36.5 dB.

However, in addition, the ambiguous echo will be moving at a rate given by equation (9) and the null must track the ambiguity at an appropriate update rate. The limits on the integrals of equation (9) can be altered to evaluate the degree of suppression when the null is not perfectly aligned with the ambiguity (Figures 3b and 3c), from which the necessary update rate can be estimated.

The required degree of suppression will depend on the levels of the ambiguous responses with respect to the wanted echo, and the acceptable residual level after cancellation. The worst case will occur when the wanted signal is furthest in range and the ambiguities are closest. Over the ocean, the backscatter coefficient will also be greater nearer to normal incidence,
which will exacerbate this effect.

The ambiguity suppression in the cases of Figures 3b and 3c are, respectively, -30.4 dB and -22.4 dB. Adopting a figure of -30 dB for the required suppression, we see that, in the worst case, an update rate of 100 µs will be necessary.

The approximation assumed for the null pattern deserves some comment. The sin^2 pattern is what would be obtained from an array of two elements (or subarrays), and would be used to suppress a single ambiguity. This pattern is relatively broad, and is therefore well suited to an ambiguity of finite angular extent. If more ambiguities are to be suppressed, each (single null) pattern will be narrower, and hence the degrees of suppression evaluated above will be reduced. In such situations, more than one degree of freedom per ambiguity may be necessary - but this will, of course, be at the expense of greater complexity in the adaptive array processor.

The foregoing arguments have given some idea of the performance required of the adaptive algorithm: an update interval ~ 100 µs will be necessary, a single null per ambiguity appears adequate (at least for the suppression of a small number of ambiguities), and the system will have to operate at the full signal bandwidth (~ 20 MHz). The next stage is to consider the choice of adaptive algorithm.

3. ADAPTIVE ALGORITHMS

Adaptive algorithms are described in detail in references [3], [4] and [5]. Figure 4 shows a block diagram of a generic adaptive array processor. The array output is formed from the sum of the set of element (or subarray) inputs a_1, a_2, ..., a_n, each weighted in amplitude and phase by weights W_1, W_2, ..., W_n, derived by the adaptive algorithm. The algorithm may use as inputs the element signals, the array output, as well as prior information to (for example) steer a beam in the direction of a wanted signal. The implementation may be analogue, digital, or a combination of the two. The scheme of Figure 4 may be replicated to provide further (independent) outputs, as will be required for the separate beams in this application.

Figure 4. Block diagram of a generic adaptive array algorithm.

Several types of algorithm have been studied for this application, and two have emerged as suitable. The implementation of these will now briefly be described.

3.1 Howells-Applebaum Algorithm

Historically, the Howells-Applebaum, or correlation-loop processor, was one of the first adaptive array algorithms developed, dating back to the 1950s. A single loop is shown schematically in Figure 5, using the same notation as Compton [3].

Figure 5. A single loop of the Howells-Applebaum correlation loop algorithm.

The complex weight for the element signal is derived by correlating the element signal, with the array output (which will contain some small uncancelled error term), so as to maximise the ratio of the desired signal to the undesired (interfering) signal. A constraint u^*_d ensures that the adapted output has gain in the (a priori known) direction of a wanted signal. The correlator output is filtered by a lowpass filter, which determines the dynamic performance of the loop. In Figure 5, G is a gain constant, u^*_d is the i^th element of the steering vector U^*_d and μ is an arbitrary scalar constant. Compton [3] gives Applebaum's original proof that the vector of array weights obtained from a set of such loops is optimal.
The Howells-Applebaum loop is conventionally implemented in analogue form. The scheme of Figure 5 is replicated for each beam required. The process may be implemented either at RF or at IF, though in practice IF implementation is likely to be most convenient.

3.2 The Direct Matrix Inversion Algorithm

In this method the covariance matrix is evaluated, and inverted directly to yield the optimal weighting vector, subject to the constraint of a beam in a wanted direction. The method is fast, since there is no 'convergence' involved, but the computational burden required for inversion of the matrix may be substantial.

Using the same notation as Hudson [4], the covariance matrix is formed as:

$$\mathbf{R}_S = \sum_{k=1}^{n} \mathbf{x}_k \mathbf{x}_k^H$$

and from this the optimal weighting vector $\mathbf{W}_S$ is calculated:

$$\mathbf{W}_S = \lambda \mathbf{R}_S^{-1} \mathbf{s}^*$$

where $\mathbf{s}^*$ is the steering vector.

The weighting vector is used as part of a digital element weighting system in which the weighting vector is transmitted to on-line digital multipliers. The algorithm is shown in block diagram form in Figure 6, and is simply a hardware implementation of equations (10) and (11). The subarray signals are digitised, and the covariance matrix formed. This is inverted, and the optimal weighting vector calculated and applied to the set of subarray signals. The scheme is replicated for each beam required, though the operation of forming and inverting the covariance matrix is common to all such processes, so only needs to be done once.

Figure 6. Implementation of the Direct Matrix Inversion algorithm. The process is replicated for each independent beam.

The degree of computational complexity required for this algorithm can be assessed as follows. Assume that we require to suppress $n$ ambiguities, each by means of a single null. This requires $(n+1)$ elements (or subarrays), each followed by its own receiver channel and I/Q digitisation, as shown in Figure 6. To form the $(n+1)$ x $(n+1)$ covariance matrix will require one complex multiplication per (unsmoothed) element of the matrix, i.e. $(n+1)^2$ complex multiplications for the whole matrix, per data sample. Compton [3] shows that a minimum of $2n$ samples are necessary to form the smoothed covariance matrix element, which means that $2(n+1)^2$ complex multiplications will be needed to form the covariance matrix.

Hudson [4] describes a fast recursive matrix inversion algorithm, which requires about $(n+1)^2$ complex multiplications per data sample. For each of the $(n+1)$ beams, the weighting vector is then calculated by multiplication by the constraint vector, which requires a total of $(n+1)^2$ complex multiplications.

This amounts to a total of $3(n+1)^2$ complex multiplications per update interval of 1.00 μs. For $n=2$, 810 k complex operations per second are required; for $n=3$, 1.92 M complex operations per second are required. Clearly this cannot be realised on board the satellite, especially since such a processor would have to be space-qualified, although such performance is well within the capabilities of special-purpose terrestrial processing hardware [6].

Another more reasonable option, however, would be to telemeter the digitised subarray outputs and perform the covariance matrix formation, inversion and weighting processing on the ground (this is only possible because the algorithm is not recursive). The telemetry data rate would be increased by a factor $(n+1)$.

4. CONCLUSION

We have described a technique which is capable of suppressing range ambiguities in SARs, and hence allowing a considerably widened swath width. Two adaptive algorithms appear suitable; either the Howells-Applebaum algorithm implemented onboard the satellite, or the Direct Matrix Inversion algorithm implemented as part of the ground segment processing. Further simulation work will be necessary to confirm and demonstrate the validity of these results.

5. ACKNOWLEDGEMENT

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References


