Combined input shaping and feedback control for double-pendulum systems

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Abstract

A control system combining input shaping and feedback is developed for double-pendulum systems subjected to external disturbances. The proposed control method achieves fast point-to-point response similar to open-loop input-shaping control. It also minimizes transient deflections during the motion of the system, and disturbance-induced residual swing using the feedback control. Effects of parameter variations such as the mass ratio of the double pendulum, the suspension length ratio, and the move distance were studied via numerical simulation. The most important results were also verified with experiments on a small-scale crane. The controller effectively suppresses the disturbances and is robust to modelling uncertainties and task variations.

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1. Introduction

Double-pendulum systems are difficult to control when they are subjected to external disturbances. The control challenge is exacerbated by the difficulty of sensing the swing angle of the second pendulum. One of the most important classes of double-pendulum systems are those arising in crane operations. Cranes are highly flexible and lightly-damped systems. Manipulation of payloads can be challenging in the presence of swing induced by intentional motion of the crane and external disturbances, such as wind forces. The crane control system, which usually includes a human operator, must be robust to external disturbances. In addition, it must move the payload from one location to another with low settling time and minimal transient payload swing in order to achieve high throughput.

Input-shaping methods and feedback control have been developed and used for control of a wide range of flexible systems. Input-shaping is a type of Finite Impulse Response (FIR) filtering. It places zeros near the location of flexible poles of the system, leading to pole-zero cancellation [1,2]. The impulse amplitudes in an input shaper are determined by a set of constraint equations [3–5]. The command signal convolved with the input-shaper impulses suppresses unwanted excitation arising from intended motion. Numerous variations of this method have been demonstrated successfully to reduce payload sway on single-pendulum configurations [6–8], and on double-pendulum configurations [9–17].

Simulation based analyses of advanced input-shaping methods on double-pendulum cranes, and the capability to achieve negligible residual vibration due to induced motion by the actuator, are presented in [11]. Masoud and Alhazza [16]...
presented a frequency-modulation approach to tune the controller frequency such that a single mode input shaper can eliminate residual vibration in double-pendulum systems. Tuan and Lee [18] presented simulation based analyses of non-linear robust controllers using sliding mode control for sway reduction in double-pendulum cranes. Shah and Hong [19] suggested that modified input-shaping methods are also effective in mitigating residual oscillations in damped systems with external forces acting on them as observed in underwater fuel transport systems for nuclear power plants.

Feedback control methods have also been well studied, and implemented to reject external disturbances acting on cranes [20–22]. Blajer and Kotodziejczyk [22] presented a feedback based methodology for control of cranes while avoiding obstacles in a curvilinear path. The proposed methodology used precise path information and approximation of path using smooth splines to generate a feed-forward control law in combination with feedback control loop. These techniques can successfully reject external disturbances, but do not perform particularly well with motion-induced oscillations of the payload.

A combination of input shaping and feedback control is necessary for precise positioning of cranes subject to disturbances. A combination of feedback control and input shaping has been proposed for single-pendulum crane configurations [7,8,23,24], and benchmark mass-spring system [25]. Such controllers combine the benefit of both techniques and enable robust controller design. However, the previous work on the combined control methodology is limited mostly to single-pendulum configurations, and less effort has been directed at eliminating payload swing for double-pendulum crane configuration subject to external disturbances. Note that feedback control of double-pendulum cranes is very challenging because it is difficult to accurately measure the payload motion. In this work, the feedback loop is implemented based on the trolley position, and deflection of the hook which can be tracked from an overhead camera. The velocity command generated by the feedback loop is input-shaped to mitigate motion-induced vibrations.

In this work, a combined input-shaping and feedback controller is applied to double-pendulum bridge cranes. The system dynamics are described in Section 2. The mathematical model and controller implementation in simulation is described in Section 3. The experimental setup and results are presented in Section 4. The results from robustness analysis of the proposed method are discussed in Section 5. Finally, a summary is presented in Section 6.

2. System dynamics

Fig. 1 shows a schematic diagram of a planar double-pendulum bridge crane. The motors provide the actuation force for motion along the trolley direction. A cable of length $L_1$ is suspended below the trolley and supports the hook of mass, $m_1$. A rigging cable is attached to the hook, measuring $L_2$, and supports a payload of mass $m_2$. Assuming the lengths of cables to be constant, the governing linearized equations of motion are:

$$\begin{align*}
\dot{\theta}_1 &= \frac{1}{m_1 L_1^2} \left[ u(t) - m_1 g L_1 \cos \theta_1 - m_2 g L_2 \cos \theta_2 \right] \\
\dot{\theta}_2 &= \frac{1}{m_2 L_2^2} \left[ -m_2 g L_2 \cos \theta_2 \right]
\end{align*}$$

Fig. 1. Schematic diagram of a double-pendulum system.
\[ \ddot{\theta}_1(t) = -\left(\frac{g}{L_1}\right)\dot{\theta}_1 + \left(\frac{gR}{L_1}\right)\dot{\theta}_2 - \frac{u(t)}{L_1} \]
\[ \ddot{\theta}_2(t) = \left(\frac{g}{L_1}\right)\dot{\theta}_1 - \left(\frac{g}{L_2} + \frac{gR}{L_2}\right)\dot{\theta}_2 + \frac{u(t)}{L_1} \]

where, \(\theta_1\) and \(\theta_2\) are the angles of the two masses, \(R\) is the ratio of the payload mass to the hook mass, \(u(t)\) is the acceleration of the trolley, and \(g\) is the acceleration due to gravity. The linearized frequencies of the double-pendulum system are [26]:

\[ \omega_{1,2} = \frac{g}{\sqrt{2}} \sqrt{(1 + R) \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \pm \beta} \]
\[ \beta = \sqrt{(1 + R)^2 \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^2 - 4 \left(\frac{1 + R}{L_1 L_2}\right)} \]

The frequencies depend on the ratio of the cable lengths and the mass ratio. Therefore, these parameters are useful in the design of the shaper.

For constant total length of the double-pendulum, the low frequency component \((\omega_1)\) remains fairly constant with variation in the rigging length; while the second mode \((\omega_2)\) has a strong dependence on the parameters, as shown in Fig. 2. It has been previously reported that the second mode contribution is increasingly significant for double-pendulum systems with smaller mass ratios and length ratios near unity [10]. This showed that any oscillation control scheme would require more robustness to the second vibration mode, implying the use of a robust input shaper.

For the case examined in this paper, both the baseline mass ratio \((R)\) and length ratio \((L_2/L_1)\) were set at 1. The other baseline physical parameters of the system are given in Table 1. Using the mean of the linearized frequencies found previously, an extra insensitive (EI) shaper [9] was found to have sufficient range to suppress both oscillation modes, while also having relatively short shaper duration. The average-mode EI shaper is given by:

\[ A = \begin{bmatrix} 0.27 & 0.46 & 0.27 \\ 0 & 0.62 & 1.24 \end{bmatrix} \]

where, \(A_i\) are the amplitudes of the impulses, and \(t_i\) are the time locations of the impulses. Fig. 3 shows the shaper sensitivity to mass ratio error for cable and rigging lengths of 0.75 m. The horizontal axis shows mass ratio error, or the fraction of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload Mass ((m_2)), kg</td>
<td>0.69</td>
</tr>
<tr>
<td>Hook Mass ((m_1)), kg</td>
<td>0.69</td>
</tr>
<tr>
<td>Cable Length ((L_1)), m</td>
<td>0.75</td>
</tr>
<tr>
<td>Rigging Length ((L_2)), m</td>
<td>0.75</td>
</tr>
<tr>
<td>Max Trolley Velocity, (\text{ms}^{-1})</td>
<td>0.30</td>
</tr>
<tr>
<td>Max Trolley Acceleration, (\text{ms}^{-2})</td>
<td>1.0</td>
</tr>
</tbody>
</table>
model mass ratio to actual mass ratio and the vertical axis displays the vibration relative to that caused by the unshaped motion. While there is be a significant change in second (high) mode frequency with system parameters as shown in Fig. 2, an input shaper design depends on the actual operating regime of the pendulum system. For the cases considered in this study, assuming realistic operation of the system, a single EI shaper computed using baseline operating conditions of unity mass and length ratios is selected. In previous study [13], it was shown that a single mode shaper is effective in suppressing motion-induced oscillations as long as the second mode does not become very significant. Therefore, an average of the two frequency modes is used to design an average shaper which achieves high effectiveness in suppressing residual vibration and also keeps the duration of the shaper short for practical applications. The final shaper design results in less than 8% relative vibration for modelling errors in mass ratio as high as −90% and +110%. Thus, the EI shaper is robust to significant modelling uncertainties. The baseline shaper parameters for unity length and mass ratio are chosen to remove the bias towards any other system configuration.

3. Control system structure

MATLAB & Simulink® were used for model development. The Simulink® block diagram is shown in Fig. 4. There are 10 blocks in the diagram. Blocks 1 and 2 model the motor actuator limits and generate the actual motor velocity for a given command velocity. Block 3 provides the trolley acceleration, and Block 4 generates a disturbance with variable magnitude and duration. These are input into Block 5 - crane dynamics; the output is payload deflection. Payload deflection is multiplied by a proportional gain to generate a part of the command velocity in Block 6. This feedback loop is responsible for damping the payload oscillation. It is important to note that the deflection feedback controller is only using information about the hook (partial state feedback).

![Fig. 4. Simulation block diagram.](image-url)
Blocks 7 through 10 form the second feedback loop, and generate the other part of the command velocity. Block 7 integrates the actual velocity to obtain position, and Block 8 generates an error. This position error is input to a PD control in Block 9, which is then EI shaped by Block 10. This feedback loop is responsible for driving the trolley to the destination, with minimal induced vibration.

This dual-loop feedback with shaper-in-the-loop simulation has some important properties. First, the deflection feedback loop guarantees tunable transient vibration response and zero residual vibration even under a transient or constant disturbance. This could not be achieved with input shaping alone. Secondly, all intentional trolley movements generate negligible vibration via shaping. This could not be achieved with pure feedback control. Finally, the disturbance can be configured to model many types of wind loads (square wave for gusts, step for sustained wind, sinusoid etc.).

Initial controller gains were chosen by balancing settling time and overshoot of the trolley position, and developing suitable gain envelopes. As described previously, the feedback controller has two parts: the proportional-derivative feedback of the trolley position and the proportional feedback of the hook deflection. Gains for each controller were varied independently and the resulting closed-loop systems were simulated. Fig. 5 displays the settling times as a function of position proportional gain and deflection proportional gain with constant position derivative gain. Gain combinations that result in a low settling time are in the middle of the plot in blue color, while gain combinations that settle in 10 s or longer are colored yellow. The middle area in blue color represents desirable gain envelopes. Position derivative gains were investigated by analyzing a multitude of such figures. The gains were further fine-tuned on the experimental setup. Consequently, the gains for the position feedback were chosen to be 0.500 and 0.0625 for proportional and derivative gains, respectively. For the deflection feedback, a proportional gain of 1.14 was chosen. This design point is shown in Fig. 5. The gains were selected such that the system operation was close to the fastest settling time conditions. However, the absolute fastest gains were not selected because that puts the machine near the conditions where the actuator system would start to chatter, i.e. putting the actuator system close to the limit-cycle condition. This gain combination resulted in low transient vibration, rapid settling time, and good stability. It is challenging to definitely prove the closed loop stability are the system because of the nonlinear actuator limits. However, simulations were performed for a wide-range of conditions. All reasonable conditions resulted in stable behavior. Note that because of the actuator limits, the system will go into limit-cycle conditions, rather than full-blown instability.

4. Experimental setup and results

A double-pendulum structure was fabricated and installed in the Advanced Crane Control Laboratory at the Georgia Institute of Technology. The experimental crane is shown in Fig. 6. The double-pendulum consists of a metal shaft with adjustable collars to vary the mass of the payload. The payload shaft is connected to the pendulum hook through a permanent magnet. The rigging length and the length of the suspension of the hook can be adjusted, as shown in the close-up view on the right. An overhead camera is used to track the motion of the hook only. For post-experiment data analysis, an additional camera was placed to the side to simultaneously record the motion of the suspended payload. The dimensions of both the hook and payload are 0.063 m diameter. The hook is approximately 2 in. in height, while the payload mass can be varied by

![Fig. 5. Variation in settling time [s] with controller gains. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).](image-url)
stacking thin steel plates. The experiments are intended to simulate the disturbance caused by wind forces and characterize the response of the controller. However, due to the lack of a consistent mechanism to implement wind forces, the disturbance in the experiments is applied as a pulse by physically impacting the payload manually. The initial angle of deflection due to the application of the disturbance is maintained approximately the same to maintain consistency between different test cases.

Fig. 7 compares the simulated unshaped-, simulated combined-control, and experimental combined-control of the trolley for a 0.75 m move and no disturbance. The unshaped velocity command was generated using feedback control only. It was found by adding the trolley position PD controller output (bypassing the shaper in Fig. 4) and the hook deflection proportional controller output. The combined controller command was found similarly, except that the position control output was first convolved with the input shaper given in (3). The graphs in Fig. 7 show the shaper and feedback loop...
working together to position the crane trolley and damp the oscillations. The input shaper minimizes deflection due to the initial acceleration.

The feedback controller works to dampen the vibration, while also moving the trolley to the desired position. It settles at the destination within about 8 s. Small payload oscillations near the destination are a result of partial state feedback and no direct control of the payload. Nevertheless, the simulated linear system and experimental results show good agreement. The combined controller significantly improves performance over the unshaped case; it exhibits approximately 60% reduced transient vibration and reduces settling time by 3 s.

Fig. 8 compares the simulated and experimental responses of a payload disturbance while the system is at equilibrium. In both simulation and experiment, the payload was disturbed with a pulse-like force parallel to the trolley axis, such that the hook would deflect approximately 10 degrees. The time instance of the disturbance is shown by the vertical line. The controller successfully returns the trolley to the initial position, and simultaneously damps the oscillations. This control action is completed in about 8 s. Again, the experimental and simulated results agree fairly well.

Fig. 9 compares the simulated and experimental responses of the controlled trolley with an external disturbance applied during a 0.75 m movement. The payload was disturbed in the opposite direction of trolley movement by approximately 10 degrees after 2.5 s. The disturbance time instance is shown as a solid vertical line. Although the controller manages to settle the crane oscillation, the experimental response is slower than the previous test cases. This can be attributed to limited actuator acceleration, as evident in the saw-tooth-like velocity command occurring after the disturbance was applied, as shown in the top of Fig. 9. The trolley could not respond fast enough to decrease the hook deflection to the same extent as the case shown in Fig. 8.

5. Robustness analysis

A robustness study was also conducted. Various system parameters such as mass ratio, modelling frequency error, move distance, and disturbance magnitude were varied in simulation studies. Fig. 10(a) shows the variation in settling time for different mass ratio values resulting from a 0.75 m move. The controller gains were kept constant at the design conditions.
for all cases, but the input shaper parameters were updated with changes in the linearized frequencies. As expected, higher mass ratios resulted in longer average frequency, longer shaper durations, and longer settling times. This behavior was observed in both simulations and experiments. Fig. 10(b) shows the results for same mass ratios, but with a constant input shaper. This study shows the robustness of the proposed combined controller to modelling error. It can be observed that the settling time remains approximately constant. This is a result of the over-damped nature of the trolley position response, which dominates the payload damping for the majority of the transient behavior. In other words, the controller is able to damp payload oscillations before arriving at the destination. The maximum payload deflection remained less than 6 degrees for all the cases shown in Fig. 10(a), and less than 3 degrees for all the cases in Fig. 10(b).
Fig. 11. Robustness to trolley move distance [27].

Fig. 12. Robustness to disturbance magnitude [27].

Fig. 13. Robustness to bang-bang command.
Fig. 11 shows the controller robustness to variation in trolley move distance. The settling time remains nearly constant for trolley move distances up to about 0.8 m and increases with trolley move distance after that point. For short trolley move distances, the oscillation damping seems to dominate the settling time. For large trolley move distances, the trolley movement dominates. The maximum payload deflection for the cases considered in this test varied between 0.8 degrees to 7 degrees.

Fig. 12 shows how the settling time increases with the magnitude of the disturbance. The variation in disturbance pulse magnitude causes the change in initial deflection magnitude which is shown on the x-axis of the plot. This was performed in order to maintain consistency with previous test cases where the external disturbance was introduced using similar approach. The controller provides good settling time performance, even as the disturbance gets very large. As the disturbance magnitude increases, so does the actuator effort, maximum trolley displacement, nonlinearity (pendulum dynamics, actuator saturation, etc.), and settling time. This behavior was found in both simulation and experiment. It should be noted that the settling time remained below 10 s in all the cases.

Fig. 13 shows the variation in settling time for a bang-bang type trolley disturbance. From equilibrium, the trolley was given an unshaped velocity command of 0.3 ms⁻¹ for a specified duration; the resulting physical trolley velocity is trapezoidal with slope of 1 ms⁻². As no repeatable external disturbance was available, the disturbances for this test were generated internally through trolley movement. The simulated disturbance causes oscillation of varying magnitude depending on the duration of the bang-bang command input. After the command was completed, the combined controller was enabled, and the system settled at the new trolley position. As expected, disturbances shorter than the physical velocity rise time settled quickly, and settling time scaled directly with disturbance time. However, commands longer than the 0.3 s rise time maintained an approximately constant settling time. In other words, trolley movement at constant velocity after the initial acceleration has negligible effect on settling time. The maximum payload deflection for all the cases considered in this case varied between 0.29 degrees for shortest command duration to 10.3 degrees for longest command duration.

6. Conclusions

A control system methodology involving combined input shaping and feedback control was developed for control of double-pendulum systems. The proposed controller effectively rejects external disturbances, while also providing rapid point-to-point motion. The controller is also robust to variation in system parameters such as mass ratio, move distance, and modelling uncertainties. The effectiveness and robustness of the control system was experimentally verified and aligned well with simulations.

References


