Mitigation of Lower Order Harmonics in a Grid-Connected Single-Phase PV Inverter

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Abstract—In this paper, a simple single-phase grid-connected photovoltaic (PV) inverter topology consisting of a boost section, a low-voltage single-phase inverter with an inductive filter, and a step-up transformer interfacing the grid is considered. Ideally, this topology will not inject any lower order harmonics into the grid due to high-frequency pulse width modulation operation. However, the nonideal factors in the system such as core saturation-induced distorted magnetizing current of the transformer and the dead time of the inverter, etc., contribute to a significant amount of lower order harmonics in the grid current. A novel design of inverter current control that mitigates lower order harmonics is presented in this paper. An adaptive harmonic compensation technique and its design are proposed for the lower order harmonic compensation. In addition, a proportional-resonant-integral (PRI) controller and its design are also proposed. This controller eliminates the dc component in the control system, which introduces even harmonics in the grid current in the topology considered. The dynamics of the system due to the interaction between the PRI controller and the adaptive compensation scheme is also analyzed. The complete design has been validated with experimental results and good agreement with theoretical analysis of the overall system is observed.

Index Terms—Adaptive filters, harmonic distortion, inverters, solar energy.

I. INTRODUCTION

RENEWABLE sources of energy such as solar, wind, and geothermal have gained popularity due to the depletion of conventional energy sources. Hence, many distributed generation (DG) systems making use of the renewable energy sources are being designed and connected to a grid. In this paper, one such DG system with solar energy as the source is considered.

The topology of the solar inverter system is simple. It consists of the following three power circuit stages:

1) a boost converter stage to perform maximum power point tracking (MPPT);
2) a low-voltage single-phase H-bridge inverter;
3) an inductive filter and a step-up transformer for interfacing with the grid.

Fig. 1 shows the power circuit topology considered. This topology has been chosen due to the following advantages: The switches are all rated for low voltage which reduces the cost and lesser component count in the system improves the overall reliability. This topology will be a good choice for low-rated PV inverters of rating less than a kilowatt. The disadvantage would be the relatively larger size of the interface transformer compared to topologies with a high-frequency link transformer [1].

The system shown in Fig. 1 will not have any lower order harmonics in the ideal case. However, the following factors result in lower order harmonics in the system: The distorted magnetizing current drawn by the transformer due to the nonlinearity in the B–H curve of the transformer core, the dead time introduced between switching of devices of the same leg [2]–[6], on-state voltage drops on the switches, and the distortion in the grid voltage itself.

There can be a dc injection into the transformer primary due to a number of factors. These can be the varying power reference from a fast MPPT block from which the ac current reference is generated, the offsets in the sensors, and A/D conversion block in the digital controller. This dc injection would result in even harmonics being drawn from the grid, again contributing to a lower power quality.

It is important to attenuate these harmonics in order for the PV inverter to meet standards such as IEEE 519-1992 [7] and IEEE 1547-2003 [8]. Hence, this paper concentrates on the design of the inverter current control to achieve a good attenuation of the lower order harmonics. It must be noted that attenuating the lower order harmonics using a larger output filter inductance is not a good option as it increases losses in the system along with a larger fundamental voltage drop and with a higher cost. The boost stage and the MPPT scheme are not discussed in this paper as a number of methods are available in the literature to achieve a very good MPPT [9]–[13].

There has been considerable research work done in the area of harmonic elimination using specialized control. In [15]–[22], multiresonant controller-based methods are used for selective harmonic elimination. The advantage of these methods is the simplicity in implementation of the resonant blocks. However, discretization and variations in grid frequency affect the performance of these controllers and making them frequency adaptive increases overall complexity [21], [22]. Also, as mentioned in [19], [21], and [22], the phase margin of the system becomes small with multiresonant controllers and additional compensation is required for acceptable operation. The study in [23]–[25] considers the use of repetitive controller-based harmonic elimination which involves complicated analysis and design. As mentioned in [26], the performance of the repetitive controller is very sensitive to frequency variations and needs structural change for better performance, which might affect the stability.
In [27]–[31], adaptive filter-based controllers are considered for harmonic compensation. The study in [27] uses an adaptive filter to estimate a harmonic and then adds it to the main current reference. Then, a multiresonant block is used to ensure zero steady-state error for that particular harmonic reference. Thus, the study in [27] uses both adaptive and multiresonant schemes increasing overall complexity. Similar approaches are found in [28] and [29] which add the harmonic current reference estimated using adaptive filters and use a hysteresis controller for the reference tracking. Usage of a hysteresis controller makes it difficult to quantify the effectiveness of this scheme. The study in [30] uses an adaptive filter-based method for dead-time compensation in rotating reference frame, which is not suitable in single-phase systems. The method proposed in [31] requires an inverse transfer function of the system and is proposed for grid-connected topology assuming the connection to be purely inductive.

The advantage of the adaptive filter-based method is the inherent frequency adaptability which would result in same amount of harmonic compensation even when there are shifts in grid frequency. The implementation of adaptive filters is simple. Thus, in this paper, an adaptive filter-based method is proposed. This method estimates a particular harmonic in the grid current using a least-mean-square (LMS) adaptive filter and generates a harmonic voltage reference using a proportional controller. This voltage reference is added with appropriate polarity to the fundamental voltage reference to attenuate that particular harmonic. This paper includes an analysis to design the value of the gain in the proportional controller to achieve an adequate level of harmonic compensation. The effect of this scheme on overall system dynamics is also analyzed. This method is simple for implementation and hence it can be implemented in a low-end digital controller.

The presence of dc in the inverter terminal voltage results in a dc current flow into the transformer primary. This dc current results in drawing of even harmonics from the grid. If the main controller used is a PR controller, any dc offset in a control loop will propagate through the system and the inverter terminal voltage will have a nonzero average value. Thus, in this paper, a modification to the conventional PR controller scheme is proposed. An integral block is used along with the PR controller to ensure that there is no dc in the output current of the inverter. This would automatically eliminate the even harmonics. This scheme is termed as proportional-resonant-integral (PRI) control and the design of the PRI controller parameters is provided. The complete scheme is verified experimentally and the results show a good correspondence with the analysis. Experimental results also show that the transient behavior of the system is in agreement with the theoretical prediction.

The organization of this paper is as follows: Section II discusses the sources of lower order harmonics in the system and the design of fundamental current control using a PRI controller.

II. ORIGIN OF LOWER ORDER HARMONICS AND FUNDAMENTAL CURRENT CONTROL

This section discusses the origin of the lower order harmonics in the system under consideration. The sources of these harmonics are not modeled as the method proposed to attenuate them works independent of the harmonic source. The fundamental current control using the proposed PRI controller is also explained.

A. Origin of Lower Order Harmonics

1) Odd Harmonics: The dominant causes for the lower order odd harmonics are the distorted magnetizing current drawn by the transformer, the inverter dead time, and the semiconductor device voltage drops. Other factors are the distortion in the grid voltage itself and the voltage ripple in the dc bus.

The magnetizing current drawn by the transformer contains lower order harmonics due to the nonlinear characteristics of the $B-H$ curve of the core. The exact amplitude of the harmonics drawn can be obtained theoretically if the $B-H$ curve of the transformer is known [32]. The phase angle of the harmonics due to the magnetizing current will depend on the power factor.

Fig. 1. Power circuit topology of the $1-\phi$ PV system for a low-voltage inverter with 40 V dc bus connected to 230 V grid using a step-up transformer.
of operation of the system. As the operation will be at unity power factor (UPF), the current injected to the grid will be in phase with the grid voltage. However, the magnetizing current lags the grid voltage by 90°. Hence, the harmonic currents will have a phase displacement of either +90° or −90° depending on harmonic order.

The dead-time effect introduces lower order harmonics which are proportional to the dead time, switching frequency, and the dc bus voltage. The dead-time effect for each leg of the inverter can be modeled as a square wave error voltage out of phase with the current at the pole of the leg [2]–[6]. The device drops also will cause a similar effect but the resulting amount of distortion is smaller compared to that due to the dead time. Thus, for a single-phase inverter topology considered, net error voltage is the voltage between the poles and is out of phase with the primary current of the transformer. The harmonic voltage amplitude for a hth harmonic can be expressed as

\[ V_{\text{error}} = \frac{4}{h\pi} \frac{2V_{\text{dc}}t_d}{T_s} \]  

where \( t_d \) is the dead time, \( T_s \) is the device switching frequency, and \( V_{\text{dc}} \) is the dc bus voltage. Using the values of the filter inductance, transformer leakage inductance, and the net series resistance, the harmonic current magnitudes can be evaluated. Again, it must be noted that the phase angle of the harmonic currents in this case will be 180° for UPF operation.

Thus, it can be observed that the net harmonic content will have some phase angle with respect to the fundamental current depending on the relative magnitudes of the distortions due to the magnetizing current and the dead time.

2) Even Harmonics: The topology under consideration is very sensitive to the presence of dc offset in the inverter terminal voltage. The dc offset can enter from a number of factors such as varying power reference given by a fast MPPT block, the offsets in the A/D converter, and the sensors. To understand how a fast MPPT introduces a dc offset, consider Figs. 2 and 3. In Fig. 2, \( d_{\text{boost}} \) is the duty ratio command given to the boost converter switch, \( V_{pv} \) and \( i_{pv} \) are the panel voltage and current, respectively, \( P_{pv} \) is the panel output power, \( V_g \) is the rms value of the grid voltage, \( \sin \theta \) is the in-phase unit vector for the grid voltage, and \( i^* \) is the reference to the current control loop from an MPPT block. As the power reference keeps on changing due to fast MPPT action, the current reference may have a nonzero average value, which is illustrated in Fig. 3 for a step change in power reference which repeats.

Assume that a certain amount of dc exists in the current control loop. This will result in applying a voltage with a dc offset across the L-filter and the transformer primary. The net average current flowing in the filter and the transformer primary loop will be determined by the net resistance present in the loop. This average current will cause a dc shift in the B–H curve of the transformer [33]–[35]. This shift would mean an asymmetric nonlinear saturation characteristic which causes the transformer magnetizing current to lose its half-wave symmetry. The result of this is occurrence of even harmonics. The dc in the system can be eliminated by using the PRI controller which is discussed next.

B. Fundamental Current Control

1) Introduction to the PRI Controller: Conventional stationary reference frame control consists of a PR controller to generate the inverter voltage reference. In this paper, a modification to the PR controller is proposed, by adding an integral block, \( G_I \) as indicated in Fig. 4. The modified control structure is termed as a PRI controller.

Here

\[ G_I = \frac{K_I}{s} \]  

\[ G_{\text{PR}}(s) = K_P + \frac{K_r s}{s^2 + \omega_o^2} \]  

The plant transfer function is modeled as

\[ G_{\text{plant}}(s) = \frac{V_{\text{dc}}}{R_s + sL_s} \]  

This is because the inverter will have a gain of \( V_{\text{dc}} \) to the voltage reference generated by the controller and the impedance offered is given by \((R_s + sL_s)\) in s-domain. \( R_s \) and \( L_s \) are the net
resistance and inductance referred to the primary side of the transformer, respectively. $L_s$ includes the filter inductance and the leakage inductance of the transformer. $R_s$ is the net series resistance due to the filter inductor and the transformer.

The PRI controller is proposed to ensure that the output current of the system does not contain any dc offset. The PRI controller introduces a zero at $s = 0$ in the closed-loop transfer function. Hence, the output current will not contain any steady-state dc offset. This is necessary in the topology considered because the presence of a dc offset would result in a flow of even harmonics as explained in Section II-A.

The following section explains the design of PR controller parameters and proposes a systematic method of selecting and tuning the gain of the integral block in the PRI controller.

2) Design of PRI Controller Parameters: The fundamental current corresponds to the power injected into the grid. The control objective is to achieve UPF operation of the inverter. The main control block diagram is shown in Fig. 4.

First, a PR controller is designed for the system assuming that the integral block is absent, i.e., $K_I = 0$. Design of a PR controller is done by considering a PI controller in place of the PR controller [36]. The PI parameters are chosen based on the plant transfer function and the required current controller bandwidth. The PI controller parameters are then plugged in for the PR controller parameters.

Let

$$G_{PI}(s) = \frac{K_p \omega_m R_s T}{s V_{dc}}. \quad (5)$$

With the PI controller as the compensator block in Fig. 4 and without integral block, the forward transfer function will be

$$G_{forw}(s) = \left( \frac{K_p \omega_m R_s}{s T} \right) \frac{V_{dc}}{R_s + s L_s}. \quad (6)$$

The pole in (6) is canceled with the zero given by the PI controller. Then, the following relations are obtained:

$$T = \frac{L_s}{R_s} \quad (7)$$

$$G_{forw}(s) = \frac{K_p \omega_m V_{dc}}{s T R_s}. \quad (8)$$

If $\omega_m$ is the required bandwidth, then $K_p$ can be chosen to be

$$K_p = \frac{\omega_m R_s T}{V_{dc}}. \quad (9)$$

Now, if the PI controller in (5) is written as

$$G_{PI}(s) = K_p + \frac{K_I}{s} \quad (10)$$

then, $K_I$ is given as

$$K_I = \frac{\omega_m R_s}{V_{dc}}. \quad (11)$$

For the PR controller, the expressions obtained in (9) and (11) are used for the proportional and resonant gain, respectively.

Thus

$$K_p = \frac{\omega_m R_s T}{V_{dc}} \quad (12)$$

$$K_I = \frac{\omega_m R_s}{V_{dc}}. \quad (13)$$

For the complete system with an integral block, i.e., the PRI controller, the PR parameters will be same as in (12) and (13).

The following procedure is used to select the value of $K_I$ in (2). The integral portion is used to ensure that there will not be any steady-state dc in the system. Hence, the overall dynamic performance of the complete system should be similar to that with the PR controller except at the low-frequency region and dc.

The closed-loop transfer function for Fig. 4 is given as

$$G_{cl,PRI} = \frac{i(s)}{i^*(s)} = \frac{G_{plant} G_{PR}}{1 + G_{plant} (G_{PR} + G_I)}. \quad (14)$$

Without the integral block, the closed-loop transfer function would be

$$G_{cl,PR} = \frac{G_{plant} G_{PR}}{1 + G_{plant} G_{PR}}. \quad (15)$$

Let (4) be modified as,

$$G_{plant} = \frac{M}{1 + s T} \quad (16)$$

where $M = V_{dc}/R_s$ and $T$ is as defined in (7).

The numerators in both (14) and (15) are the same. Thus, the difference in their response is only due to the denominator terms in both. The denominator in (14) can be obtained as

$$\text{den}_{PRI} = \left[ T s^4 + (1 + MK_p) s^3 + (\omega_o^2 T + M (K_r + K_I)) s^2 \right]$$

$$+ \frac{\omega_o^2 (1 + MK_p) s + MK_I \omega_o^2}{s (1 + s T)(s^2 + \omega_o^2)}. \quad (17)$$

Similarly, the denominator in (15) is given by

$$\text{den}_{PR} = \left[ T s^3 + (1 + MK_p s^2 + (\omega_o^2 T + MK_r) s \right]$$

$$+ \frac{(MK_p + 1) \omega_o^2}{(1 + s T) (s^2 + \omega_o^2)}. \quad (18)$$

The numerators in (17) and (18) are the characteristic polynomials of the closed-loop transfer functions given in (14) and (15), respectively.

Let the numerator polynomial in (17) be written as

$$(s + p)(as^3 + bs^2 + cs + d) = as^4 + (b + ap)s^3 + (c + bp)s^2 + (d + cp)s + dp \quad (19)$$
where \( p \) corresponds to a real pole. Equating (19) with the numerator in (17), the following relations can be obtained:

\[
\begin{align*}
    a &= T \\
    b &= 1 + MK_p - Tp \\
    c &= \omega_o^2 T + M(K_r + K_I) - (1 + MK_p)p - Tp^2 \\
    d &= MK_I \omega_o^2 / p.
\end{align*}
\]  

(20)

If \( p \) is such that it is very close to the origin and the remaining three poles in (14) are as close as possible to the poles of (15), then the response in case of the PRI controller and the PR controller will be very similar except for dc and low frequency range. Thus, the remaining third-order polynomial in (19) should have the coefficients very close to the coefficients of the numerator in (18). In that case, using (20), the following conditions can be derived:

\[
\begin{align*}
    p &< \frac{1 + MK_p}{T} \quad (21) \\
    K_I &< K_r \quad (22) \\
    K_I &= p(K_p + 1/M). \quad (23)
\end{align*}
\]

Thus, (21)–(23) can be used to design the value of \( K_I \). Fig. 5 shows the comparison between the Bode plots of the system with the PRI controller and the PR controllers validating the design procedure for the values given in Table II. As it can be observed, the responses differ only in the low frequency range. The system with the PRI controller has zero gain for dc while the system with the PR controller has a gain of near unity.

The step response of the closed-loop system with the PRI controller can be seen in Fig. 6. As can be observed, increasing \( K_I \) has an effect of decreasing the settling time up to a certain value. Beyond that, the system becomes underdamped and settling time increases with increase in \( K_I \). This plot can be used to tune the value of \( K_I \) further, after the design from (21) to (23).

### III. ADAPTIVE HARMONIC COMPENSATION

In this section, first the LMS adaptive filter is briefly reviewed. Then, the concept of lower order harmonic compensation and the design of the adaptive harmonic compensation block using this adaptive filter are explained. Next, complete current control along with the harmonic compensation blocks is presented. Finally, the stability considerations are discussed.

#### A. Review of the LMS Adaptive Filter

The adaptive harmonic compensation technique is based on the usage of an LMS adaptive filter to estimate a particular harmonic in the output current. This is then used to generate a counter voltage reference using a proportional controller to attenuate that particular harmonic.

Adaptive filters are commonly used in signal processing applications to remove a particular sinusoidal interference signal of known frequency [37]. Fig. 7 shows a general adaptive filter with \( N \) weights. The weights are adapted by making use of the LMS algorithm.

For Fig. 7, coefficient vector is defined as

\[
\mathbf{w} = [w_0 \ w_1 \ldots w_{N-1}]^T.
\]  

(24)

Input vector and filter output are given in

\[
\mathbf{x}(n) = [x(n) \ x(n-1) \ldots x(n-N+1)]^T
\]  

(25)

\[
y(n) = \mathbf{w}^T \mathbf{x}(n).
\]  

(26)

The error signal is

\[
e(n) = d(n) - y(n).
\]  

(27)
Here, \(d(n)\) is the primary input. A frequency component of \(d(n)\) is adaptively estimated by \(y(n)\). Now, a performance function is defined for the LMS adaptive filter as
\[
\zeta = e^2(n). \tag{28}
\]
In any adaptive filter, the weight vector \(\overline{w}\) is updated such that the performance function moves toward its minimum. Thus
\[
\overline{w}(n + 1) = \overline{w}(n) - \mu \nabla (e(n)^2). \tag{29}
\]
In (29), \(\mu\) is the step size. The convergence of the adaptive filter depends on the step size \(\mu\). A smaller value would make the adaptation process very slow whereas a large value can make the system oscillatory. \(\nabla\) is defined as the gradient of the performance function with respect to the weights of the filter.

The final update equation for weights of an LMS adaptive filter can be shown to be [37]
\[
\overline{w}(n + 1) = \overline{w}(n) + 2\mu e(n)\overline{x}(n). \tag{30}
\]

Thus, from a set of known input vector \(\overline{x}(n)\), a signal \(y(n)\) is obtained by the linear combination of \(\overline{x}(n)\) and the weight vector \(\overline{w}(n)\) as in (26). Signal \(y(n)\) is an estimate of the signal \(d(n)\) and the weight vector is continuously updated from (30) such that the LMS error \(e(n) = d(n) - y(n)\) is minimized.

This concept can be used to estimate any desired frequency component in a signal \(d(n)\). The adaptive filter used for this purpose will take the reference input \(\overline{x}(n)\) as the sine and cosine terms at that desired frequency. The weight vector will contain two components which scale the sine and cosine and add them up to get an estimated signal \(y(n)\). The weights will then be adapted in such a way as to minimize the LMS error between \(d(n)\) and \(y(n)\). In steady state, estimated signal \(y(n)\) will equal the frequency component of interest in \(d(n)\).

### B. Adaptive Harmonic Compensation

The LMS adaptive filter discussed previously can be used for selective harmonic compensation of any quantity, say grid current. To reduce a particular lower order harmonic (say \(i_k\)) of grid current:

1. \(i_k\) is estimated from the samples of grid current and phase-locked loop (PLL) [38] unit vectors at that frequency;
2. a voltage reference is generated from the estimated value of \(i_k\);
3. generated voltage reference is subtracted from the main controller voltage reference.

Fig. 8 shows the block diagram of the adaptive filter that estimates the \(k\)th harmonic \(i_k\) of the grid current \(i\). The adaptive block takes in two inputs \(\sin(k\omega_n t)\) and \(\cos(k\omega_n t)\) from a PLL. These samples are multiplied by the weights \(W_{\cos}\) and \(W_{\sin}\). The output is subtracted from the sensed grid current sample, which is taken as the error for the LMS algorithm. The weights are then updated as per the LMS algorithm and the output of this filter would be an estimate of the \(k\)th harmonic of grid current.

The weights update would be done by using the equations given next, where \(T_s\) is the sampling time, \(e(n)\) is the error of \(n\)th sample, and \(\mu\) is the step size
\[
e(n) = i(n) - i_k(n) \tag{31}
\]

Now, a voltage reference has to be generated from this estimated current. In this paper, the proportional gain method is used as it is very simple for both design and implementation and is verified to meet harmonic requirements. Fig. 9 shows the scheme used for harmonic voltage reference generation from estimated harmonic current.

The overall current control block diagram with the adaptive compensation is shown in Fig. 10. Note that the fundamental current control is done using the transformer primary current and the harmonic compensation block uses the secondary current, which is the current injected into the grid.

Fig. 10 shows only one adaptive harmonic compensation block for the \(k\)th harmonic. If say dominant harmonics third, fifth, and seventh need to be attenuated, then three adaptive filters and three gain terms \(k_{adapt}\) are required and the net voltage reference added to the output of the PRI controller will be the sum of the voltage references generated by each of the block. Thus, depending on the number of harmonics to be attenuated, the number of blocks can be selected.

Note that \(n\) in Fig. 10 is the transformer turns ratio from secondary to primary. \(i_{sec.k.t}\) is the net \(k\)th harmonic current in the secondary, which is estimated using the LMS adaptive filter. This is mainly due to the harmonics in the magnetizing current and the dead-time effect. A single-phase PLL is used to generate the reference sine–cosine signals synchronized with the grid voltage for the adaptive filter. Next, computation of the adaptive gain \(k_{adapt}\) is discussed.

1. **Computation of \(k_{adapt}\)**: Based on the estimated net \(k\)th harmonic in the grid current, the voltage reference \(v_{k.ref}\) is generated by multiplying the estimated harmonic with \(k_{adapt}\). The effect of this voltage reference is that it results in an amplified voltage at that harmonic frequency at the inverter terminals and this will inject a current at that frequency in the primary side. The reflected secondary current will oppose the original current that was present in the secondary and hence there will be a net
Fig. 10. Complete ac current control structure of the inverter.

Fig. 11. Block diagram for calculating $k_{\text{adapt}}$.

reduction in that particular harmonic in the grid current. Consequently, the primary side current will be more distorted. The amount of reduction of the harmonic in grid current will depend on $k_{\text{adapt}}$.

To calculate $k_{\text{adapt}}$, the control block diagram shown in Fig. 11 is used. This block diagram is derived using Fig. 10 by considering the control variable to be regulated as the $k$th harmonic in secondary current. While deriving this harmonic control block diagram, the fundamental reference $i_{\text{pri}}^\star$ is set to zero and $G_{\text{PRI}} = G_{\text{PRI}} + G_I$. Here, $i_{\text{pri}}^*$ is the $k$th harmonic in primary current, $i_{\text{sec},k}$ is the corresponding reflected secondary current. The net $k$th harmonic in the secondary is given by $i_{\text{sec},k} - i_{\text{sec},k}(0)$, which is estimated by the adaptive filter to give $i_{\text{sec},k,t}$. $i_{\text{sec},k}(0)$ is the $k$th harmonic current flowing when there was no compensation.

Let $G(s)$ be the transfer function between $v_{k,\text{ref}}$ and $i_{\text{pri}}^*$. This can be expressed from Fig. 11 as in (34). Here, $G_{\text{plant}}(s)$ is the plant transfer function as given in (4)

$$G_{\text{AF}}(s) = \frac{G_{\text{plant}}(s)}{1 + G_{\text{plant}}(s)G_{\text{PRI}}(s)}.$$  

(34)

$G_{\text{AF}}(s)$ is the equivalent transfer function of the adaptive filter tracking $k$th harmonic of the grid current. In order to model $G_{\text{AF}}(s)$, consider an adaptive filter which tracks a dc value in a signal. This dc tracking adaptive filter can be modeled as a first-order transfer function with unity gain and with a time constant $T_a$ which depends on the parameter $\mu$. This transfer function is designated as $G_{\text{AF},0}(s)$ and is given in (35). In order to obtain the transfer function of the adaptive filter tracking $k$th harmonic, low pass to bandpass transformation [36] is used to transform (35). This gives $G_{\text{AF}}(s)$ as in (36)

$$G_{\text{AF},0}(s) = \frac{1}{1 + k_{\text{adapt}}G_{\text{PRI}}(s)} \frac{G_{\text{AF}}(s)}{n}.$$  

(35)

$$G_{\text{AF}}(s) = \frac{2s}{T_a s^2 + 2s + (k\omega_o)^2 T_a}.$$  

(36)

Thus

$$\frac{i_{\text{sec},k,t}}{i_{\text{sec},k,t}^*}(s) = \frac{k_{\text{adapt}}G(s)G_{\text{AF}}(s)/n}{1 + k_{\text{adapt}}G(s)G_{\text{AF}}(s)/n}. $$  

(37)

For the $k$th harmonic, let the steady value for the transfer function in (37), evaluated at frequency $k\omega_o$ have a magnitude $\alpha$, with $\alpha < 1$. Then

$$\left|\frac{i_{\text{sec},k,t}(j(k\omega_o))}{i_{\text{sec},k,t}^*(j(k\omega_o))}\right| = \alpha.$$  

(38)

As $G_{\text{AF}}(jk\omega_o) = 1$

$$k_{\text{adapt}} = \frac{\alpha n}{1 - \alpha |G(jk\omega_o)|}.$$  

(40)

The transfer function $G(jk\omega_o)$ for harmonics can be approximated as

$$|G(jk\omega_o)| \approx \frac{1}{|G_{\text{PRI}}(jk\omega_o)|}.$$  

(41)

Using (41) in (40), the final expression for $k_{\text{adapt}}$ can be obtained as

$$k_{\text{adapt}} = \frac{\alpha}{1 - \alpha nK_p}.$$  

(42)
For a given value of $\alpha < 1$, it can be shown that, the original $k$th harmonic in the grid current gets reduced by a factor of $1/(1 - \alpha)$. Thus, $k_{\text{adapt}}$ can be chosen from (42) depending on the amount of reduction required. The residual distortion after adaptive compensation can be determined as

$$i_{\text{sec}, k, t} = i_{\text{sec}, k(0)} \times (1 - \alpha).$$  \hspace{1cm} (43)

### C. Interaction Between the PRI Controller and the Adaptive Compensation Scheme

It can be recalled that while designing $K_p$, $K_r$, and $K_I$, the control block diagram considered in Fig. 4 did not include the effect of adaptive compensation. In fact, from Fig. 10, it can be observed that the primary current control is linked to the adaptive compensation section and the actual transfer function for the primary current control including the model for adaptive compensation is

$$G_{\text{cl}, a} = \frac{i_{\text{pri}}}{i_{\text{pri}}} = \frac{G_{\text{PR}} G_{\text{plant}}}{1 + G_{\text{plant}} (G_{\text{PR}} + G_{I} + G_{\text{AF}} k_{\text{adapt}}/n)}. \hspace{1cm} (44)$$

To ensure that the fundamental control loop performance is not affected appreciably, the Bode plots of the transfer function $G_{\text{cl}, a}$ given in (44) and the one used in (14) are compared. Fig. 12 shows the two Bode plots for the design values as in Table II. As it can be observed, there is no appreciable difference in the two plots.

Note that the extra term appearing in the actual transfer function will modify the overall response near the harmonic frequency range, third harmonic in Fig. 12. At all other frequency range, the response is practically the same.

As the value of $k_{\text{adapt}}$ determines the amount of compensation, the stability of the closed-loop transfer function is analyzed as $k_{\text{adapt}}$ varies. This is done by looking at the Routh array of the denominator polynomial in the closed-loop transfer function as given in (44). Now, (44) as a function of $k_{\text{adapt}}$ is determined by the transfer function

$$G_{\text{cl}, a} = \frac{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s}{a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \hspace{1cm} (45)$$

The coefficients in (45) are provided in Table I.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_5$</td>
<td>12.86</td>
<td>$a_6$</td>
<td>$151 \times 10^{-6}$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>3405</td>
<td>$a_5$</td>
<td>12.9</td>
</tr>
<tr>
<td>$b_3$</td>
<td>1.286 x 10^4</td>
<td>$a_4$</td>
<td>3982 + 19.05$k_{\text{adapt}}$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>2.340 x 10^4</td>
<td>$a_3$</td>
<td>1.297 x 10^4</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.127 x 10^4</td>
<td>$a_2$</td>
<td>10^6(2782 + 1.880$k_{\text{adapt}}$)</td>
</tr>
<tr>
<td>$k_{\text{adapt}}$</td>
<td>$10^6$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As can be observed from (46), there is no sign change in the first column for positive $k_{\text{adapt}}$. This means that the system will be stable for all positive values of $k_{\text{adapt}}$, which is selected using (42). Thus, the interaction between the PRI loop and the adaptive compensation scheme would not affect the stability of the system, and also as observed from Fig. 12 the response for the design values is practically unaffected.

### IV. Experimental Results

#### A. System Parameters

The circuit topology shown in Fig. 1 was built in the laboratory for a maximum power rating of 150 W. The power circuit and control parameters are listed in Table II. All the design related plots and the experimental results have the parameters as listed in Table II.

#### B. Description of the Hardware

The photograph of the experimental setup of the laboratory built prototype converter is shown in Fig. 13. The photograph shows the main circuit board which contains the power circuit, the filter inductor, the line frequency grid interface transformer, and the field-programmable gate array (FPGA)-based controller board. A sensor interface circuit board is stacked below the main circuit board. In addition to the power circuit, the main circuit board contains gate drive circuit,
TABLE II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value/Part Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{dc}$</td>
<td>DC bus voltage</td>
<td>40V</td>
</tr>
<tr>
<td>$1:n$</td>
<td>Transformer turns ratio</td>
<td>1:15</td>
</tr>
<tr>
<td>$\omega_{br}$</td>
<td>Bandwidth of current controller</td>
<td>$84.8 \times 10^3$ rad/s</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Net series resistance referred to primary</td>
<td>0.28Ω</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Net series inductance referred to primary</td>
<td>1.41mH</td>
</tr>
<tr>
<td>$s_1 - s_4$, $s_{boost}$</td>
<td>Power MOSFETs</td>
<td>IRF Z44 ($V_{DS,max} = 60V$, $I_{D,max} = 50A$)</td>
</tr>
<tr>
<td>$C_{dc}$</td>
<td>DC bus capacitance</td>
<td>6600μF, 63V</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Device switching frequency</td>
<td>40kHz</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional term</td>
<td>4</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Resonant term</td>
<td>594</td>
</tr>
<tr>
<td>$K_I$</td>
<td>Integral term</td>
<td>100</td>
</tr>
<tr>
<td>$K_{adapt}$</td>
<td>Gain in harmonic compensation block</td>
<td>25.6</td>
</tr>
<tr>
<td>$T_\alpha$</td>
<td>Time constant in $G_{AF,0}(s)$ and $G_{AF}(s)$</td>
<td>0.05s</td>
</tr>
</tbody>
</table>

The result of the comparison is given in Table III which shows the number of multiplications and additions required for the fundamental current control and control of one harmonic, say third harmonic. For multiple harmonic control, the numbers mentioned there would increase linearly. As given in Table III, the proposed method has higher resource requirement for the fundamental current control as it uses an integral block in addition to the fundamental PR controller. But, it can be observed that the resource utilization is lesser for the proposed method in the harmonic current control case. Thus, it can be observed from Table III that overall the proposed method uses less resources.

C. Experimental Results

This section contains the experimental results validating the design procedure proposed in this paper. All the experimental results correspond to one of the four cases of current control that are listed in Table IV.

Case 1 has just a PR controller and will have the highest lower order harmonic distortion. Case 2 contains a PR controller and adaptive harmonic compensation. Case 3 contains only a PRI controller but the LMS adaptive filter is disabled. Case 4 contains both the methods proposed in this paper, i.e., the PRI controller and adaptive harmonic compensation using the LMS filter and the proportional controller. This case will have the least lower order harmonic distortion.

First set of experimental results are shown in Figs. 14, 15, and 16. Here, the control loop does not have a dc offset and hence the grid current does not contain any significant even harmonics. The distortion is due to the lower order odd harmonics caused predominantly by the distorted transformer magnetizing current.

Fig. 14(a) shows the grid current and sensed grid voltage with voltage sensor gain of 0.01 V/V for the current control method of case 1 as indicated in Table IV. The presence of lower order harmonics can be seen from Fig. 14(a). Fig. 14(b) shows the same set of waveforms when the proposed control scheme is used, which corresponds to case 4 with adaptive compensation applied to third harmonic alone. The improvement in the wave shape can be observed due to the attenuation of the third harmonic.
The harmonic spectrum of the grid current waveform in Fig. 14 is shown in Fig. 15. The reduction in the third harmonic can be observed from the spectrum in Fig. 15. The summary of the total harmonic distortion (THD) of the grid current waveform in Fig. 14 is given in Table V. As mentioned in the Table V, the THD in the grid current has been brought to less than 5% by just using third harmonic compensation. The third harmonic was reduced from a value of 7.38% to 3.47%. If necessary, by adding adaptive compensation blocks for the higher harmonics, the THD of grid current can be further improved.

For the same situation, Fig. 16 shows the primary current and the sensed grid voltage. Primary current for case 1 is of a better quality, as can be seen from Fig. 16(a). This is because the dominant cause for distortion in this system is the distorted transformer magnetizing current. This was drawn from the grid in case 1. From Fig. 16(b), it is clear that the harmonics are added to the primary current and it is more distorted. In other words, the distortion has been transferred from the grid current to the primary-side current, which improves the grid current as seen in Fig. 14(b).

The control loop was not having any offset for the results shown in Figs. 14 and 15. In other words, the performance would have been the same even when the PR controller was used in place of the proposed PRI controller. To show the effectiveness of the PRI controller, an offset was added to the control loop. As explained in Section II, this offset will introduce even harmonics in the grid current. Thus, the next set of waveforms shown in Figs. 17 and 18 is for the case when the control loop contains a dc offset. Here, the distortion in the uncompensated case, i.e., case 1 is very pronounced due to the presence of significant even harmonics in addition to the odd harmonics as can be seen in Fig. 17(a). Fig. 17(b) shows the grid current for the same situation but with full compensation, i.e., case 4. It can be clearly

---

**TABLE III**

<table>
<thead>
<tr>
<th>Method</th>
<th>Multiplications</th>
<th>Additions</th>
<th>Multiplications</th>
<th>Additions</th>
<th>Multiplications</th>
<th>Additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental current control</td>
<td>4</td>
<td>4</td>
<td>9</td>
<td>8</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Harmonic current control</td>
<td>4</td>
<td>4</td>
<td>13</td>
<td>13</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>8</td>
<td>22</td>
<td>21</td>
<td>30</td>
<td>29</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>No dc offset compensation and no adaptive harmonic compensation</td>
</tr>
<tr>
<td>Case 2</td>
<td>No dc offset compensation but adaptive harmonic compensation is implemented</td>
</tr>
<tr>
<td>Case 3</td>
<td>DC offset compensation is implemented but no adaptive harmonic compensation</td>
</tr>
<tr>
<td>Case 4</td>
<td>Both dc offset compensation and adaptive harmonic compensation are implemented</td>
</tr>
</tbody>
</table>

---

Fig. 14. Comparison of grid current when there is no dc offset in control loop. (a) Case 1: No dc offset compensation and no adaptive harmonic compensation. (b) Case 4: Both dc offset compensation and adaptive harmonic compensation are implemented. [CH2: grid current (scale: 1 div = 1 A); CH1: sensed grid voltage (scale: 1 div = 5 V), horizontal scale: 1 div = 5 ms.]

Fig. 15. Comparison of grid current harmonic spectrum for Case 1—No dc offset compensation and no adaptive harmonic compensation—and Case 4—Both dc offset compensation and adaptive harmonic compensation are implemented—when there is no dc offset in a control loop.
Fig. 16. Comparison of primary current. (a) Case 1: No dc offset compensation and no adaptive harmonic compensation. (b) Case 4: Both dc offset compensation and adaptive harmonic compensation are implemented. [CH2: primary current (scale: 1 div = 10 A); CH1: sensed grid voltage (scale: 1 div = 5 V), horizontal scale: 1 div = 5 ms.]

Table V
Comparison of grid current THD and its third harmonic content for Cases 1 and 4 when there is no dc offset in the control loop

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Without compensation (Case 1)</th>
<th>With compensation (Case 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid current THD</td>
<td>8.20%</td>
<td>4.97%</td>
</tr>
<tr>
<td>Magnitude of third harmonic</td>
<td>7.34%</td>
<td>3.47%</td>
</tr>
</tbody>
</table>

seen that Fig. 17(a) has even harmonics whereas in Fig. 17(b) the even harmonics are practically negligible. This is confirmed in the spectrum of grid current shown in Fig. 18. In Table VI, the dc in primary current is compared when the primary current control loop has a dc offset. As it can be observed, the PRI controller practically eliminates the dc component.

The transient response of the system is also studied experimentally by considering the startup transient. Fig. 19(a) shows the transient response of the primary current for case 1, i.e., with the PR controller alone and Fig. 19(b) shows the transient response for case 3, i.e., with the PRI controller alone. As it can be observed from Fig. 19, the startup transient is practically the same for both PR and PRI controllers, which would also assert that the design procedure used for the PRI controller meets its objectives. The correction of a dc offset occurs in about three line cycles in case of the PRI controller.

Fig. 20(a) shows the startup transient for case 2 and Fig. 20(b) shows the same for case 4. It can be concluded that the inclusion of adaptive compensation has practically no effect on the main control loop as was deduced using the Bode plot shown in Fig. 12. Thus, the transient response of the system with the PRI controller and the adaptive compensation is as per the theoretical expectation.

The seamless transition in the grid current can be observed from Fig. 21. Here, when the enable signal is made high, the
adaptive compensation and the integrator of the PRI controller are enabled, i.e., a switch over from case 1 to case 4 occurs. This switch over does not produce any abrupt transients in the output current. The smooth transition of the wave shape can be observed from Fig. 21.

Fig. 22 shows the output of an adaptive filter estimating the third harmonic in the primary current. As it can be observed, the third harmonic content in the primary current increases after the compensation has been enabled. The building up of the output in the LMS adaptive filter can be observed from this figure to occur in about 30 ms.

V. CONCLUSION

Modification to the inverter current control for a grid-connected single-phase photovoltaic inverter has been proposed in this paper, for ensuring high quality of the current injected into the grid. For the power circuit topology considered, the dominant causes for lower order harmonic injection are identified as the distorted transformer magnetizing current and the dead time of the inverter. It is also shown that the presence of dc offset in control loop results in even harmonics in the injected current for this topology due to the dc biasing of the transformer. A novel solution is proposed to attenuate all the dominant lower order harmonics in the system. The proposed method uses an LMS adaptive filter to estimate a particular harmonic in the grid current that needs to be attenuated. The estimated current is converted into an equivalent voltage reference using a proportional
controller and added to the inverter voltage reference. The design of the gain of a proportional controller to have an adequate harmonic compensation has been explained. To avoid dc biasing of the transformer, a novel PRI controller has been proposed and its design has been presented. The interaction between the PRI controller and the adaptive compensation scheme has been studied. It is shown that there is minimal interaction between the fundamental current controller and the methods responsible for dc offset compensation and adaptive harmonic compensation. The PRI controller and the adaptive compensation scheme together improve the quality of the current injected into the grid.

The complete current control scheme consisting of the adaptive harmonic compensation and the PRI controller has been verified experimentally and the results show good improvement in the grid current THD once the proposed current control is applied. The transient response of the whole system is studied by considering the startup transient and the overall performance is found to agree with the theoretical analysis. It may be noted here that these methods can be used for other applications that use a line interconnection transformer wherein the lower order harmonics have considerable magnitude and need to be attenuated.

REFERENCES


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