Direct Torque Control of an IPM-Synchronous Motor Drive at Very Low Speed Using a Sliding-Mode Stator Flux Observer

Gilbert Hock Beng Foo, Student Member, IEEE, and M. F. Rahman, Senior Member, IEEE

Abstract—Direct torque control of interior permanent-magnet synchronous motors is known to deliver fast torque and flux dynamic responses. However, the poor flux estimation at very low speeds has been its largest drawback. A multitude of flux observers have been proposed for flux estimation, but most of them fail to fare in the low-speed region. This paper proposes a sliding-mode stator flux observer for improved flux estimation at very low speeds. Unlike conventional flux observers, this observer does not require any speed adaptation mechanism and is immune to speed estimation error. A novel stator resistance estimator is incorporated into the sensorless drive to compensate the effects of stator resistance variation. Global asymptotic stability of both the flux observer and stator resistance estimator is guaranteed by the Lyapunov stability analysis. DC-offset effects are mitigated by introducing a small integral component in the observer gains. Simulation and experimental results at very low speeds, including 0 and 2 r/min, without signal injection confirm the effectiveness of the proposed method.

Index Terms—Active flux, direct torque control (DTC), interior permanent-magnet synchronous motor (IPMSM), sliding-mode flux observer.

I. INTRODUCTION

SINCE the advent of direct torque control (DTC) for induction machines in the 1980s, as proposed by Depenbrock [1] and Takahashi and Noguchi [2], its research has been becoming ever more prevalent in the society. In the late 1990s, the DTC was successfully implemented on interior permanent-magnet synchronous motors (IPMSMs), as reported in [3]. The block diagram of the DTC is depicted in Fig. 1.

When compared to conventional vector-controlled drives, DTC possesses several advantages, such as elimination of coordinate transformation, lesser parameter dependence, and faster dynamic response [3]. As the torque and flux are regulated directly and independently, DTC features fast responses. Furthermore, due to the absence of coordinate transformation, DTC is inherently sensorless. Nevertheless, the inability to accurately estimate the stator flux at low speeds is its main drawback. Usage of an encoder for stable operation at low speeds seems to negate the benefits of the DTC. In addition, the presence of this position sensor increases the cost while reducing the reliability of the system.

In recent years, many researchers have proposed several solutions to this problem. They can be broadly classified as follows:

1) open-loop back-EMF-based estimators [3]–[6];

Fig. 1. Block diagram of the DTC.

\[ \lambda_d, \lambda_q \]

Stator flux linkage in the \( dq \) reference frame.

\[ \lambda_f \]

Permanent magnet flux linkage.

\[ \tilde{\lambda} = (\tilde{\lambda}_d, \tilde{\lambda}_q)^T \]

Stator flux estimation error.

\[ \omega_{re} \]

Rotor speed.

\[ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \]

\( dq \)-axes inductances.

\( R_s \)

Stator resistance.

\( T \)

Electromagnetic torque.

\( v = (v_d, v_q)^T \)

Stator voltage in the \( \alpha\beta \) reference frame.

\( v_{d}, v_{q} \)

Stator voltage in the \( dq \) reference frame.

\( \theta_{re} \)

Rotor angle.

\[ \lambda = (\lambda_d, \lambda_q)^T \]

Stator flux linkage in the \( \alpha\beta \) reference frame.
2) closed-loop adaptive observers based on advanced models [7]–[16];
3) estimation based on high-frequency signal injection, exploiting the saliency property of an IPMSM [17]–[23].

Every solution has its own benefits and limitations. In general, most of them (except the latter) fail to deliver satisfactory performance at very low speeds. Although signal injection methods allow sensorless operation even at standstill, it increases the noise in the system while reducing its overall efficiency. Furthermore, the dynamics of the system are shown to be sluggish when this method is adopted for sensorless control.

This paper proposes a novel sliding-mode stator flux observer that is capable of accurate flux estimation at very low speeds without signal injection. This observer uses the active flux concept [24]–[26] that makes sensorless control simpler to be performed on an IPM machine. With the adoption of the active flux, the proposed observer can be implemented in a dual reference frame that in turn allows it to be inherently sensorless. The observer does not require any speed adaption mechanism, and hence, any inaccuracy due to speed estimation errors is eliminated [27]. This significantly improves the flux estimation at very low speed. A similar observer was proposed in [28] for direct-torque and flux-controlled (DTFC) IPM motor drives. Nevertheless, effects of dc-measurement offset parameter variations, which pose significant problems at very low speeds, were not taken into account. Furthermore, the observer gains selection, which may become critical at very low speeds, were not elaborated.

The proposed sliding-mode flux observer in this paper is superior to that in [28]. An online stator resistance estimator is incorporated into the proposed flux observer to mitigate any effect of stator resistance variation at low speed. The effect of variations in \( L_q \) is accounted for by the sliding-mode term in the proposed observer. The stability of both the flux observer and stator resistance estimator is proven via the Lyapunov stability analysis. The observer gains selection procedure is also included. An integral component is combined with the observer gain to overcome the undesirable effects of dc-measurement offset at low speeds. Simulation and experimental results of the sensorless DTC drive at very low speeds, including standstill and 2 r/min, are included to verify the effectiveness of the proposed approach.

II. SLIDING-MODE OBSERVER (SMO)

The key factor that affects the performance of a sensorless drive is the accuracy of the stator flux observer. Due to its robustness to parameter variations and disturbance rejection capabilities, adaptive observers in the stationary \((\alpha,\beta)\) [11]–[13] or rotating \((dq)\) [14], [15] frame are commonly used. However, these observers are always realized in a single reference frame that results in a rotor-speed-dependant term. As a result, the observer has to be speed adaptive. In a digital realization, the speed adaptation is usually performed as the last step of the estimation process. Hence, the speed estimate is affected by cumulative errors, noise, and delays. When the inaccurate speed value is fed back to the observer, the flux and speed estimation gradually worsen. This can easily lead the drive to instability, especially at low speeds.

A. Design of the Observer

The speed dependency term can be removed if both the rotating \((dq)\) and stationary \((\alpha,\beta)\) frames are used. If the stator flux is defined as the state variable and the stator current as the output, a closed-loop SMO without speed adaptation can be designed, as shown in Fig. 2. Based on the machine equations in the rotor \((dq)\) reference frame, the stator flux observer can be mathematically expressed as follows:

\[
\frac{d}{dt} \lambda = -R_s i + v + K \hat{i} + K_{SMO} \text{sign}(i)
\]

\[
\hat{T} = T^{-1}(\hat{\theta}_{re})L^{-1} T(\hat{\theta}_{re}) \lambda + \frac{L_d}{L_d} \left( \frac{\cos \hat{\theta}_{re}}{-\sin \hat{\theta}_{re}} \right)
\]

where

\[
K \text{ and } K_{SMO} \text{ are the gains of the observer, and } \hat{\theta} \text{ denotes the estimated quantities. The observer employs both linear and nonlinear feedback terms. The linear and nonlinear gains determine the error dynamics and robustness, respectively. The estimated rotor angle is calculated from}
\]

\[
\hat{\theta}_{re} = \tan^{-1} \left( \frac{\hat{\lambda}_{\alpha,\beta}}{\hat{\lambda}_{\alpha,\alpha}} \right)
\]
B. Lyapunov Stability Analysis

Assuming orientation, the current errors are given by

$$\tilde{i} = T^{-1} L^{-1} T \dot{x}.$$  \hspace{1cm} (4)

Furthermore, the state error dynamics are as

$$\dot{\tilde{x}} = -\tilde{R}_s \dot{i} - K \tilde{i} - K_{SMO} \text{sign}(\tilde{i})$$ \hspace{1cm} (5)

where $\tilde{R}_s = R_s - \hat{R}_s$. Define a Lyapunov candidate function as

$$V = \frac{1}{2} \left( \dot{\tilde{x}}^T L^{-1} T L^{-1} T \dot{x} + \dot{\tilde{x}}^T \tilde{R}_s \dot{\tilde{x}} \right) > 0$$ \hspace{1cm} (6)

where $\gamma > 0$. Assuming that the stator resistance remains constant within one sampling interval

$$V = \lambda^T T^{-1} L^{-1} T \dot{x} + \lambda^T \tilde{T} \dot{x} + \omega_r \lambda^T T^{-1} L^T \dot{x} - \frac{\dot{\tilde{x}}^T \tilde{R}_s \dot{\tilde{x}}}{\gamma}.$$ \hspace{1cm} (7)

Substituting (4) and (5) into (7) yields

$$\dot{V} = -\tilde{T}^T K \dot{x} + \omega_r \tilde{T}^T J L T \dot{x} - \dot{\tilde{x}} \left( i_\alpha \dot{i}_\alpha + i_\beta \dot{i}_\beta + \frac{\dot{\tilde{x}}}{\gamma} \right) - i K_{SMO} \text{sign}(\tilde{i}).$$ \hspace{1cm} (8)

Define $K = k_1 I + k_2 J$. Equation (8) can then be simplified to

$$\dot{V} = -\tilde{T}^T \left[ k_1 I + J (k_2 I - \omega_r L) \right] \tilde{x} - \dot{\tilde{x}} \left( i_\alpha \dot{i}_\alpha + i_\beta \dot{i}_\beta + \frac{\dot{\tilde{x}}}{\gamma} \right) - i K_{SMO} \text{sign}(\tilde{i}).$$ \hspace{1cm} (9)

For global asymptotic stability, $\dot{V} < 0$. Hence, the following equations can be deduced, which are as follows:

$$[k_1 I + J (k_2 I - \omega_r L)] \tilde{x} > 0$$ \hspace{1cm} (10)

$$-i K_{SMO} \text{sign}(\tilde{i}) > 0$$ \hspace{1cm} (11)

$$\left( i_\alpha \dot{i}_\alpha + i_\beta \dot{i}_\beta + \frac{\dot{\tilde{x}}}{\gamma} \right) = 0.$$ \hspace{1cm} (12)

Equation (10) stipulates that the eigenvalues of $[k_1 I + J (k_2 I - \omega_r L)]$ must lie entirely in the right-half plane. Thus, the individual gains $k_1$ and $k_2$ can be carefully selected via the pole placement method, outlined as follows.

C. Observer Gain Selection

The classical approach to observer gain selection is to design the observer poles proportional to the motor poles [9]. This approach allows the observer to be dynamically faster than the motor, but is susceptible to noise. This problem can be circumvented by designing the observer poles with identical imaginary parts as the motor poles, but shifted to the left in the complex plane [10]. This approach is adopted in this paper. The observer still possesses faster dynamics than the machine. Nevertheless, the noise immunity of the observer is improved because the other pole, already having a large magnitude, is less amplified.

It can be shown that poles of the machine as a function of rotor speed are given by

$$p_{1,2}(\omega_r) = -\frac{R_s}{L_d}, -\frac{R_s}{L_q}.$$ \hspace{1cm} (13)

On the other hand, the poles of the observer are governed by the eigenvalues of $[k_1 I + J (k_2 I - \omega_r L)]$ in (10). Shifting the observer poles to the left by $k$ yields

$$p_{o1,2}(\omega_r) = -\frac{R_s}{L_d} - k, -\frac{R_s}{L_q} - k$$ \hspace{1cm} (14)

where $k > 0$. Imposing this condition on the observer results in the following individual gains $k_1$ and $k_2$ as a function of rotor speed:

$$k_1 = \frac{R_s}{2} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + k$$

$$k_2 = \frac{1}{2} \left( \omega_r (L_d + L_q) + \text{sign}(\omega_r) \left( L_d - L_q \sqrt{\omega_r^2 - \left( \frac{R_s}{L_d L_q} \right)^2} \right) \right).$$ \hspace{1cm} (15)

Since the actual speed $\omega_r$ is not available, the estimated one is used instead. Fig. 3 illustrates the machine and observer poles in the complex plane for $k = 150$. On the other hand, Fig. 4 shows the machine and observer poles in the complex plane when constant gains $k_1 = 150$ and $k_2 = 50$ are used instead. It is evident that the imaginary component of the observer poles increases with speed and this induces unwanted oscillations at high speeds.

If

$$K_{SMO} = \begin{pmatrix} K_{SMO,1} & 0 \\ 0 & K_{SMO,2} \end{pmatrix}$$

Equation (11) yields

$$K_{SMO,1} |i_\alpha| + K_{SMO,2} |i_\beta| > 0.$$ \hspace{1cm} (16)

Hence, $K_{SMO,1}, K_{SMO,2} > 0$. Fig. 5 depicts the torque and flux estimation errors due to +20% detuning in $L_q$, when $K_{SMO,1} = K_{SMO,2} = 0$. Fig. 6 illustrates smaller estimation errors when the sliding-mode gains $K_{SMO,1} = K_{SMO,2} = 0.1$ are inserted. Larger values of $K_{SMO,1}$ and $K_{SMO,2}$ further increase
the robustness of the observer, but may generate unwanted chattering. 0 < K_{SMO,1}, K_{SMO,2} < 1 is practical for a real drive.

D. Online Stator Resistance Estimation

The performance of the flux observer relies heavily on the stator resistance parameter. Any mismatch may severely affect the operation of the drive. The high-speed performance is marginally affected because the back EMF of the machine is significantly larger than its resistive voltage drop. Nevertheless, the low-speed performance is very unsatisfactory.

Extensive simulation has been performed to investigate the effect of stator resistance variation on the performance of the observer. Fig. 7 illustrates the torque and stator flux estimation errors due to +20% variation in $R_s$ when the machine is running at 500 r/min. On the other hand, Fig. 8 illustrates the torque and stator flux estimation errors due to +20% variation in $R_s$ when the machine is running at 300 r/min. In both cases, rated load was applied at $t = 1$ s. The drive is stable, but the estimation errors are significant. It is observed in simulations that variations in $R_s$ at very low speeds render the observer unstable.

To mitigate the undesirable effects of stator resistance variations on the sensorless drive, an online resistance estimator is proposed. Based on (12)

$$\dot{\hat{R}_s} = -\gamma \int (i_\alpha \hat{i}_\alpha + i_\beta \hat{i}_\beta) \, dt. \tag{17}$$

A larger selection of the adaptation gain $\gamma$ decreases the response time of the estimator, but induces unwanted oscillations. As the stator resistance changes very slowly in a real drive, a lower value of $\gamma \approx 1$ is preferred.

The effectiveness of the proposed resistance estimator is investigated in simulations. Fig. 9 depicts the performance of the estimator due to 20% increase in $R_s$ when the machine is running at 500 r/min. Fig. 10 illustrates the estimation algorithm
due to 20% increase in $R_s$ when the machine is running at 50 r/min. In both cases, half-rated load was applied to the motor shaft. The stator resistance estimator is very effective at both low and high speeds, as it is able to identify the actual stator resistance value within a short time frame.

E. DC-offset Compensation

Another critical problem of the low-speed operation of the flux observer is the dc-measurement offset. Fig. 11 shows the torque and flux estimation errors due to a 0.5-V offset voltage in the dc-bus voltage $V_{dc}$ channel in simulation. In reality, $v_α$ and $v_β$ are reconstructed from the dc-bus voltage $V_{dc}$ and the switching status. Any dc-measurement offset in $V_{dc}$ is reflected in both $v_α$ and $v_β$. The machine was initially accelerated from standstill to 1000 r/min. At $t = 1$ s, rated torque was applied to the shaft of the motor. At $t = 2$ s, the machine was decelerated to 40 r/min. From Fig. 11, it is obvious that the high-speed performance is marginally affected, but the low-speed performance is ruined.

The effects of the dc offset can be alleviated by introducing an integral component in the observer gain as follows:

$$K = \left( k_1 + \frac{k_I}{s} \right) I + k_2 J \quad (18)$$

where $k_I$ is a small positive gain. $k_I \approx 0.5$ is practical for a real drive. Fig. 12 shows the torque and flux estimation errors with the same dc-offset voltage and test conditions, as before, but with observer gains in (18). The errors at low speed are now eliminated, proving the effectiveness of the dc-offset compensation.
Fig. 13. Block diagram of the proposed sensorless DTC scheme.

TABLE I
SWITCHING TABLE OF THE CLASSICAL DTC

<table>
<thead>
<tr>
<th>λ</th>
<th>τ</th>
<th>θ1</th>
<th>θ2</th>
<th>θ3</th>
<th>θ4</th>
<th>θ5</th>
<th>θ6</th>
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<tbody>
<tr>
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<td>τ = 1</td>
<td>V2(110)</td>
<td>V3(010)</td>
<td>V4(011)</td>
<td>V5(001)</td>
<td>V6(101)</td>
<td>V1(100)</td>
</tr>
<tr>
<td>τ = 0</td>
<td>V6(101)</td>
<td>V1(100)</td>
<td>V2(110)</td>
<td>V3(101)</td>
<td>V4(011)</td>
<td>V5(001)</td>
<td>V2(110)</td>
</tr>
<tr>
<td>λ = 0</td>
<td>τ = 1</td>
<td>V3(010)</td>
<td>V4(011)</td>
<td>V5(001)</td>
<td>V6(101)</td>
<td>V1(100)</td>
<td>V2(110)</td>
</tr>
<tr>
<td>τ = 0</td>
<td>V5(001)</td>
<td>V6(101)</td>
<td>V1(100)</td>
<td>V2(110)</td>
<td>V3(100)</td>
<td>V4(011)</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 14. Regions θ1 – θ6 of the stator flux linkage vector.

F. Torque and Rotor Speed Estimation

The electromagnetic torque is estimated from

\[
\hat{T} = \frac{3}{2} P (\hat{\lambda}_\alpha i_\beta - \hat{\lambda}_\beta i_\alpha).
\]

(19)

The rotor speed is only required for speed control. It can be calculated based on the derivative of (2)

\[
\hat{\omega}_{re} = \frac{\lambda_{a,\alpha}(k-1) \hat{\lambda}_{a,\beta}(k) - \hat{\lambda}_{a,\beta}(k-1) \hat{\lambda}_{a,\alpha}(k)}{T_s \left(\hat{\lambda}_{a,\alpha}(k) + \hat{\lambda}_{a,\beta}(k)\right)}
\]

(20)

where \(T_s\) is the sampling period and \(k\) and \(k-1\) denote two consecutive sampling instants. The speed signal is low-pass filtered to remove the noise. This is a compact, yet effective, speed estimation scheme.

III. EXPERIMENTAL RESULTS

The effectiveness of the proposed sensorless DTC drive scheme was tested experimentally. The block diagram of the sensorless drive is shown in Fig. 13. The switching logic of the DTC is implemented, as shown in Table I. Variables \(\lambda\) and \(\tau\) are the outputs of the flux and torque hysteresis controllers, respectively. \(\lambda = 1\) implies that the estimated flux is smaller than its reference value and vice versa. The same thing applies to \(\tau\) for...
torque control. $\theta_1 - \theta_0$ represent the region in which the stator flux vector lies, as defined in Fig. 14.

A DS1104 DSP card was used to carry out the real-time algorithm. A three-phase insulated-gate bipolar transistor (IGBT) intelligent power module is used for an inverter. Coding of real-time control software was done using C language. The pulsewidth modulated (PWM) signals were generated on the DS1104 board. In experiments, the sampling period of the drive was set to 50 $\mu$s. A dc machine, whose armature current is separately regulated, is used to emulate the load. The IPM machine, whose parameters are tabulated in Table II, was used in this experiment. An incremental encoder was used to obtain the position signal that was solely used for comparison and not for control purposes. The reference flux value was selected according to the maximum torque per ampere (MTPA) trajectory to increase the efficiency of the drive system [4]. The control parameters of the drive are shown in Table III.

The performance of the sensorless drive, especially, at low speeds is severely affected by the nonlinearities of the inverter. To sustain very-low-speed sensorless operation, accurate forward voltage drop and dead-time compensation are mandatory [29]. The high-frequency signal injection method [18] was used to estimate the initial rotor position only and not used subsequently for control purposes. The $d$-axis inductance $L_d$ and the permanent magnet flux $\lambda_f$ stays relatively constant with variation in operating current. On the other hand, the $q$-axis inductance $L_q$ decreases as the operating current increases. Hence, the variation of $L_q$ with current has to be taken into account and can be represented by the polynomial in Fig. 15.

Full-load dynamics at zero-speed operation with rated torque is illustrated in Fig. 16. The actual speed, estimated torque and flux, and speed estimation error are shown. From Fig. 16, we

![Figure 15: Variation of the $q$-axis inductance as a function of operating current.](image1)

![Figure 16: Full-load steps at standstill.](image2)

![Figure 17: Sensorless half full-load dynamics at 2 r/min.](image3)
can observe that the observer is capable of persistent full-load operation at standstill. Half full-load dynamics at 2 r/min is depicted in Fig. 17, while steady-state operation at 5 r/min with full load is shown in Fig. 18. The same quantities are shown. The speed and torque ripples are now higher, but the system is stable. Low-speed reversal from −10 to +10 r/min with full load is depicted in Fig. 19. The actual speed, estimated torque and flux, and speed estimation error are shown. The estimated speed follows the actual speed very closely during the transient and steady state.

The dynamic response during acceleration from standstill to 1000 r/min is shown in Fig. 20. The machine was originally operated at zero speed with no load and was then accelerated to 1000 r/min. The estimated speed tracks the actual speed closely during the transient and steady state.

The speed reversal dynamics from −1000 to 1000 r/min is illustrated in Fig. 21. Again, the estimated speed follows the actual speed very well during the reversal, confirming the effectiveness of the torque, flux, and speed observer.

The effectiveness of the stator resistance estimator is depicted in Fig. 22. The stator resistance $R_s$ is increased by 40% at 50 and 500 r/min, and the responses of the resistance estimator are displayed in Fig. 22(a) and (b), respectively. In both cases, the machine is operated with half full load, and it can be observed...
that the resistance estimator is more than satisfactory, as the stator resistance $R_s$ can be identified within a short interval.

IV. CONCLUSION

In this paper, a sliding-mode stator flux observer for DTC of IPMSM drives was presented. Due to the adoption of the active flux concept, the stator flux observer was implemented in a dual reference frame. As a consequence, it does not require speed adaptation and is not susceptible to speed estimation errors, especially, at low speed. A novel online resistance estimator was proposed to further compensate for the effects of stator resistance variation. The stability of the proposed observer and stator resistance estimator were proven by the Lyapunov stability analysis. The effects of dc-measurement offsets were mitigated by incorporating an integral compensating term in the observer gain. The proposed sliding-mode flux observer is capable of delivering high performance over a wide speed range, including very low speeds. Simulation and experimental results at very low speeds, including standstill and 2 r/min, without signal injection demonstrate the effectiveness of the proposed sensorless drive.

REFERENCES


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