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Research on Application of Single Moving Mass in the Reentry Warhead Maneuver

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Abstract

Moving mass is one of the control actuators, whose configuration in the warhead is the basis of application in engineering for moving mass control technology. The servo forces, which drive the moving mass to move, affect the translational and rotational motion of the missile. It not only could increase the moving range of the center of system mass, but also reduce the requirement for the roll channel. Firstly, a single moving mass control mode, which consists of the rotation of rail around the longitudinal body-axis and the translation of the moving mass along the rail, is presented. Then, the 8-DOF dynamic model is established and the coupling characteristics between the attitude control and the servo loop control are analyzed in this paper. Based on the above analysis, the attitude tracking control law is designed by using sliding mode theory. Finally, the problem about the terminal guidance of reentry vehicle is studied by numerical simulation: the state of moving mass is controlled by the servo force, and the missile attitude is controlled by adjusting the position of moving mass so that the excepted overload designed by using the proportion guidance could be tracked exactly. The simulation results verify the validity of the proposed method.

Keywords: Single moving mass; Reentry warhead; Actuator layout; Control system design

1. Introduction

Maneuver is one of the effective penetration measures for the reentry warhead. As the dynamics pressure is very high after the reentry warhead into the dense atmosphere, its maneuver could be achieved by using lift, whose control principle is adjusting the body attitude relative to the incoming flow. The aerodynamic rudder is a typical attitude control actuator; for example, the U.S. Pershing II reentry warhead is controlled by using the aerodynamics rudder. The warhead attitude could be controlled by adjusting the aerodynamic rudder deflection, which could change the control moment of the rudder surface relative to the center of mass. Moving mass is another attitude control actuator. The warhead could achieve the maneuver by moving the actuator mass in the body of

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so the MMTC problems of reentry warheads are more concerned.

The configuration of the moving mass in the axisymmetric reentry warheads is one of the key problems that the MMTC technology applies in the engineering. Whether the control actuator is within the warhead is the significant difference between the moving mass control and rudder control. Though the warhead could have a good aerodynamics shape by setting the actuator in the body, it makes more difficult to design the internal layout of the warhead. Taking into account the limit of the internal space and overall parameters such the mass of moving mass, it needs to be considered how many moving mass should be configured, how the moving mass rail should be placed and so on. Frost and Castello [10,11] investigated the ability of an internal rotating mass unbalance to actively control both fin and spin-stabilized projectiles. But for a traditional STT (Skid To Turn) vehicle, the above configurations could only supply one channel control ability, i.e. either longitudinal or lateral control ability. Rogers and Castello [12,13] considered a translating moving mass control mechanism applicable to both fin- and spin-stabilized configurations. But the c.m. (center of mass)’s change of the system is a circle around the body x-axis and not arbitrary, so the control authorities of above configurations are limited. Double moving mass is a commonly used configuration at the moment. The double moving mass control style could be divided into the orthogonal layout style and non-orthogonal layout style according to whether the rail rotates or not. The characteristics of orthogonal layout style are that the rail is fixed and the moving mass is moving. The expected move of the system’s center of mass is achieved by controlling the position of two moving masses in two rails, which are fixed on the body orthogonally. On the contrary, the characteristics of non-orthogonal layout style are that the rail is rotary and the moving mass is fixed on the end of rail. The expected move of the system’s center of mass is achieved by controlling the rotation angle of two rails around the longitudinal axis of the warhead. Taking axisymmetric reentry warhead as the research subject, a single moving mass control style is proposed in this paper. The control style combines the advantages of double orthogonal control style and non-orthogonal style, i.e., the MMTC could be achieved by the movement of the moving mass and the rotation of the rail.

By controlling the rotation angle of the rail and the position of moving mass in the rail, this single moving mass control style could not only increase the moving range of the center of system mass, but also reduce the requirement for the roll channel, combining the advantages of two double moving mass control styles.

Dynamic analysis and optimization of the overall parameters is another key problem for the MMTC technology applying in the engineering. Robinett [14] studied the ability for inertial force, principal axis misalignment and other physical objects to impact the attitude of the warhead body. The analytical expressions of the required trim moment for ballistic missiles maneuvering are given in case of ignoring the dynamic characteristics of moving mass. The open literatures mainly focused on the problems about trajectory and attitude dynamics. At present, the dynamics problems of body-loop and servo-loop considering the dynamics characteristics of moving mass should be discussed further. Based on the above problems, this paper takes the single moving mass control style as the research subject, and develops the system coupling dynamics analysis. The servo force drives the moving mass to move in order to adjust the state of moving mass, but its impact on the body rotation and translation can’t be ignored. Therefore it makes the dynamics analysis become more difficult for this missile.

The design of the attitude control system is a research focus of the moving mass control technology. PID control, feedback linearization, sliding mode control and other methods have been adopted. Because the servo force that drives the moving mass to move will have a greater impact on the rotation and translation of the warhead, the design of guidance and control loop considering the dynamic characteristics will be the focus of future research. This paper is aimed at three key problems of the moving mass configuration, dynamic analysis and the design of control loop. Combining with the internal relation between three key problems, the research is carried out deeply.

2. Configuration of attitude control actuator

As the attitude control actuator of the warhead, the configuration of moving mass is not only the focus of the overall design, but also affects the structure of missile dynamics modeling. The layout of equipments in the warhead and dynamic model of the warhead will have big differences if the configuration of moving mass is different. For the axisymmetric and non-spinning reentry warhead, in order to track the command overload of the pitch and yaw channel, double moving mass control style was adopted usually. According to whether the rail rotates or not, the double moving mass control style could be divided into the orthogonal layout style and non-orthogonal layout style.

The characteristics of orthogonal layout style, which is shown in Fig.1, are that the rail is fixed and the moving mass is moving. Two rails are orthogonal to each other and parallel to two axes of the body coordinate system. By controlling the state of moving mass in the rail, the attitude tracking control of pitch and yaw channel could be achieved exactly.

Fig.1. Diagram of orthogonal layout style with double moving mass
The characteristics of non-orthogonal layout style are that the rail is rotary and the moving mass is fixed on the end of rail, as shown in Fig.2. The attitude control of the warhead is achieved by controlling the rotation angle of two rails around the longitudinal axis of the warhead.

Fig.2. Diagram of non-orthogonal layout style with double moving mass

The common denominator between two layout styles is that the rails are perpendicular to the longitude axis of warhead. For the orthogonal layout style, the two rails don’t intersect in order to avoid collision of the moving masses. However, it is not necessary to consider this problem for non-orthogonal layout style because the rails can intersect. Compared with the non-orthogonal layout style, adopting the orthogonal style can reduce the difficulty of designing equipments but has a higher requirement for the roll channel. A greater roll torque will be produced in the rotary process of the rails if the mass of moving mass is large. In the process of maneuvering, the expected centroid position of the moving mass is arbitrary, so it is difficult to ensure the roll torque produced by the rotation of two rails could cancel each other out. The Larger roll torque will greatly increase the difficulty of rolling channel design.

To solve the drawbacks of the above two layout styles, a single moving mass control style is proposed in this paper, as shown in Fig.3. The characteristics of this layout style are that the rail can rotate and the moving mass can move along the rail. Compared with the orthogonal layout style with double moving mass, this style could reduce the difficulty of designing equipments. By controlling the rotation angle of rail and the position of moving mass in the rail, this layout style could reduce system requirements for the roll channel.

Fig.3. Diagram of single moving mass layout style

3. System dynamics modeling

The servo actuator of traditional orthogonal layout style is mainly linear motor, whose control variable is the driving force of the linear motor. However, the actuators of the proposed single moving mass layout style in this paper include both linear motor and the rotating motor that controls the rail to rotate, and the control variables are the driving force of the linear motor and driving torque of the rotating motor. Because the actuators and control variables of two layout styles are different, the decoupling dynamic model established in this paper is different from the dynamic model of traditional control style.

The internal structure of the system with a single moving mass is shown as Fig.3. The system S whose centroid is denoted by $S^*$ is composed of the warhead body $B$, whose centroid is denoted by $B^*$ and the internal moving mass denoted by $p$. In this section, according to the characteristics of study object, the system dynamics model will be established based on the momentum theorem and angular momentum theorem.

3.1. Nomenclature

The following notation is used in deriving the equations of motion:

1. $m_B, m_p, m_s$ are the mass of the warhead body B, the moving mass $p$ and the system $S$, respectively.
2. $\mu = m_p/m_s$ is the mass ratio of the moving mass relative to the system and $g$ is the gravity acceleration vector.
3. $I_{bpd}$ is the $3 \times 3$ inertia tensor of the body B about its centre mass.
4. $r_{bp} = [l \cos \phi \; \sin \phi]^T$ is the position vectors of the actuator mass in the body frame centered at the vehicle center mass.
5. If $\delta \geq 0$, $\phi$ is equal to the rotation angle of rail. If $\delta < 0$, $\phi$ differs 180 degrees from the rotation angle of rail, as shown in Fig.3. $\delta$, $\dot{\delta}$ and $\phi$, $\phi$ are the velocity, acceleration of the actuator mass along the rail and angular velocity and acceleration of the rail rotation.
6. $r_b$ and $r_p$ are the position vectors of the system centriod S and the actuator mass p with respect to the inertial reference frame, respectively. $V_b$ is the inertial velocity vector of the center of mass of the vehicle body B. $\omega = [\omega_x \; \omega_y \; \omega_z]^T$ is the inertial angular velocity vector of the body B.
7. $\frac{d^2 V_b}{dt^2}, \frac{d^2 \omega}{dt^2}, \frac{d^2 r_p}{dt^2}, \frac{d^2 r_w}{dt^2}$ are the first or second derivative of the vectors with respect to the body fixed reference frame, respectively. $\frac{d^2 r_b}{dt^2}, \frac{d^2 r_p}{dt^2}, \frac{d^2 r_w}{dt^2}$ are the second derivative of the vectors with respect to the inertial reference frame, respectively.

3.2. Forces and Moments Model

Because the reentry warhead maneuvers by using the aerodynamic force instead of the thrust during the reentry process, it can be regarded as a system of particles with constant mass that only impacted by the force of gravity and aerodynamics. There are the constraint forces interacting with each other between the warhead body and the actuator mass, which are related to the control forces and torques of the servo actuator. The detailed instructions for the above forces and torques model are as follow.
1) \( \mathbf{F}_a \) is the vector of aerodynamic force at the center of pressure in the body frame. \( \mathbf{M}_a \) is the vector of aerodynamic moments about the center of mass of the warhead body. The aerodynamic forces and moments are expressed in the body frame by using the expressions:

\[
\begin{align*}
\mathbf{F}_a &= \begin{bmatrix} -X \\ Y \\ Z \end{bmatrix} = -C_v \rho S L \mathbf{v} + C_{v\alpha} \rho S L v \alpha \\
\mathbf{M}_a &= \begin{bmatrix} M_{Bx} \\ M_{By} \\ M_{Bz} \end{bmatrix} = \begin{bmatrix} m_v \rho L \alpha_x^{\alpha} \\ m_v \rho \beta \alpha + \frac{m_v}{\rho} L \alpha_x^{\alpha} \\ m_v \rho \beta \alpha - \frac{m_v}{\rho} L \alpha_x^{\alpha} \end{bmatrix}
\end{align*}
\]

\( q = \rho \left| \mathbf{v} \right|^2 / 2 \)

where \( C_v \) is the drag coefficient, and \( C_{v\alpha} \) and \( C_{v\beta} \) are the partial derivatives of the normal force coefficients with respect to the angle of attack and sideslip angle, respectively. \( m_v^{\alpha} \) and \( m_v^{\beta} \) are the partial derivatives of the pitching moment and the yawing moment coefficients with respect to the angle of attack and sideslip angle, respectively. \( m_v^{\alpha} \), \( m_v^{\beta} \), and \( m_v^{\rho} \) are the aerodynamic damping coefficients, respectively. \( q \) is the dynamic pressure, \( \rho \) is the atmospheric density, \( S \) is the cross-sectional area, \( L \) is the reference length, \( v \) is the total velocity, i.e. the magnitude of the inertial velocity vector.

2) \( \mathbf{N} \) is the force exerted by the warhead body on the actuator mass, whose body-axis components are given by:

\[
\mathbf{N} = \left[ N_x \quad F_c \cos \phi - M_s \sin \phi / \delta \quad F_c \sin \phi + M_s \cos \phi / \delta \right]
\]

where \( N_x \) is the force in the direction of the body-axis, \( F_c \) exerted by the warhead body on the actuator mass, \( F_c \) is the control force exerted by the servo motor on the actuator mass, \( M_s \) is the control moment exerted by the servo motor on the rail, which are shown as Fig. 3.

3.3. Dynamics Equations

1. Eight-degree-of freedom dynamics Equations

According to the momentum theorem, the system translational dynamics are given by Eq.(4):

\[
m_x \frac{d^2 r_x}{dt^2} = \sum F_x \Rightarrow
m_x \left( \frac{d^2 r_x}{dt^2} + \mathbf{a} \times \mathbf{V} + \mu \frac{d^2 r_y}{dt^2} \right) = \mathbf{F}_a + m_x \mathbf{g}
\]

The motions of the actuator mass are described by Eq.(5):

\[
m_y \frac{d^2 r_y}{dt^2} = \sum F_y \Rightarrow
m_y \left( \frac{d^2 r_y}{dt^2} + \mathbf{a} \times \mathbf{V} + \frac{d^2 r_y}{dt^2} \right) = \mathbf{N} + m_y \mathbf{g}
\]

According to the angular momentum theorem, the system rotational dynamics are given by Eq.(6):

\[
\frac{d^2 r_y}{dt^2} = \sum M_y \Rightarrow
I_{y\beta} \frac{d \omega}{dt} + \alpha \mathbf{a} \times (I_{y\beta} \cdot \omega) = M_y - \mu r_y \mathbf{F}_a
\]

Expanding the Eq.(4)~(6) with respect to the body frame, the eight-degree-of freedom differential equations become

\[
\begin{align*}
\frac{d^2 r_y}{dt^2} + \mu \frac{d \omega}{dt} \frac{d r_y}{dt} + \mu \frac{d \omega}{dt} \times r_y = & \frac{b}{d} \mathbf{V} + \mu \frac{d \omega}{dt} \mathbf{r} + \mathbf{a} \times (\omega \times \mathbf{r}) \\
I_{y\beta} \frac{d \omega}{dt} + \alpha \frac{d r_y}{dt} \cdot \mathbf{a} & = M_y - \mu r_y \mathbf{F}_a \\
& - (1 - \mu) r_y \cdot m_x \left( 2 \omega \times \frac{d r_y}{dt} + \mathbf{a} \times (\omega \times \mathbf{r}) \right) \\
\end{align*}
\]

where

\[
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

The differential terms of Eqs.(7) ~ (9) are coupled to each other, so in order to make it easier to design the attitude controller, the decoupling body attitude and the actuator mass dynamics equations are derived next.

2. Attitude dynamics equations of the warhead body

Combining Eq.(4) and Eq.(5), the following equation could be derived:

\[
\frac{d^2 r_y}{dt^2} = \frac{1}{1 - \mu} \left( \frac{N - F_y}{m_y} \right)
\]

Substituting Eq.(10) into Eq.(6):

\[
I_{y\beta} \frac{d \omega}{dt} + \alpha \mathbf{a} \times (I_{y\beta} \cdot \omega) = M_y - r_y \times \mathbf{N}
\]

Expanding the left-hand sides of Eq.(10) with respect to the body frame:

\[
\begin{align*}
\frac{d^2 r_y}{dt^2} + \sqrt{2} \omega \times \frac{d r_y}{dt} + \frac{d \omega}{dt} r_y + \omega \times (\omega \times \mathbf{r}) = & \frac{1}{1 - \mu} \left( \frac{N - F_y}{m_y} \right) \\
\end{align*}
\]

The expression of \( N_x \) is derived by expanding the vectors of Eq.(12):

\[
N_x = F_x + (1 - \mu) m_x \left( \delta \omega \sin \phi - \omega \cos \phi \right) - \mu X
\]

where the expression of \( F_x \) is as follow:
\[ \begin{align*}
F_i = & (1 - \mu) m_\nu E_1 \left[ \frac{d^2 r_\nu}{dt^2} + 2 \omega \times \frac{dr_\nu}{dt} + \omega \times (\omega \times r_\nu) \right] \\
= & (1 - \mu) m_\nu \left[ 2 \omega_1 (\delta \sin \phi - \delta \phi \cos \phi) - 2 \omega_1 (\delta \cos \phi - \delta \phi \sin \phi) \\
& - \omega_1 (\omega \times \delta \omega \cos \phi - \omega_1 \omega \times \delta \omega \sin \phi) \right]
\end{align*} \]

where \( E_1 = [1 \ 0 \ 0] \) is a basis unit vector. The decoupling attitude dynamics equations of the warhead body can be given by substituting Eq.(13) into Eq.(11):

\[
\begin{pmatrix}
I_{w, \nu} + \Delta I
\end{pmatrix} \begin{pmatrix}
\dot{\omega}_\nu \\
\dot{\omega}_\nu \\
\dot{\omega}_\nu
\end{pmatrix} = M_\nu - \omega \times (I_{w, \nu} \cdot \omega) + \begin{pmatrix}
0 & 0 & 0 \\
0 & \mu X & 0 \\
0 & 0 & -\mu X
\end{pmatrix} \begin{pmatrix}
\delta \cos \phi \\
\sin \phi \delta \cos \phi \\
\sin \phi \delta \sin \phi
\end{pmatrix}
- \begin{pmatrix}
\delta \sin \phi & -l \sin \phi & -l \cos \phi / \delta \\
\delta \cos \phi & l \cos \phi & -l \sin \phi / \delta \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
F_i \\
F_i \\
F_i
\end{pmatrix}
= M_\nu + M_\nu + M_\nu + M_\nu + M_\nu + M_\nu 
\]

The specifications of the moment symbols are provided in the Appendix A.

3. Dynamics Equations of the Actuator Mass

First of all, define a matrix \( E_{23} \) as follows:

\[
E_{23} = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\( E_{23} \) consists two basis unit row vectors. The result of multiplying a column vector that has three elements by \( E_{23} \) is a vector which only contains the last two elements of the column vector. The expression which contains the second derivative of \( \delta \) and \( \phi \) can be given by multiplying Eq.(12) by the matrix \( E_{23} \):

\[
\begin{pmatrix}
\cos \phi & -\delta \sin \phi \\
\sin \phi & \delta \cos \phi
\end{pmatrix} \begin{pmatrix}
\dot{\delta} \\
\dot{\phi}
\end{pmatrix} = E_{23} \left( -2 \omega \times \frac{dr_\nu}{dt} - \omega \times (\omega \times r_\nu) \right)
- \begin{pmatrix}
-\delta \sin \phi & 0 \\
\delta \cos \phi & -l
\end{pmatrix} \begin{pmatrix}
\omega_\nu \\
\omega_\nu
\end{pmatrix}
+ \begin{pmatrix}
-2 \delta \phi \sin \phi - \delta \phi^2 \cos \phi \\
2 \delta \phi \cos \phi - \delta \phi^2 \sin \phi
\end{pmatrix} \begin{pmatrix}
\omega_\nu \\
\omega_\nu
\end{pmatrix}
- \begin{pmatrix}
1
\end{pmatrix} \begin{pmatrix}
Y \\
Z
\end{pmatrix}
+ \begin{pmatrix}
1
\end{pmatrix} \begin{pmatrix}
\frac{\cos \phi}{\sin \phi} \cdot \delta \phi / \delta \\
\sin \phi \cos \phi / \delta
\end{pmatrix} \begin{pmatrix}
F_i \\
M_\phi
\end{pmatrix}
\]

Then the attitude dynamic Eq.(14) can be transformed into:

\[
\begin{pmatrix}
\dot{\omega}_\nu \\
\dot{\omega}_\nu \\
\dot{\omega}_\nu
\end{pmatrix} = H \begin{pmatrix}
F_i \\
M_\phi
\end{pmatrix}
\]

(16)

where

\[
H = \begin{pmatrix}
T_\phi - \omega \times (I_{w, \nu} \cdot \omega) & 0 & 0 \\
F_\delta \sin \phi & + & 0 & \mu X & 0 & \delta \cos \phi \\
- \delta F_\phi \cos \phi & - & \mu X & 0 & \delta \sin \phi
\end{pmatrix}
\]

\[
\Gamma = \begin{pmatrix}
I_{w, \nu} + \Delta I
\end{pmatrix}^{-1} \left( \begin{pmatrix}
0 & 0 & 0 \\
- \delta \sin \phi & -l \sin \phi / \delta \\
l \cos \phi & -l \cos \phi / \delta
\end{pmatrix} \right)
\]

At last, the decoupling dynamics equation of the actuator mass can be given by substituting Eq.(16) into Eq.(15):

\[
\begin{pmatrix}
\delta \\
\phi
\end{pmatrix} = f + B \begin{pmatrix}
F_i \\
M_\phi
\end{pmatrix}
\]

(17)

where

\[
\begin{pmatrix}
-\delta \sin \phi & 0 \\
\delta \cos \phi & -l
\end{pmatrix} \begin{pmatrix}
1
\end{pmatrix} \begin{pmatrix}
Y \\
Z
\end{pmatrix}
+ \begin{pmatrix}
-2 \delta \phi \sin \phi - \delta \phi^2 \cos \phi \\
2 \delta \phi \cos \phi - \delta \phi^2 \sin \phi
\end{pmatrix} \begin{pmatrix}
\omega_\nu \\
\omega_\nu
\end{pmatrix}
+ \begin{pmatrix}
1
\end{pmatrix} \begin{pmatrix}
\cos \phi & -\sin \phi / \delta
\end{pmatrix} \begin{pmatrix}
F_i \\
M_\phi
\end{pmatrix}
\]

4. Analysis and Comparison with Double Moving Mass Layout Style

In order to compare with the double moving mass layout style [9], the further simulation and analysis are presented in this section. The simulation assumptions as follows: Assume that for the double moving mass control style and the proposed single moving mass control style, the initial and final positions of the equivalent center of mass of the actuator mass are same. For the double moving mass control style, two rails begin to rotate according to cosine rule from the location of the negative z-axis to the designated location shown in Tab.1 during 0.1s. But for the proposed single moving mass control style, except the rotation of the rail, the actuator mass also moves according to cosine rule to the maximum displacement along the rail. The specific simulation conditions are shown in Tab.1 and Fig.4-5.

Tab.1. Simulation conditions of two control styles

Fig.4. Schematic of the single moving mass control style

Fig.5. Schematic of the double moving mass control style

The servo control forces and torques for two different control styles are shown in Fig.6. The servo control torque required for the double moving mass control style is larger, because a greater roll torque will be produced in the rotary process of the rails if the mass of moving mass is large. For the double moving mass control style, the average of the servo control torques, which drive the rails to rotate, are 1975 N·m and 658.3N·m, respectively, while this average is only 1097.6 N·m for the single moving mass control style. Therefore, in the case of achieving the same angle of attack, the control ability of the proposed single moving mass style is superior to that of the double moving mass style.

Fig.6. Servo control forces and torques for two control styles
Under the condition that the roll motion is uncontrolled, the effects on the roll channel for two control styles are shown in Fig.7. For double moving mass style, the roll torques produced by the rails’ rotation are larger than that for single moving mass style, and have the greater effects on roll angular velocity, which greatly increased the difficulty of designing the roll channel.

Fig.7. Effects on the roll channel for two control styles

The above simulation analysis shows that the single moving mass control style has a better control ability and a less effects on the roll channel, so the style is convenient for control system design.

5. Control system design

The loop-decoupling dynamic model of the warhead described in Sec. 3. This section will present the design of control law. The sliding mode control technique is used to design nonlinear control laws based on the simplified model of the warhead. Firstly, consider the following nonlinear MIMO system:

\[ x = f(x) + B(x)u + d \]
\[ y = Cx \]

where \( x = [\alpha, \beta, \omega_x, \omega_y, \delta, \phi, \dot{\phi}]^T \) is the state of system, \( u = [F_c, M_c] \) is the control input, and \( y = [n_x, n_y] \) is the output of system. \( n_x \) and \( n_y \) are the output overloads of pitch channel and yaw channel, respectively. \( d \) denotes the function uncertainty, which is due to the modeling errors and external disturbances.

The purpose of control system is: designing the controller to make the output of Eq.(19) asymptotically stably track the given continuous and time-varying reference variable \( y_r = [n_x(t), n_y(t)] \):

\[ \lim_{t \to \infty} \|y_r - y\| = 0 \]  (20)

The flow chart of control system for the moving mass actuated warhead is shown in Fig.8:

Fig.8. Flow chart of attitude control for the tracking channel

In the process of designing controller, the overloads tracked can be transformed into AOA and sideslip angle. With the development of the nonlinear control scheme, the sliding mode control methodology applied in the flight control system has been researched widely. It is assumed that the warhead’s roll channel has been controlled stably by differential ailerons, and the roll angle, the roll angular velocity are small and bounded. The pitch and yaw channels are controlled by driving the actuator mass to move and the rail to rotate within the warhead. An approach to flight path control is presented in this section. The control system is decomposed into inner and outer feedback loop. A sliding mode control system in the outer loop is designed for controlling the attitude of warhead and achieving the tracking of the guidance commands. The feedback linearization methodology in the inner loop is designed for controlling the position of the actuator mass and tracking the required displacement of the actuator mass along the rail, and the rotational angle of rail by the outer loop.

First, defining the states \( x_1, x_2, x_3, x_4 \in \mathbb{R}^2 \), and the control input \( u \) as follows:

\[
\begin{align*}
    x_1 &= [\alpha \quad \beta]^T, \\
    x_2 &= [\omega_x \quad \omega_y]^T, \\
    x_3 &= [\delta \quad \phi]^T, \\
    x_4 &= [\dot{\delta} \quad \dot{\phi}]^T, \\
    \delta &= [\delta_x \quad \delta_y]^T = [\delta \cos \phi \quad \delta \sin \phi]^T
\end{align*}
\]  (21)

The following assumptions are used in the design and analysis procedure.

**Assumption 1.** The coupling effects between the pitch and yaw channels generated by \( \Delta I \) are negligible.

The additional inertia tensor \( \Delta I \), which is generated by the motion of the actuator mass, causes the coupling between the pitch and yaw channels. However, it can be seen from the expressions of \( \Delta I \) in Appendix A that the coupling effects could be ignored because the mass and displacement of the actuator mass are very small quantities relative to the mass and size of warhead.

**Assumption 2.** The inertia moments \( M_{I_x}, M_{I_y}, M_{I_z} \) in Eq.(14) could be regarded as disturbances of the control loop.

For the general reentry maneuvering warhead, the moving mass control style is usually adopted below 40km where the aerodynamic drag is very large. The control moment related to aerodynamic drag and the displacement of the actuator mass is much larger than other inertia moments in Eq.(14). So we could regard the other inertia moments as disturbances of the control loop.

**Assumption 3.** The roll channel has been controlled stably by differential ailerons.

The moments that affect the roll channel include the conventional aerodynamic moment, gyro moment, and the control torque \( M_{I_z} \), which has an effect on the roll channel. The roll angle couldn’t be stable only by the aerodynamic damping, which requires that another control style is introduced to control the roll channel. Differential ailerons are adopted in this paper.

With above assumptions, the warhead dynamics equations can be simply expressed in state-space form as follows:

\[ \dot{x}_1 = f_1(x_1) + B_1(x_1)x_2 \]  (22)
\[ \dot{x}_2 = f_2(x_1, x_2) + B_2(x_1, x_2)\delta \]  (23)
\[ \delta = [x_1(1) \cos(x_1(2)) \quad x_1(1) \sin(x_1(2))] \]  (24)
\[ x_3 = x_4 \]  (25)
\[ \dot{x}_4 = f_3(x_1, x_2, x_3) + B_3(x_1, x_2, x_3)u \]  (26)
where the definitions of all the matrices in Eq.(22) can be found in Reference [15]. The definitions of \( x_1 \) and \( x_2 \) in Eq.(34) are shown in Eq.(21). The expressions of all the matrices in Eq.(26) are same to Eq.(18). The expressions of all the matrices in Eq.(23) are as follows:

\[
\begin{align*}
 f_1(x_1, x_2) &= \begin{bmatrix}
 \frac{(I_z - I_x)}{I_y} \omega_{x_1} \omega_{x_2} \\
 \frac{(I_x - I_z)}{I_y} \omega_{x_1} \omega_{x_2} \\
 \frac{(I_x - I_z)}{I_y} \omega_{x_1} \omega_{x_2}
\end{bmatrix} + \begin{bmatrix}
 m_c g \xi(x_1^2) \omega_{x_1} + \frac{q S m_{L}}{I_y} \beta \\
 m_c g \xi(x_1^2) \omega_{x_1} + \frac{q S m_{L}}{I_y} \alpha \\
 0
\end{bmatrix} \\
 B_1(x_1, x_2) &= \begin{bmatrix}
 0 \\
 \frac{\mu g S c}{I_y} \\
 -\frac{\mu g S c}{I_y}
\end{bmatrix}
\end{align*}
\]

where \( I_x, I_z \) are moments of inertia of the body, respectively.

Without loss of generality, the parameter uncertainty, input uncertainty, and function uncertainty are taken into account in the proposed control system. Generally speaking, the uncertainties can be divided into structured uncertainty and unstructured uncertainty. The structured uncertainty is caused by the parameter uncertainty: neglecting the principal axis misalignment will bring about the parameter uncertainty of inertia tensor of system. Besides, as the warhead disturbed by the free flow, AOA and sideslip angle, and aerodynamic coefficients can’t be accurately known, which also brings about the uncertainty. The unstructured uncertainty includes the unknown disturbances and the modeling errors. These nonlinear negative factors could be compensated by the designed controller with strong robustness.

To design the sliding mode controller, an appropriate sliding surface should be selected in consideration of the desired warhead dynamics. Let us define the sliding surface as

\[
s = e^T e + c_1 \int_0^t e dt \tag{27}
\]

where \( e = y - y_r = x_3 = (\alpha, \beta)^T \) is the tracking error of output, \( c_1, c_2 \) are the gain matrices. By selecting the appropriate \( e, c_1, e_2 \), the tracking error of output can obtain the desired dynamic characteristics.

In order to ensure the movement performance in the reaching mode, the exponential reaching law [16,17] is applied. The sliding mode control law is designed as follows:

\[
\delta = (B_1 B_2)^T \begin{bmatrix} 1 \\ B_1 & -B_1 x_1 - B_1 f_1 - c_1 e - c_1 e - k \end{bmatrix} s - e_1 \text{sgn}(s) \tag{28}
\]

where \( k \) is a positive definite diagonal matrix, and \( e_1 > 0 \). The Appendix B provides some more details on the design process of sliding mode controller.

According to Eq.(17), the designed control variables \( u_x \) and \( u_z \) of the outer attitude tracking control loop are not the direct output of the inner loop which contains the position \( \delta \) of the actuator mass and the rotation angle \( \phi \) of the rail. The relations between \( \delta \), \( \phi \) and \( u_x \), \( u_z \) could be described as follows according to Eq.(21):

\[
\delta_x = \sqrt{u_x^2 + u_z^2}, \quad \delta = -\sqrt{u_x^2 + u_z^2}
\]

where \( 1 \) is a positive definite diagonal matrix, and

\[
\begin{align*}
\phi &= \begin{cases}
\frac{\pi}{2} & u_x = 0, u_z > 0 \\
\frac{-\pi}{2} & u_x = 0, u_z < 0 \\
\pi + \arctan(u_x/u_z) & u_x < 0, u_z > 0 \\
-\pi + \arctan(u_x/u_z) & u_x > 0, u_z < 0
\end{cases}
\end{align*}
\]

According to the above equation, the value of \( \delta \) may be positive or negative, so there will be two different solutions \( (\delta, \phi) \) and \( (\delta, \phi) \) corresponding to a group of control variable \( (u_x, u_z) \), where \( \delta_x \) is the positive value of the control of the position of the actuator mass and \( \phi \) is the rotation angle of the rail associated with \( \delta_x \) in Eq.(29). \( \delta_x \) is the negative value of the position of the actuator mass and \( \phi \) is the rotation angle of the rail associated with \( \delta_x \). The difference between \( \phi \) and \( \phi \) is 180 deg.

Then which group of \( (\delta, \phi) \) is selected as the input variables of the inner servo loop is discussed. Let us define \( \Delta \phi \) as the angle difference between the command angle of last moment and the current command angle. In order to make the absolute of \( \Delta \phi \) minimal, the selection strategy of the control command is given in Eq.(30)

\[
\begin{align*}
\delta &= \delta_x, \phi = \phi_x \quad |\Delta \phi| \leq \frac{\pi}{2} \\
\delta &= \delta_x, \phi = \phi_x \quad |\Delta \phi| > \frac{\pi}{2}
\end{align*}
\]

The following section will design the control law of the inner loop in brief. First, it is assumed that the system states are observed. The feedback linearization methodology can be used to design the control law. According to Eq.(17), the dynamic differential equation of the actuator mass is expressed as follows:

\[
\delta = \ddot{x}_1 = f + B \begin{bmatrix} F_c \\ M \end{bmatrix} \tag{31}
\]

So the control force and torque is designed as the following equation:
\[
\begin{bmatrix}
F_c \\
M_c
\end{bmatrix} = B^{-1}(-f + \dot{x}_n - 2\dot{\xi}_n \omega_n (x_3 - \dot{x}_n) - \omega_n^2 (x_3 - \dot{x}_n))(32)
\]

Where \( \xi_n \) is the designed damping ratio and \( \omega_n \) is the designed angular frequency.

6. Simulation results and discussion

In order to demonstrate the performance of the proposed control law, simulations are presented in this section. The initial conditions of the engagement are given as follows:

- Mass of the system (warhead body and the actuator mass): \( m_s = 1000 \text{kg} \)
- Reference area: \( S_h = 0.5 \text{m}^2 \); Reference length: \( L = 4 \text{m} \)
- Moment of inertia: \( I_x = 1000 \text{kg} \cdot \text{m}^2 \), \( I_y = 1000 \text{kg} \cdot \text{m}^2 \), \( I_z = 2100 \text{kg} \cdot \text{m}^2 \)
- Mass ratio of the actuator mass relative to system: \( \mu = 0.1 \)
- Offset of the actuator mass along the axial line: \( l = 0 \text{m} \)
- Initial attitude angle: \( (r, \alpha, \beta) = (5^\circ, 0^\circ, 0^\circ) \); Initial angular velocity: \( (\omega_r, \omega_\alpha, \omega_\beta) = (360, 0, 0) \text{rad/s} \)
- Flight height: \( h = 10 \text{km} \); Mach number: \( Ma = 10 \)

In particular, the air density is looked up from the Standard Atmospheric Model using the current flight altitude [18]. The needed moment coefficients and atmospheric coefficients are also determined through a looked up table using the current flight status (i.e., Mach number, altitude).

Figs.9-11 show the simulation results of tracking the step command AOA and sideslip angle taking into account the modeling error. The time history of achieved AOA and sideslip angle are shown in Fig.9. As expected, the proposed flight control system could guarantee the achieved angles track the commanded angles effectively. The dynamics response of angles is well: the respond is fast and the magnitude of overshoot is small.

As indicated in many literatures, the mass ratio \( \mu \) will determined the control authority of moving mass control. The control authority will increase proportionately to the mass ratio. In order to get the desired control authority, a large mass fraction is needed. For our conceptual design, we select a relatively bigger mass ratio \( \mu \) in order to generate more modest control authority. According to our simulation results, if the mass ratio is too small, even less than 0.01, it will take much larger displacement of moving mass and much longer rail to achieve the equivalent control authority. Taking into account the limit of the internal space of the warhead, it is unrealistic in practice. For reentry warhead, it’s unnecessary to add other extra parts as the moving mass. We can only select some inherent components of the warhead such as ammunitions or mechanical equipments as the moving mass.

Fig.9. Time history of AOA and sideslip angle

Fig.10 shows the responses of the displacement of the actuator mass and the rotation angle of the rail. Fig.11 shows the responses of the force and torque of the servo actuator. In order to guarantee an effective tracking, the displacement of the actuator mass and the rotation angle of the rail have a significant shock in the initial phase, but also accompanied by the substantial change of the force and torque of the servo actuator. But as indicated in Fig.11, the servo torque and force reach the steady state values in order to keep the force balance with the force of gravity of the moving mass and make the displacement of the actuator mass and the rotation angle of the rail keep the commanded state. The force of the servo actuator is below 5kN, and the torque of the servo actuator is smaller, which meets the requirement for the servo motor in engineering application.

Fig.10. Time history of the displacement of the actuator mass and the rotation angle of the rail

Fig.11. Time history of the force and torque of the servo actuator

The validity of the proposed sliding mode control law is demonstrated in the above numerical simulations for the pitch and yaw channel control system. In order to demonstrate the performance of guidance and control system during the reentering process, the 8-DOF numerical simulations of the complete trajectory are presented below.

- Initial coordinate of the warhead: \([-112000 60000 14000] \text{m} \)
- Initial velocity of the warhead: 4760 m/s;
- Initial roll angle: 30°; Initial AOA and sideslip angle: 0°
- Longitude of the target: 139° east; Latitude of the target: 35° north

Fig.12. Time history of achieved and commanded overloads

Fig.13. Responses of the force and torque of the servo actuator

Fig.12 and 13 show the time history of achieved and commanded overloads and responses of the force and torque of the servo actuator in the process of reentry.

As indicated in Fig.12 and 13, the control ability of the moving mass increases with the decrease of flight height. The performance of tracking the overloads is well, and the force and torque of the servo actuator are in the valid range. The warhead can reach the slowly moving target on the ground accurately with a position error of 39.0929m. The above results and analysis show that the proposed attitude control law can resist the external disturbances effectively, and has good robustness.
7. Conclusions

Focused on the axisymmetric reentry warhead, this paper studies the attitude control problems with the moving mass as the actuator. Combining the advantages of two typical configurations of double moving mass, a single moving mass layout style is proposed in this paper. A more common control model is established with considering the servo actuator's effects on the translation and rotation of the warhead. The robust sliding mode controller is designed for the attitude tracking control loop (outer loop), and the controller based on the feedback linearization methodology is designed for the servo control loop (inner loop). To track the commanded linearization methodology is designed for the servo actuator's effects on the translation of the warhead. The robust sliding mode control law is designed as follows: the exponential reaching law is applied.

Appendix A: Specifications of the attitude dynamics equation

The detailed descriptions of all variables in Eq.(14) are shown as follows in this appendix.

\[
\Delta I = \begin{bmatrix}
0 & 0 & 0 \\
(1-\mu)m_y\delta^2 \sin^2 \phi & -(1-\mu)m_y\delta^2 \sin \phi \cos \phi & 0 \\
0 & -(1-\mu)m_y\delta^2 \sin \phi \cos \phi & (1-\mu)m_y\delta^2 \cos^2 \phi
\end{bmatrix}
\]

\[
M = -2(1-\mu)m_y \times \left[ I \times \left( \omega \times \frac{dx}{dt} \right) \right]
\]

\[
M = -2(1-\mu)m_y \times \left[ I \times \left( \omega \times (\omega \times \rho) \right) \right]
\]

\[
M = -(1-\mu)m_y \times \left( E \times (\omega \times (\omega \times \rho)) \right)
\]

\[
M = -(1-\mu)m_y \times \left[ E \times (\omega \times (\omega \times \rho)) \right]
\]

\[
M = -\omega \times (I - \omega \times \omega), \quad M = \begin{bmatrix}
0 & 0 & 0 \\
\mu & 0 & 0 \\
0 & \mu & 0
\end{bmatrix}
\]

\[
M = -\left[ I \times \left( \omega \times \omega \right) \right] F
\]

\[
M = -\left[ I \times \left( \omega \times \omega \right) \right] F
\]

\[\Delta I\] is the additional inertia tensor which is generated by the motion of the actuator mass. \(M_\delta\) is the normal gyroscopic moment; \(M_\beta\) is the moment related to the motion of the actuator mass and the angular velocity of the warhead body; \(M_r\) is the reaction moment caused by the eccentric motion of the actuator mass; \(M_p\) is the reaction moment exerted by the servo actuator in the body of warhead; which represents the servo loop's coupling effect on the attitude control loop. \(M_i\) is the aerodynamic drag moment caused by the displacement of the actuator mass, which is the main control moment of all, known as the control moment.

Appendix B: Detailed design of sliding mode controller

The dynamics Eqs. (22) and (23) can be rewritten in abbreviate form as

\[
\begin{align*}
\dot{x}_1 &= f_1 + B_1 x_1 + d_1 \\
\dot{x}_2 &= f_2 + B_2 u + d_2
\end{align*}
\]

where \(f_i, B_i\) are the state matrices of normal model. \(d_1\) and \(d_2\) denotes the function uncertainty, which is due to the modeling errors and external disturbances.

Let us take the second derivative of \(x_1\) with respect to time as

\[
\ddot{x}_1 = f_1 + B_1 x_1 + B_2 (f_2 + d_2) + B_1 B_2 u + d_1
\]

The motion of the closed-loop system using the sliding mode control law is composed of two modes. The first mode is a reaching mode where the states beginning from arbitrary initial state are attracted towards the sliding surface \(s = 0\). In the second mode, i.e., sliding mode, the states slide along the sliding surface \(s = 0\), and the state error \(e\) converges to zero because \(s = 0\). In the presence of uncertainty discontinuous control law is used for accomplishing sliding motion. Once the sliding surface has been chosen, a controller should be designed to make \(s = 0\) be an attractive surface.

Let us differentiate Eq.(27) with respect to time and then substitute Eq.(34)

\[
\ddot{s} = y_1 - \left[ f_1 + B_1 x_1 + B_2 (f_2 + d_2) + B_1 B_2 u + d_1 \right] + c \dot{e} + \dot{c} e
\]

where \(B_1 B_2 \neq 0\) and \(e_0 > 0\).

In order to ensure the movement performance in the reaching mode, the exponential reaching law is applied. The sliding mode control law is designed as follows:

\[
\delta = (B_1 B_2)^{-1} \left[ y_1 - f_1 - B_1 x_1 - B_2 f_2 - c \dot{e} - \dot{c} e - k e - \epsilon_1 \text{sgn}(s) \right]
\]

where \(k_1\) is a positive definite diagonal matrix, and \(\epsilon_1 > 0\).

Assumption: The uncertain functions satisfy for \(X \in \Omega\)

\[
\| -B_1 d_2 - d_1 \|_c \leq \rho_i
\]

where for \(m \in R^n\), \(\| m \|_c = \max_{1 \leq i \leq n} | m_i |\), \(\| m \| = \sum_{i=1}^n | m_i |\)

\[
\| m \|_c = \left( \sum_{i=1}^n | m_i |^2 \right)^{1/2}
\]

The sign of \(\rho_i\) is positive. The inequality Eq.(37) essentially restricts the uncertainty due to the modeling errors and external disturbances.
For the sliding mode controller design, consider a Lyapunov function
\[ V = \frac{s^T s}{2} \]  
(38)
The derivative of \( V \) along the solution of Eq.(35) is given by
\[ \dot{V} = s^T \left[ -Bd_z - d_1 s - k_i e \right] \text{sgn}(s) \]  
(39)
Substituting Eq.(37) into Eq.(39):
\[ \dot{V} \leq -k_i ||e||^2 - \epsilon_i ||e|| + \rho ||d|| \]  
(40)
In view of Eq.(40), the gain \( \epsilon_i \) is chosen as
\[ \epsilon_i > (\rho_i + \rho^*) \]  
(41)
Using Eq.(41) in Eq.(40) yields
\[ \dot{V} \leq -k_i ||e||^2 - \rho^* ||e|| \leq 0 \]  
(42)
From Eq.(42), it can be stated that the equilibrium state is stable. Finally, it can be shown that the error state \( e \) tends to zero as \( t \) tends to \( \infty \) by LaSalle-Yoshizawa theorem [19].

Note that the control law in Eq.(36) includes discontinuous functions which can cause chattering phenomenon. As a practical matter, chattering is undesirable because it involves very high control action and may excite high frequency un-modeled dynamics. The discontinuity in the control law can be dealt with by adopting thin boundary layer around the sliding surface.

\[ \text{sat}(s) = \begin{cases} 
-1 & s_i < -1 \\
\frac{s_i}{\Lambda_i} & ||s|| \leq 1 \\
1 & s_i > 1 
\end{cases} \]  
(43)
where \( \Lambda_i (i=1,2,3) \) is the thickness of the boundary layer.

Acknowledgments

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References

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- Equivalent mass point of moving masses

Initial position

Final position
Fig. 6

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Angle of Attack $\alpha$ (deg)

Time (sec)

Angle of Sideslip $\beta$ (deg)

Time (sec)
Fig. 11
<table>
<thead>
<tr>
<th>Parameters at the initial moment</th>
<th>Single moving mass control style ($m = 200, \text{kg}$)</th>
<th>Double moving mass control style ($m = m_1 = 100, \text{kg}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle between the rail and the positive y-axis of body frame</td>
<td>0</td>
<td>$\phi_1 = -\pi/2, \phi_2 = -\pi/2$</td>
</tr>
<tr>
<td>Displacement of the actuator masses in the rail</td>
<td>0 m</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Parameters at the final moment</td>
<td>$\pi/3$</td>
<td>$\phi_1 = 0, \phi_2 = 2\pi/3$</td>
</tr>
<tr>
<td>Angle between the rail and the positive y-axis of body frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement of the actuator masses in the rail</td>
<td>0.2 m</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Rotations rule of the rails</td>
<td>$\phi = \frac{\pi}{6} - \frac{\pi}{6} \cos(10, \text{m})$</td>
<td>$\phi_1 = \frac{\pi}{4} - \frac{\pi}{4} \cos(10, \text{m})$</td>
</tr>
<tr>
<td>Motions rule of the actuator masses in the rail</td>
<td>$\delta = 0.1 - 0.1 \cos(10, \text{m})$</td>
<td>$\delta_1 = \delta_2 = 0.2 \text{m}$</td>
</tr>
</tbody>
</table>