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To cite this article: Bin Xu, Fuchun Sun, Chenguang Yang, Daoxiang Gao & Jianxin Ren (2011): Adaptive discrete-time controller design with neural network for hypersonic flight vehicle via back-stepping, International Journal of Control, 84:9, 1543-1552

To link to this article: http://dx.doi.org/10.1080/00207179.2011.615866

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Adaptive discrete-time controller design with neural network for hypersonic flight vehicle via back-stepping

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(Received 6 June 2011; final version received 10 August 2011)

In this article, the adaptive neural controller in discrete time is investigated for the longitudinal dynamics of a generic hypersonic flight vehicle. The dynamics are decomposed into the altitude subsystem and the velocity subsystem. The altitude subsystem is transformed into the strict-feedback form from which the discrete-time model is derived by the first-order Taylor expansion. The virtual control is designed with nominal feedback and neural network (NN) approximation via back-stepping. Meanwhile, one adaptive NN controller is designed for the velocity subsystem. To avoid the circular construction problem in the practical control, the design of coefficients adopts the upper bound instead of the nominal value. Under the proposed controller, the semiglobal uniform ultimate boundedness stability is guaranteed. The square and step responses are presented in the simulation studies to show the effectiveness of the proposed control approach.

Keywords: hypersonic flight vehicle; back-stepping; neural network; discrete-time

1. Introduction

Hypersonic flight vehicles (HFVs) are intended to present a reliable and more cost efficient way to access space. The recent success of NASA’s X-43A experimental airplane in flight testing has affirmed the feasibility of this technology. The longitudinal model of the dynamics is known to be unstable, non-minimum phase with respect to the regulated output, and affected by significant model uncertainty. Therefore, HFVs are extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. For the controller design for hypersonic aircraft, it must guarantee stability and provide a satisfied control performance. Different considerations are included, such as the aerothermoelastic effects (Wilcox, MacKunis, Bhat, Lind, and Dixon 2009) and the control constrain (Serrani, Zinnecker, Fiorentini, Bolender, and Doman 2008, 2009). With different modelling analyses many methods have been applied on HFV control. Based on the input–output linearisation using Lie derivative notation, the sliding mode control (Xu, Mirmirani, and Ioannou 2004) is investigated for altitude and velocity control and the adaptive control (Du, Wu, Jiang and Wen 2010) is applied for attitude control. Robust control (Wang and Stengel 2000) took the genetic algorithm to search a design parameter space of the nonlinear-dynamic-inversion structure. The adaptive control (Gibson, Crespo, and Annaswamy 2009) and robust control (Hu, Sun, and Liu 2010) are investigated by linearising the model at the trim state. In Gibson et al. (2009), the control structure combines feedforward input, nominal feedback and adaptive feedback terms. The sequential loop closure controller design (Fiorentini, Serrani, Bolender, and Doman 2008, 2009) is based on the decomposition of the equations into functional subsystems. The method followed the approach that combined robust adaptive dynamic inversion with back-stepping arguments to obtain control architecture. Back-stepping design (Kokotovic 1991; Chen, Ge and How 2010; Chen, Jiao, Li, and Li 2010; Li, Chen, and Li 2011) is an effective way to deal with the system in strict-feedback form, into which the altitude subsystem of HFV is transformed in Gao and Sun (2011). Due to the system uncertainty, fuzzy logic system Gao and Sun (2011) and neural networks (NN; Liu and Lu 2009; Hu et al. 2010) are employed to compensate the unknown nonlinearities and modelling errors, which are caused by changes of flight conditions. With the same control problem, one high gain observer based neural controller (Xu, Gao, and Wang 2011) is proposed for
HFV with only one NN to approximate the lumped uncertainty.

With the development of hardware, computer control is drawing more and more attention. However, most of the research of HFV control is focusing on the continuous time and fewer works have been found in the literature for discrete control of HFV. For the modelling, the discrete HFV model is obtained by Taylor expansion in this article. By proper assumptions the discrete model is transformed into the strict-feedback form where some theoretical results have been studied. In Chen and Narendra (2001), robust estimation is formulated to design the optimal controller where the robust linear controller and neural adaptive control are switched by nonlinear identifiers. In Zhang, Wen, and Chai Soh (2001), the parametric uncertainty and unknown function are considered and the controller overcomes the overparametrisation problem by employing the prediction errors. In Yang, Ge, Xiang, Chai, and Lee (2008), the strict-feedback system is transformed into the prediction model and the general result with error feedback and NN compensation is given. Considering the peculiar features of HFV, it is difficult to get the prior information of the parameters for the linear estimation (Chen and Narendra 2001; Zhang et al. 2001). For the nonlinearity and the coupling, the nominal part of the dynamics should be considered for the feedback control design to provide good performance.

In this article, for the controller design we designed the controller step by step with the back-stepping scheme. The key idea of back-stepping is to start with a system which is stabilisable with a known feedback law for a known Lyapunov function, and then to add to its input an integrator. We take consideration of the nominal nonlinearity for feedback design and the NN is taken to approximate the system uncertainty. In order to avoid the circular construction of the control inputs, the upper bound is taken instead of the nominal value for the coefficients design.

This article is organised as follows. Section 2 describes the longitudinal dynamics of a generic HFV. The strict-feedback form is formulated and the discrete analysis model is obtained in Section 3. A brief description of higher order neural network (HONN) is explained in Section 4. Section 5 presents the adaptive neural controller design via back-stepping scheme and the stability analysis. The simulation result is included in Section 6. Section 7 presents several comments and final remarks.

The main contributions of the article lie in:

(i) The discrete-time controller design for HFV via back-stepping is presented and the simulation result shows the effectiveness.

(ii) The upper bound is adopted for the controller coefficients design. It solves the difficulty of circular construction.

(iii) A Lyapunov function is introduced for stability analysis and control design via back-stepping scheme.

2. Hypersonic vehicle modelling

The control-oriented model of the longitudinal dynamics of a generic hypersonic aircraft considered in this study is given by Wang and Stengel (2000). This model comprises five state variables $X = [V, h, \alpha, \gamma, q]^T$ and two control inputs $U_c = [\delta_e, \beta]^T$ where $V$ is the velocity, $\gamma$ is the flight path angle, $\alpha$ is the attack angle, $q$ is the pitch rate, $\delta_e$ is elevator deflection and $\beta$ is the throttle setting.

\[ \dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2}, \]
\[ \dot{h} = V \sin \gamma, \]
\[ \dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{V^2}, \]
\[ \dot{\alpha} = q - \dot{\gamma}, \]
\[ \dot{q} = \frac{M_{yy}}{I_{yy}}, \]

where $T$, $D$, $L$ and $M_{yy}$ represent thrust, drag, lift-force and pitching moment, respectively; $m$, $I_{yy}$ and $\mu$ represent the mass of aircraft, moment of inertia about pitch axis and gravity constant, respectively; $r$ is the radial distance from the centre of the earth and $r = h + R_E$. The related definitions are given as:

\[ \ddot{\alpha} = \frac{1}{2} \rho V^2, \quad L = \bar{q} S C_L, \quad D = \bar{q} S C_D, \quad T = \bar{q} S C_T, \]
\[ M_{yy} = \bar{q} S \left[ C_M(\alpha) + C_M(\delta e) + C_M(q) \right], \]
\[ C_L = 0.6203 \alpha, \]
\[ C_D = 0.6450 \alpha^2 + 0.0043378 \alpha + 0.003772, \]
\[ C_M(\alpha) = -0.035 \alpha^2 + 0.036617 \alpha + 5.3261 \times 10^{-6}, \]
\[ C_M(q) = (\bar{q} \tilde{\alpha} / 2 V) \times (-6.796 \alpha^2 + 0.3015 \alpha - 0.2289) \]

where $\rho$ denotes the air density, $S$ is the reference area, $\tilde{\alpha}$ is the reference length and $R_E$ is the radius of the earth. $C_x, x = L, D, T, M$ are the force and moment coefficients. The control inputs are involved in the following items:

\[ C_T = \begin{cases} 0.02576 \beta, & \text{if } \beta < 1, \\ 0.0224 + 0.00336 \beta, & \text{otherwise}, \end{cases} \]
\[ C_M(\delta e) = 0.0292 (\delta e - \alpha). \]
The engine dynamics are modelled by a second-order system:

$$\ddot{\beta} = -2\xi\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c.$$  \hfill (6)

From Equations (1)–(5), the velocity is mainly related to throttle setting and the rate of change of altitude is mainly related to the elevator deflection. So the dynamics can be decoupled into two functional subsystems. Given the tracking reference $V_d$ and $h_d$, we design the velocity and altitude controller separately in Section 5.

3. System transformation

3.1 Strict-feedback formulation

Assumption 1: Since $\gamma$ is quite small during the cruise phrase, we can take $\sin \gamma \approx \gamma$ in (2) for simplification. The thrust term $T\sin \alpha$ in (3) can be neglected because it is generally much smaller than $L$.

Assumption 2: The rate of change of velocity is slow and the magnitude of velocity is small so that the velocity can be approximately considered to be constant during the sampling period.

($\Sigma A$) Velocity subsystem

The velocity subsystem (1) can be rewritten as follows:

$$\dot{V} = f_V + g_V u_V,$$
$$u_V = \beta_c,$$

where $f_V = -(D/m + \mu \sin \gamma / r^2) + \tilde{q}S \times 0.0224 \cos \alpha / m$, $g_V = \tilde{q}S \times 0.00336 \cos \alpha / m$ if $\beta_c > 1$. Otherwise, $f_V = -(D/m + \mu \sin \gamma / r^2)$, $g_V = \tilde{q}S \times 0.02576 \cos \alpha / m$.

($\Sigma B$) Altitude subsystem

Define $X = [x_1, x_2, x_3, x_4]^T$, $x_1 = h$, $x_2 = \gamma$, $x_3 = \theta$, $x_4 = g$, $\theta = \alpha + \gamma$, $u_A = \delta_e$. With Assumptions 1 and 2, the dynamics (2)–(5) can be written as the strict-feedback form:

$$\dot{x}_1 = V\sin x_3 \approx Vx_2 = f_1(x_1) + g_1(x_1)x_2,$$
$$\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)x_3,$$
$$\dot{x}_3 = f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)x_4,$$
$$\dot{x}_4 = f_4(x_1, x_2, x_3, x_4) + g_4(x_1, x_2, x_3, x_4)u_A,$$

where

$$f_1 = 0, \quad g_1 = V,$$
$$f_2 = -(\mu - V^2)\cos \gamma / (V^2) - \tilde{q}S \times 0.6203 / (mV) \times \gamma,$$
$$g_2 = \tilde{q}S \times 0.6203 / (mV), \quad f_3 = 0, \quad g_3 = 1,$$
$$f_4 = \tilde{q}S(C_M(\alpha) + C_M(q) - 0.0292\alpha) / I_{yy},$$
$$g_4 = 0.0292\tilde{q}S / I_{yy}.$$

Assumption 3: $f_i$ and $g_i$ are unknown smooth functions. $f_i$ can be decomposed into the nominal part $f_{iN}$ and the unknown part $\Delta f_i$. There exist constants $\tilde{g}_i$ and $g_i$ such that $\tilde{g}_i \geq g_i \geq g_i > 0$, $i = 2, 4, V$.

Remark 1:

- In Section 5, $f_{iN}$ will be considered as the feedback item while $\tilde{g}_i$ will be employed to avoid the circular design of control inputs. To get the larger $\tilde{g}_V$, the coefficients of $g_V$ with $\beta_c < 1$ are considered.
- Though some items are constants such as $f_1$ and $f_3$, the formulation (8) is retained to make the expression and control design explicit.

3.2 Discrete-time model

By first-order Taylor expansion with sample time $T$, systems (7)–(8) can be approximated by a discrete-time model as

$$V(k + 1) = V(k) + T[\hat{f}_V(k) + g_V(k)u_V(k)], \quad (9)$$
$$x_1(k + 1) = x_1(k) + T[\hat{f}_1(k) + g_1(k)x_2(k)],$$
$$x_2(k + 1) = x_2(k) + T[\hat{f}_2(k) + g_2(k)x_3(k)],$$
$$x_3(k + 1) = x_3(k) + T[\hat{f}_3(k) + g_3(k)x_4(k)],$$
$$x_4(k + 1) = x_4(k) + T[\hat{f}_4(k) + g_4(k)u_A(k)].$$

It is noted that systems (9) and (10) are in strict-feedback form. It should be mentioned that the first-order Taylor expansion is just for analysis and the controller will be implemented on the real physical system (1)–(5) with (6).

4. HONN approximation

HONN (Ge, Huang, Lee, and Zhang 2001) is one kind of linearly parametrised NN. The structure of HONN is expressed as follows:

$$U(\theta, X) = \omega^T \theta(X) \omega, \quad \theta(X) \in R^N, \quad \theta_i(X) = \prod_{j \in I_i}[s(X_j)]^{\omega_i},$$
where $X \subset R^m$ is the input to HONN, $N$ is the NN nodes number, $\{I_1, I_2, \ldots, I_N\}$ is a collection of $N$ not-ordered subsets of $\{1, 2, \ldots, m\}$, specified by the designer, $d_i$'s are prescribed non-negative integers, $\omega$ is an adjustable synaptic weight vector and $s(X)$ is a monotonically increasing and differentiable sigmoidal function. In this article, it is chosen as a hyperbolic tangent function, i.e. $s(X) = (e^{x_i} - e^{-x_i})/(e^{x_i} + e^{-x_i})$.

For a desired function $U^*$, it is assumed there exists an ideal weight vector $\omega^*$ such that the smooth function vector can be approximated by an ideal NN on a compact set

$$U^* = \omega^* T\theta(X) + \varepsilon(X), \quad ||\varepsilon(X)|| < \varepsilon_M, $$

where $\varepsilon(X)$ is the bounded NN approximation error vector and $\varepsilon_M$ is the supreme of $\varepsilon(X)$.

5. Discrete control design

5.1 Adaptive NN control via back-stepping for the altitude subsystem

The desired controller can be obtained by performing the following back-stepping design procedures. The errors are defined as

$$z_1(k) = x_1(k) - x_1d(k),$$

$$z_2(k) = x_2(k) - x_2d(k),$$

$$z_3(k) = x_3(k) - x_3d(k),$$

$$z_4(k) = x_4(k) - x_4d(k),$$

where $x_1d(k) = h_1d(k)$, the desired reference trajectory, and $x_2d(k), x_3d(k), x_4d(k)$ are the desired virtual control inputs to be designed.

**Step 1:** From (14),

$$z_1(k+1) = x_1(k) + T_s [f_1(k) + g_1(k) x_2(k)] - x_1d(k+1),$$

where $x_1d(k+1) = h_1d(k+1)$ is the reference trajectory.

Take $x_2(k)$ in (18) as the virtual control input and design its desired value as

$$x_2d(k) = [T_s g_1(k)]^{-1} [-x_1(k) + c_1 z_1(k) + x_1d(k+1) - T_s f_1(k)],$$

where $0 < c_1 < 1$, $g_1(k) = V(k)$.

By combining (15), (18) and (19), the following equation can be obtained.

$$z_1(k+1) = c_1 z_1(k) + T_s g_1(k) z_2(k).$$

**Step 2:** From (15),

$$z_2(k+1) = x_2(k) + T_s [f_2(k) + g_2(k) x_3(k)] - x_2d(k+1),$$

Define $X_2(k) = [V(k), x_1(k), x_2(k), x_1d(k+1), x_2d(k+1)]^T$. The uncertainty $U_2$ is defined and approximated by NN as

$$U_2(k) = \frac{\hat{g}_2}{\hat{g}_2(k)} [-x_2(k) - T_s f_2(k) + x_2d(k+1)] - [-x_2(k) - T_s f_2(k)]$$

$$= \omega^*_2 T\theta_2(X_2(k)) + \varepsilon_2(X_2(k)), $$

(22)

where $f_2(k)$ is the nominal part of $f_2(k), x_2d(k+1)$ is the future desired control input value as in (19), $\omega^*_2$ is the optimal parameters for NN to approximate $U_2(k)$ and $\varepsilon_2(X_2)$ is the NN reconstruction error.

Take $x_3(k)$ in (21) as the virtual control input and design its desired value as

$$x_3d(k) = [T_s \hat{g}_2(k)]^{-1} [-x_2(k) - T_s f_2(k) + c_2 z_2(k) + \omega^*_2 T\theta_2(X_2(k))],$$

(23)

where $0 < c_2 < 1$ and $\omega^*_2$ is the estimation of $\omega^*_2$.

By combining (16), (21) and (23), the following equation can be obtained.

$$z_2(k+1) = x_2(k) + T_s [f_2(k) + g_2(k) x_3(k)] - x_2d(k+1)$$

$$= x_2(k) + T_s [f_2(k) + g_2(k) [z_3(k) + x_3d(k)]] - x_2d(k+1)$$

$$= T_s g_2(k) z_3(k) + \frac{g_2(k)}{\hat{g}_2} c_2 z_2(k) + \frac{g_2(k)}{\hat{g}_2} \times [\omega^*_2(k) \theta_2(X_2(k)) - U_2(k)]$$

$$= \frac{g_2(k)}{\hat{g}_2} c_2 z_2(k) + T_s g_2(k) z_3(k) + \frac{g_2(k)}{\hat{g}_2} \times [\omega^*_2(k) \theta_2(X_2(k)) - \varepsilon_2(X_2(k))],$$

(24)

where $\omega^*_2(k) = \hat{\omega}_2(k) - \omega^*_2$. The update law for the NN weights is

$$\hat{\omega}_2(k+1) = \hat{\omega}_2(k) - \lambda_2 z_2(k+1) \theta_2(X_2(k)) - \lambda_2 \hat{\omega}_2(k),$$

(25)

**Remark 2:** If the nominal part $g_2$ of $g_2$ is employed for controller design, we have the following expression:

$$z_2(k+1) = x_2(k) + T_s [f_2(k) + g_2(k) x_3(k) + (g_2(k) + \Delta g_2(k)) x_3d(k)] - x_2d(k+1)$$

$$= x_2(k) + T_s f_2(k) + T_s g_2(k) x_3d(k)$$

$$+ T_s g_2(k) z_3(k) + T_s [\Delta g_2(k) + \Delta g_2(k)] x_3d(k) - x_2d(k+1).$$

(26)

The uncertainty will include $\Delta g_2(k), \Delta g_2(k) x_3d(k)$ and $x_2d(k+1)$. The uncertainty is to be approximated by NN while NN is part of $x_3d(k)$. This will lead to the circular design. So we employ the upper bound of $g_2$ in the design to avoid this problem.
Step 3: From (16),
\[ z_3(k+1) = x_3(k) + T_s[f_3(k) + g_3(k)x_4(k)] - x_3d(k+1). \] (27)

Define \( X_3(k) = [V(k), x_1(k), x_2(k), x_3(k), x_4(k), x_1d(k+2), x_1d(k+3)]^T \). From (23), \( x_3(k+1) \) involves \( x_3(k+1), f_3(k+1), z_3(k+1) \) and \( x_3d(k+2) \). It can be concluded that \( x_3d(k+1) \) is the function of \( X_3(k) \).

The uncertainty \( U_3(k) \) is defined and can be approximated by NN as
\[ U_3(k) = x_3d(k+1) - \omega_3^* T \theta_3(X_3(k)) + \varepsilon_3(X_3(k)), \] (28)
where \( \omega_3^* \) is the optimal parameters and \( \varepsilon_3(X_3(k)) \) is the NN reconstruction error.

Take \( x_4(k) \) in (27) as the virtual control input and design its desired value as
\[ x_4d(k) = [T_s g_3(k)]^{-1}[-x_3(k) + c_3 z_3(k) + \omega_3^* T \theta_3(X_3(k))], \] (29)
where \( 0 < c_3 < 1 \) and \( \omega_3 \) is the estimation of \( \omega_3^* \).  

Remark 3: Actually the construction of \( U_3(k) \) and \( x_4d(k) \) is as same as \( U_2(k) \) and \( x_4d(k) \). The result of (28) and (29) is due to the fact that \( f_3 = 0 \) and \( g_3 = 1 \).

By combining (17), (27) and (29), the following equation can be obtained.
\[ z_3(k+1) = c_3 z_3(k) + T_s g_3(k) z_4(k) + [\dot{\omega}_3(k) \theta_3(X_3(k)) - \varepsilon_3(X_3(k))], \] (30)
where \( \dot{\omega}_3(k) = \dot{\omega}_3(k) - \omega_3^* \).

The update law for the NN weights is
\[ \dot{\omega}_3(k + 1) = \dot{\omega}_3(k) - \lambda_3 z_3(k + 1) \theta_3(X_3(k)) - \delta_3 \dot{\omega}_3(k). \] (31)

Step 4: From (17),
\[ z_4(k+1) = x_4(k) + T_s [f_4(k) + g_4(k) u_4(k)] - x_4d(k+1). \] (32)

Define \( X_4(k) = [V(k), x_1(k), x_2(k), x_3(k), x_4(k), x_1d(k+2), x_1d(k+3), x_1d(k+4)]^T \). Similarly we can deduce that the uncertainty \( U_4(k) \) is the function of \( X_4(k) \) and it can be approximated by NN as
\[ U_4(k) = [\tilde{g}_4(k)]^{-1}[x_4(k) - T_s f_4(k) + x_4d(k+1)] \]
\[ = [\tilde{g}_4(k)]^{-1}[-x_4(k) - T_s f_4(k) + x_4d(k+1)] \]
\[ = \omega_4^* T \theta_4(X_4(k)) + \varepsilon_4(X_4(k)), \] (33)
where \( \omega_4^* \) is the optimal parameters for NN to approximate \( U_4(k) \) and \( \varepsilon_4(X_4(k)) \) is the NN reconstruction error. The actual control input is designed as
\[ u_4(k) = [T_s \tilde{g}_4(k)]^{-1} \]
\[ \times [-x_4(k) - T_s f_4(k) + c_4 z_4(k) + \dot{\omega}_4^T(k) \theta_4(X_4(k))], \] (34)
where \( 0 < c_4 < 1 \) and \( \dot{\omega}_4 \) is the estimation of \( \omega_4^* \). Then
\[ z_4(k+1) = \frac{g_4(k)}{T_s} x_4z_4(k) + \frac{g_4(k)}{g_4(k)} \]
\[ \times [\dot{\omega}_4^T(k) \theta_4(X_4(k)) - \varepsilon_4(X_4(k))], \] (35)
where \( \dot{\omega}_4(k) = \dot{\omega}_4(k) - \omega_4^* \). The update law for the NN weights is
\[ \dot{\omega}_4(k + 1) = \dot{\omega}_4(k) - \lambda_4 z_4(k + 1) \theta_4(X_4(k)) - \delta_4 \dot{\omega}_4(k). \] (36)

Theorem 5.1: Considering system (10) with the controller (19), (23), (29), (34) and the update law (25), (31), (36), all the signals involved are semiglobal uniform ultimate bounded.

Proof: The following Lyapunov function is considered.
\[ L(k) = L_1(k) + L_2(k) + L_3(k) + L_4(k), \] (37)
where
\[ L_1(k) = \left(\frac{z_1(k)}{g_1}\right)^2, \quad L_2(k) = L_{21}(k) + L_{22}(k), \]
\[ L_3(k) = L_{31}(k) + L_{32}(k), \quad L_4(k) = L_{41}(k) + L_{42}(k), \]
with
\[ L_{21}(k) = z_2^2(k), \quad L_{22}(k) = \frac{\dot{\omega}_2^T(k) \tilde{g}_2(k)}{\lambda_2}, \quad L_{31}(k) = z_3^2(k), \]
\[ L_{32}(k) = \frac{\dot{\omega}_3^T(k) \tilde{g}_3(k)}{\lambda_3}, \quad L_{41}(k) = z_4^2(k) \]
\[ L_{42}(k) = \frac{\dot{\omega}_4^T(k) \tilde{g}_4(k)}{\lambda_4}. \]

Define the first difference of \( L(k) \) as
\[ \Delta L(k) = L(k+1) - L(k) \]
\[ = \Delta L_1(k) + \Delta L_2(k) + \Delta L_3(k) + \Delta L_4(k). \] (38)

The fact is known as \( 2ab \leq \rho a^2 + \frac{1}{\rho} b^2 \), where \( \rho \) is constant and positive.

The first item:
\[ \Delta L_1(k) = \frac{(z_1(k+1)/\tilde{g}_1)^2 - (z_1(k)/\tilde{g}_1)^2}{(1/\tilde{g}_1)^2} \]
\[ = \frac{(1/\tilde{g}_1)^2}{(c_1 z_1 + T_s \delta_1(k) z_2(k))^2 - z_1(k)^2} \]
\[ \leq -\tau_{11} L_1(k) + \tau_{12} L_{23}(k), \]
where
\[ \tau_{11} = \frac{1 - c_1^2 - \rho_1 c_1^2}{\tilde{g}_1^2}, \quad \tau_{12} = T_s^2 \left(1 + \frac{1}{\rho_1}\right). \]
The second item: The following items are calculated according to the appendix.

\[
\Delta L_2(k) = z_2^2(k+1) - z_2^2(k) \\
+ \frac{\tilde{\alpha}_2^2(k+1)\tilde{\omega}_2(k+1) - \tilde{\alpha}_2^2(k)\tilde{\omega}_2(k)}{\lambda_2} \\
\leq -\tau_{21}L_{21}(k) - \tau_{22}L_{22}(k) + \tau_{23} + \tau_{24}L_{31}(k),
\]

(40)

where

\[
\tau_{21} = \frac{\delta_2}{\lambda_2}, \quad \tau_{22} = 1 - c_2\rho_{22}^{-1}, \quad \tau_{23} = \frac{\lambda_2\rho_{21}\varepsilon_2^2 + \delta_2\|\omega_2^e\|^2}{\lambda_2}, \quad \tau_{24} = T_\alpha\tilde{g}_2\rho_{24}
\]

Similarly for the third and fourth item, the following conclusions can be acquired.

\[
\Delta L_3(k) = z_3^2(k+1) - z_3^2(k) \\
+ \frac{\tilde{\alpha}_3^2(k+1)\tilde{\omega}_3(k+1) - \tilde{\alpha}_3^2(k)\tilde{\omega}_3(k)}{\lambda_3} \\
\leq -\tau_{31}L_{31}(k) - \tau_{32}L_{32}(k) + \tau_{33} + \tau_{34}L_{41}(k),
\]

(41)

where

\[
\tau_{31} = \frac{\delta_3}{\lambda_3}, \quad \tau_{32} = 1 - c_3\rho_{32}^{-1}, \quad \tau_{33} = \frac{\lambda_3\rho_{31}\varepsilon_3^2 + \delta_3\|\omega_3^e\|^2}{\lambda_3}, \quad \tau_{34} = T_\alpha\tilde{g}_3\rho_{34}
\]

\[
\Delta L_4(k) = z_4^2(k+1) - z_4^2(k) \\
+ \frac{\tilde{\alpha}_4^2(k+1)\tilde{\omega}_4(k+1) - \tilde{\alpha}_4^2(k)\tilde{\omega}_4(k)}{\lambda_4} \\
\leq -\tau_{41}L_{41}(k) - \tau_{42}L_{42}(k) + \tau_{43},
\]

(42)

where

\[
\tau_{41} = \frac{\delta_4}{\lambda_4}, \quad \tau_{42} = 1 - c_4\rho_{42}^{-1}, \quad \tau_{43} = \frac{\lambda_4\rho_{41}\varepsilon_4^2 + \delta_4\|\omega_4^e\|^2}{\lambda_4}
\]

Combining (39)–(42), we have the following inequality.

\[
\Delta L(k) \leq -\tau_{11}L_1(k) + \tau_{12}L_{21}(k) - \tau_{21}L_{22}(k) - \tau_{22}L_{22}(k) + \tau_{23} + \tau_{24}L_{31}(k) - \tau_{31}L_{31}(k) - \tau_{32}L_{32}(k) + \tau_{33} + \tau_{34}L_{41}(k) - \tau_{41}L_{41}(k) - \tau_{42}L_{42}(k) + \tau_{43} \\
\leq -K_1L_1(k) - K_2L_2(k) - K_3L_3(k) - K_4L_4(k) + K_5,
\]

(43)

where the proper positive parameters \(\rho_{ij}\) can be designed to have the following results:

\[
K_1 = \tau_{11} > 0, \quad K_2 = \min(\tau_{12}, \tau_{21}, \tau_{22}) > 0, \quad K_3 = \min(\tau_{31}, \tau_{24}, \tau_{32}) > 0, \quad K_4 = \min(\tau_{41}, \tau_{34}, \tau_{42}) > 0, \quad K_5 = \tau_{23} + \tau_{33} + \tau_{43} > 0.
\]

Take \(r_A = \min(K_1, K_2, K_3, K_4)\), then \(\Delta L(k) \leq -r_AL(k) + K_5\). From Ioannou and Sun (1996), we know \(L(k)\) is bounded so \(z_i(k), i = 1, 2, 3, 4\) and \(\omega_i(k), j = 2, 3, 4\) are semiglobal uniform ultimate bounded. This completes the proof.

**Remark 4:** Summarising from 1 to \(k\)

\[
\Delta L(k) - L(1) \leq \sum_{i=1}^{k-1}(-r_AL(i) + K_5),
\]

\[
\lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k-1} L(i) \leq L(1) \lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k-1} (-r_AL(i) + K_5).
\]

Using that \(L(k)\) is bounded and that \(L(1)\) is the constant:

\[
\lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k-1} -r_AL(i) \leq L(1) \lim_{k \to \infty} \frac{1}{k} + K_5.
\]

Then we have the upper bound of the selected Lyapunov function.

\[
\lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k-1} L(i) \leq K_4 \frac{1}{r_A}.
\]

### 5.2 Adaptive NN control for the velocity subsystem

Define \(X_1(k) = [V(k), x_1(k), x_2(k), x_3(k), x_4(k), V_d(k+1)]^T\) and \(z_1(k) = V(k) - V_d(k), z_1(k+1) = V(k+1) - V_d(k+1)\)

\[
z_1(k+1) = V(k+1) - V_d(k+1) = V(k) + T_\alpha\tilde{g}_1(f_1(k) + g_1(k)u_1(k)) - V_d(k+1).
\]

The following uncertainty \(U_1(k)\) is the function of \(X_1(k)\). It is approximated by NN as

\[
U_1(k) = \frac{\tilde{g}_V(k)}{g_1(k)}[-V(k) - T_\alpha f_1(k) + V_d(k+1)] - [-V(k) - T_\alpha f_1(k) + V_d(k+1)] = \omega_{1}^T\theta_1(X_1(k)) + \varepsilon_1(X_1(k)),
\]

(46)

where \(f_1(k)\) is the nominal part of \(f_1(k), \omega_{1}^T\) is the optimal parameters for NN to approximate \(U_1(k)\) and
\( \varepsilon I(k) \) is the NN reconstruction error. The control input is designed as follows.

\[
u_I(k) = \frac{1}{T_s \hat{g}_I} \left[ -T_s f_{IN}(k) - V(k) + c_I z_I(k) + \dot{\omega}_I \theta_I(X_I(k)) \right],
\]

where \( 0 < c_I < 1 \) and \( \dot{\omega}_I \) is the estimation of \( \omega_I^* \).

Then Equation (45) can be derived as

\[
z_I(k+1) = V(k+1) - V_d(k+1)
= V(k) + T_s [f_I(k) + g_I(k) u_I(k)] - V_d(k+1)
= \frac{g_I(k)}{\hat{g}_I} c_I z_I(k) + \frac{g_I(k)}{\hat{g}_I} \\
\times [\dot{\omega}_I \theta_I(X_I(k)) - \varepsilon I(X_I(k))],
\]

where \( \dot{\omega}_I = \dot{\omega}_I(k) - \omega_I^* \).

The update law for NN weights is given as

\[
\dot{\omega}_I(k + 1) = \dot{\omega}_I(k) - \lambda_I z_I(k+1) \theta_I(X_I(k)) - \delta_I \dot{\omega}_I(k).
\]

**Theorem 5.2:** Considering system (9) with the controller (47) and the update law (49), the velocity is semiglobal uniform ultimate bounded. The proof is similar to that of Theorem 5.1 and thus omitted here.

### 6. Simulations

In this section, we verify the effectiveness and performance of the proposed adaptive neural controller. Reference commands are generated by the filter:

\[
\frac{h_d}{h_c} = \frac{\omega_{n1}^2}{(s + \omega_{n1})(s^2 + 2\xi \omega_{n2} s + \omega_{n2}^2)},
\]

where \( \omega_{n1} = 0.2, \omega_{n2} = 0.5, \xi = 0.7, \omega_n = 10 \).

The parameters for the controller are selected as \( \lambda_I = 0.05, \delta_I = 0.02, \lambda_2 = 0.05, \lambda_3 = 0.05, \lambda_4 = 0.05, \delta_2 = 0.02, \delta_3 = 0.02, \delta_4 = 0.02, T_s = 0.05s, c_I = 0.9, c_2 = 0.9, c_3 = 0.9, c_4 = 0.7, c_I = 0.86 \). From the definition of \( g_i, i = 2, 4, V \), we know the upper bound is up to the upper bound of \( V \) which is selected as 15,200.

(1) **Square signal tracking of altitude** (Figures 1–3):

The tracking performance is depicted in Figure 1(a) where the altitude controller tracks the square signal with magnitude 200 ft. Figure 1(b) depicts the velocity tracking error which is maintained in the neighbourhood of 15,060 ft/s. From Figure 2, we find that the first square response costs more control inputs than the
second one. The reason can be found in the tracking of the NN weights depicted in Figure 3 from 80 s to 100s that the NN has learned the characteristics of the HFV dynamics.

(2) Step tracking of altitude and velocity (Figures 4–6): Figure 5 depicts the response performance that the altitude controller tracks the step change with magnitude 200 ft while the velocity steps from 15,060 ft/s to 15,160 ft/s. The control inputs of the elevator deflection and the throttle setting are shown in Figure 6(a) and (b). Compared with Figure 3, the magnitude of $\omega$ is larger. This is due to that test II is the step change at altitude and velocity simultaneously so the system is changing faster than test I. From Figures 2 and 6, we can see that the magnitude of elevator deflection is quite similar but the throttle setting in test II is larger than test I. It indicates that the functional decomposition is reasonable.

7. Conclusion
The adaptive neural controller via the back-stepping scheme is proposed in this article. Considering the characteristics of HFV, the nominal part of the nonlinearity during each step is eliminated and the NN is taken to approximate the system uncertainty. The upper bound of the coefficients for the virtual control inputs is taken instead of the nominal value. It avoids the circular design of the control inputs. Simulation result shows the effectiveness of this method.

Acknowledgements
This work was supported by Sino Swiss Science and Technology Cooperation, the National Science Foundation of China (Grant Nos: 60625304, 90716021) and the National Key Project for Basic Research of China (Grant Nos: G2007CB311003, 2009CB724002).

References


**Appendix**

From (25) and \( \hat{\omega}_2(k) = \hat{\omega}_2(k) - \omega_2^* \), we have

\[
\hat{\omega}_2(k + 1) = \hat{\omega}_2(k) - \lambda_2 T \theta_2(X_2(k)) z_2(k + 1) - \delta_2 \hat{\omega}_2(k). \tag{A1}
\]

Noting the following facts

\[
\phi_2^2(X_2(k)) \leq N_2, \tag{A2}
\]

where \( N_2 \) is the number of the NN rules.

From (24), we have

\[
\hat{\omega}_2(k) \theta_2(X_2(k)) z_2(k + 1) = \frac{\hat{g}_2}{g_2(k)} z_2^1(k + 1) + \epsilon_2 z_2(k + 1) + \epsilon_2(k) z_2(k + 1)
\]

\[
- T_1 \hat{g}_2 z_2(k + 1), \tag{A3}
\]

\[
2 \epsilon_2(k) z_2(k + 1) \leq \rho_{21} \epsilon_2^2 + \rho_{21}^1 z_2^2(k + 1). \tag{A4}
\]

\[
2 \epsilon_2(k + 1) z_2(k) \leq \rho_{22} \epsilon_2^2 + \rho_{22}^1 z_2^2(k), \tag{A5}
\]

\[
2 \hat{\omega}_2(k) \theta_2(X_2(k)) z_2(k + 1) \leq \hat{\omega}_2^1(k) \theta_2(X_2(k)) z_2(k + 1) + \| \hat{\omega}_2(k) \|^2 - \| (\hat{\omega}_2(k) - \omega_2) \|^2
\]

\[
\leq \hat{\omega}_2^1(k) \theta_2(X_2(k)) z_2(k + 1) + \| \omega_2^* \|^2, \tag{A6}
\]

\[
2 \hat{\omega}_2(k) \theta_2(X_2(k)) z_2(k + 1) \leq \rho_{23} \| \hat{\omega}_2(k) \|^2 + \rho_{23}^1 \| \theta_2(X_2(k)) z_2(k + 1) \|^2
\]

\[
\leq \rho_{23} \| \hat{\omega}_2(k) \|^2 + \rho_{23}^1 N_2 z_2^2(k + 1), \tag{A7}
\]

\[
2 T_1 \hat{g}_2 z_2(k + 1) \leq T_1 \hat{g}_2 \rho_{24} z_2^1(k) + T_1 \hat{g}_2 \rho_{24}^1 z_2^1(k + 1). \tag{A8}
\]
\[ \tilde{\alpha}_2^2(k + 1) \tilde{\omega}_2(k + 1) \]
\[ \leq \tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) + \lambda_2^2 N_2 \tilde{\omega}_2^2(k + 1) + \delta_2^2 \| \tilde{\omega}_2(k) \|^2 \]
\[ - 2\lambda_2 \left( \frac{\tilde{g}_2}{\tilde{g}_2(k)} \right)^2 \tilde{\omega}_2^2(k + 1) - c_2 \tilde{z}_2(k) \tilde{z}_2(k + 1) + e_2(k) \tilde{z}_2(k + 1) \]
\[ - T \tilde{g}_2 \tilde{z}_2(k) \tilde{z}_2(k + 1) \]
\[ - \delta_2 (\tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) + \| \tilde{\omega}_2(k) \|^2 - \| \omega_2(k) \|^2) \]
\[ + \lambda_2 \delta_2 (\rho_{23} \| \tilde{\omega}_2(k) \|^2 + \rho_{23}^{-1} N_2 \tilde{\omega}_2^2(k + 1)) \]
\[ \leq \tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) + \lambda_2^2 N_2 \tilde{\omega}_2^2(k + 1) + \delta_2^2 \| \tilde{\omega}_2(k) \|^2 \]
\[ - 2\lambda_2 \left( \frac{\tilde{g}_2}{\tilde{g}_2(k)} \right)^2 \tilde{\omega}_2^2(k + 1) + c_2 \lambda_2 \rho_{22} \tilde{\omega}_2^2(k + 1) \]
\[ + c_2 \lambda_2 \rho_{22}^{-1} \tilde{\omega}_2^2(k) + \lambda_2 \rho_{21} \tilde{e}_2^2 M + \lambda_2 \rho_{21}^{-1} \tilde{z}_2^2(k + 1) \]
\[ - \delta_2 (\tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) + \| \tilde{\omega}_2(k) \|^2 - \| \omega_2(k) \|^2) \]
\[ + \lambda_2 \delta_2 (\rho_{23} \| \tilde{\omega}_2(k) \|^2 + \rho_{23}^{-1} N_2 \tilde{\omega}_2^2(k + 1)) + T \tilde{g}_2 \rho_{24} \tilde{z}_2^2(k) \]
\[ + T \tilde{g}_2 \rho_{24}^{-1} \tilde{z}_2^2(k + 1) \]
\[ \leq (1 - \delta_2) \tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) + (c_2 \lambda_2 \rho_{22}^{-1} M + T \tilde{g}_2 \rho_{24}) \tilde{z}_2^2(k) \]
\[ + T \tilde{g}_2 \rho_{24} \tilde{z}_2^2(k) + \lambda_2 \rho_{21} \tilde{e}_2^2 M + \delta_2 \| \tilde{\omega}_2(k) \|^2 \]
\[ + \delta_2 (\delta_2 - 1 + \lambda_2 \rho_{23}) \| \tilde{\omega}_2(k) \|^2 \]
\[ + \left( \lambda_2^2 N_2 - 2\lambda_2 \left( \frac{\tilde{g}_2}{\tilde{g}_2(k)} \right)^2 \right) \tilde{\omega}_2^2(k + 1) + c_2 \lambda_2 \rho_{22} + \lambda_2 \rho_{21}^{-1} \tilde{\omega}_2^2(k + 1) \]
\[ + \lambda_2 \delta_2 \rho_{23}^{-1} N_2 \tilde{\omega}_2^2(k + 1). \] (A9)

Then we have
\[ \Delta L_2(k) = L_2(k + 1) - L_2(k) \]
\[ = \tilde{\alpha}_2^2(k + 1) + \frac{\tilde{\alpha}_2^2(k + 1) \tilde{\omega}_2(k + 1)}{\lambda_2} - \tilde{\alpha}_2^2(k) \]
\[ - \frac{\tilde{\alpha}_2^2(k) \tilde{\omega}_2(k)}{\lambda_2} \]
\[ \leq - \delta_2 \tilde{\alpha}_2^2(k) \tilde{\omega}_2(k) \left( 1 - c_2 \rho_{22}^{-1} + \frac{T \tilde{g}_2 \rho_{24}}{\lambda_2} \right) \tilde{z}_2^2(k) \]
\[ + \lambda_2 \rho_{21} \tilde{e}_2^2 M + \delta_2 \| \tilde{\omega}_2(k) \|^2 + T \tilde{g}_2 \rho_{24} \tilde{z}_2^2(k) \]
\[ + \delta_2 (\delta_2 - 1 + \lambda_2 \rho_{23}) \| \tilde{\omega}_2(k) \|^2 \]
\[ + (\lambda_2 N_2 + c_2 \rho_{22} + \rho_{21}^{-1} \delta_2 \rho_{23}^{-1} N_2 - 1) \tilde{z}_2^2(k + 1), \] (A10)

provided the following inequalities hold:
\[ \lambda_2 N_2 + c_2 \rho_{22} + \rho_{21}^{-1} \delta_2 \rho_{23}^{-1} N_2 - 1 \leq 0, \]
\[ \delta_2 + \lambda_2 \rho_{23} - 1 \leq 0. \]

The following inequality is obtained
\[ \Delta L_2(k) \leq - \tau_{21} L_2(k) - \tau_{22} L_2(k) + \tau_{23} + \tau_{24} L_3(k), \] (A11)

where
\[ \tau_{21} = \frac{\delta_2}{\lambda_2}, \quad \tau_{22} = 1 - c_2 \rho_{22}^{-1} + \frac{T \tilde{g}_2 \rho_{24}}{\lambda_2}, \]
\[ \tau_{23} = \frac{\lambda_2 \rho_{21} \tilde{e}_2^2 M + \delta_2 \| \tilde{\omega}_2 \|^2}{\lambda_2}, \quad \tau_{24} = \frac{T \tilde{g}_2 \rho_{24}}{\lambda_2}. \]