Assessing the Occurrence Pattern of Large Ionospheric TEC Gradients over the Brazilian Airspace

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ABSTRACT: We investigate the occurrence pattern of equatorial plasma bubbles and the corresponding ionospheric gradients over a section of the Brazilian airspace in 2014/2015. The GPS-derived total electron content (TEC) data from a chain of receiver stations were used in this study to compute the TEC gradients along the southern crest of the equatorial ionospheric anomaly region over Brazil. Here, we present a few illustrative examples to delineate the general qualitative features of equatorial plasma bubbles in this region, and the varying degree of TEC gradient magnitudes associated with these bubbles. We also inferred the overall probability distribution function of the computed TEC gradient magnitudes in this region, which extend up to ~1000 mm/km at the GPS L1 frequency. Copyright © 2016 Institute of Navigation.

INTRODUCTION

In this work, we characterize the occurrence pattern of large/steep gradients in ionospheric total electron content (TEC) over a representative section of the Brazilian airspace. The vast majority of these TEC gradients are associated with equatorial plasma bubbles that occur regularly in this region during the evenings [1, 2]. Based on our observations, these TEC gradients can be further categorized into two separate groups: (1) those associated with the side walls of bubbles, and (2) those associated with plasma density irregularities in the interior of bubbles. In relation to these two TEC gradient categories, the basic anatomy of an equatorial plasma bubble is illustrated schematically in the lower panel of Figure 1. The TEC gradients associated with the side walls of bubbles can be very large in magnitude (exceeding 600 mm/km at GPS L1 frequency). On the other hand, the TEC gradients associated with the plasma density irregularities usually are not very large in magnitude (typically 200–300 mm/km at GPS L1 frequency), but they are much more prevalent and extensive spatially. In addition, these plasma density irregularities are capable of causing scintillations as well as loss-of-lock in GPS signals [3–5]. Hence, these gradients could potentially pose serious threats to the operation of ground-based augmentation systems (GBAS) and/or space-based augmentation systems (SBAS) for GNSS [see, e.g., [6]]. In aviation, such threats might carry significant level of risk and thus need to be given proper attention.

The nature of the abovementioned threats to GBAS/SBAS operations differs considerably between midlatitude and equatorial scenarios, which is illustrated schematically in Figure 1. In midlatitude regions, anomalously large values of TEC gradients are typically associated with plumes of storm-enhanced density (SED) that appear during periods of increased geomagnetic activities [7, 8]. Over the conterminous United States (CONUS), a nominal upper bound of 425 mm/km had been previously established for this type of storm-time TEC gradients [9]. It should also be noted that because geomagnetic storms are relatively rare events, TEC gradients of this magnitude do not appear so often in midlatitude regions. In contrast, equatorial plasma bubbles/depletions occur much more frequently on a regular basis. These bubbles are primarily generated by the Rayleigh-Taylor instability at the bottomside ionosphere shortly after sunset in the equatorial regions [10–12], and they occur almost every evening during the scintillation season (September–March) over the South American sector [13, 14]. For this reason, we can expect equatorial threats to be more dynamic and longer sustained (yet more predictable seasonally) than their midlatitude counterparts.

To study the occurrence pattern of large TEC gradients over the Brazilian airspace, we have gathered
Fig. 1—A conceptual illustration of potential threats to GBAS/SBAS operation, in both midlatitude and equatorial scenarios. The gray slab represents the ionospheric plasma layer, where darker shades correspond to a denser ionospheric plasma. The associated total electron content (TEC) of the ionosphere as a function of the horizontal x-coordinate is indicated schematically by the magenta curve.

Fig. 2—The location of six GPS receiver stations used in this study, in relation to a typical electron density map in the region (a horizontal cut at 350 km altitude) derived from a model ionosphere.

and analyzed eight months of TEC data from six GPS receiver stations in this region. These GPS receiver stations are aligned roughly east–west at a geographic latitude of ~ 15°S, and they are all located beneath the southern crest of the equatorial ionospheric anomaly in the region. This setup is
illustrated in Figure 2. The TEC data used in this study were collected during the time period between August 2014 and March 2015, which spans the scintillation season in the South American sector. In our analysis, we inferred the TEC gradients from the GPS data, and subsequently compiled a histogram/probability distribution function of the TEC gradient magnitudes. Here, we found that the probability distribution function of the TEC gradient magnitudes follows a power law distribution \( f \propto |\nabla \text{TEC}|^n \) with a clear/visible break, and the condition \( n < 0 \) holds for the exponent. This break (at \( \sim 200 \text{ mm/km} \)) marks a boundary between two separate regimes, with different exponents \( n_1 \) and \( n_2 \) in each respective regime.

In the following sections below, we discuss the experimental setup and the TEC gradient calculations in detail; followed by a number of representative examples as well as a summary of the overall statistics. We also comment on a plausible physical explanation for the observed power law distribution and some of its implications.

**TEC GRADIENT CALCULATIONS**

Figure 2 shows a map of the geographic locations for the six GPS receiver stations that we used in this study. From west to east, they are CUIB (15.6°S 56.1°W), MTBA (15.8°S 52.3°W), BRAZ (15.9°S 47.9°W), BOMJ (13.3°S 43.4°W), SSA1 (12.98°S 38.5°W), and SAVO (12.94°S 38.4°W), respectively. Note also that the stations SSA1 and SAVO are only \( \sim 10 \text{ km} \) apart, while the other stations are 400–500 km apart from their adjacent neighbors in this list. Overlaid on the map is a color-coded plot of the electron density values at an altitude of 350 km above Earth’s surface, as derived from the International Reference Ionosphere (IRI) 2007 model [15]. More specifically, the model ionosphere was set up to represent the background plasma condition on 25 December 2014 at 23:30 UTC. One may readily recognize that the selected stations are situated beneath the southern crest of the equatorial ionospheric anomaly at approximately 10° latitude south of the geomagnetic equator. We expect that the occurrence of large/steep TEC gradients would be quite prevalent around this crest area, and thus, would be representative of the worst regional condition. In this study, we calculated the TEC gradients from the daily data in the time intervals 00:00-11:20 UTC and 20:40-24:00 UTC. These two UTC-intervals cover a local time window from 5:00 pm until 8:00 am every evening/morning along this receiver chain, which captures the full life cycle of the equatorial bubbles.

At a basic level, there are two independent ways of estimating the TEC gradient values using ground-based GPS receiver data [see, e.g.,[16]]. The first method uses a pair of closely-spaced receiver stations, looking at the same GPS satellite to calculate the difference in TEC values between the two neighboring ionospheric piercing points (IPP) at any given time. Here, the IPP had been set by convention at an altitude of 350 km above Earth’s surface. The TEC gradient in this case is given by the TEC difference divided by the distance of separation between the two IPPs. In short, this station-pair method gives us an instantaneous estimate of the TEC gradient along a fixed direction determined by the line segment connecting the two stations (denoted here as \( \nabla \nabla \text{TEC} \)). Meanwhile, the second method uses a single GPS receiver station to infer the spatial TEC gradient values based on the observed temporal rate of change in TEC. As the IPP changes location as a function of time, we can use the IPP speed \( v_{IPP} \) to relate the time-step between two consecutive epochs to the corresponding distance of IPP separation. In contrast to the first method, this single-station method gives us an estimate of the TEC gradient along the direction parallel to the IPP trajectory (denoted as \( \nabla \parallel \text{TEC} \)). Finally, note also that in this study, we always work with the equivalent vertical TEC — and not the slant TEC. The two are related through a geometric factor [see, e.g.,[17]]:

\[
\text{TEC} = \text{STEC} \sqrt{\frac{R_E^2 \cos^2 \epsilon}{(R_E + h)^2}} = \frac{1}{1 - \frac{R_E^2 \cos^2 \epsilon}{(R_E + h)^2}}
\]

where TEC, STEC, and \( \epsilon \) denote the equivalent vertical TEC, slant TEC, and the satellite’s elevation angle, respectively. \( R_E = 6371 \text{ km} \) is Earth’s mean radius, and \( h = 350 \text{ km} \) is the selected altitude for the ionospheric shell. Since TEC < STEC, the TEC gradients quoted in this study generally serve as a lower bound; and the actual gradient (along the slant direction) at smaller elevation angles would be larger.

For the sake of clarity, these two independent methods of TEC gradient calculation can also be expressed mathematically as follows:

\[
\nabla \perp \text{TEC} = \frac{\text{TEC}_{\text{station 1}} - \text{TEC}_{\text{station 2}}}{\delta S_{12}} \bigg|_{\text{station pair}}
\]

\[
\nabla \parallel \text{TEC} = \frac{1}{v_{IPP}} \left| \frac{d \text{TEC}}{dt} \right|_{\text{single station}} = \frac{1}{v_{IPP}} \left| \frac{d \text{TEC}}{ds} \right|_{\text{single station}}
\]

The first method (i.e., station-pair method, Equation (2)) in general allows us to isolate the spatial TEC gradient unambiguously. However, it also requires some attention on the receiver bias estimates to minimize the overall error/mismatch between the two stations. On the other hand, the second method (i.e., single-station method, Equation (3)) is not highly sensitive to errors in
the receiver bias calculation. However, it does not explicitly distinguish between spatial variations in TEC (denoted symbolically as $\delta$TEC), and any changes in TEC that are inherently temporal (denoted symbolically as $\partial$TEC/$\partial t$) because it utilizes data points from consecutive epochs. In principle, the use of single-station method can be well justified since the inherent diurnal changes in TEC due to ion production and/or recombination in the ionosphere is negligibly small over the time-step/lag between two consecutive epochs (typically 15-30 s long). However, we must note that the background ionospheric plasma drift/circulation speed can potentially inflate as well as deflate the estimated TEC gradient values obtained via the single-station method [9].

In this study, we applied the single-station method for calculating the TEC gradients along the southern crest of the equatorial ionospheric anomaly region within the Brazilian airspace. This approach was adopted since the method is not highly sensitive to errors in the receiver bias estimate, and thus, is more suitable to combine with an automated bulk data processing. Under this framework, the above-mentioned geographical region-of-interest can be adequately covered using a set of well-separated single stations, such as those we selected for this study (cf. Figure 2). We generally consider this option more advantageous as we are no longer constrained by the availability of closely-spaced station pairs within the Brazilian sector; and hence, larger sets of data/statistics can be accumulated. We must note here that although the stations SSA1 and SAVO are sufficiently close together (~10 km apart) to be used for the station-pair method, we will only compute TEC gradients using the single-station method for the sake of consistency in our statistics. The results of this computation are subsequently sorted/binned to derive an overall probability distribution function of the TEC gradient magnitudes over this region.

**REPRESENTATIVE CASES**

Here, we discuss several case examples of equatorial plasma bubble observations and their associated ionospheric gradients based on the GPS TEC measurements. In particular, we highlight the two distinct categories of equatorial ionospheric TEC gradients found in the data: (1) those associated with the side boundary walls of the bubbles, and (2) those associated with plasma density irregularities in the interior of the bubbles. These illustrative examples shall provide a general idea on the qualitative features observed during a typical GPS encounter with equatorial plasma bubbles, as well as the expected size of the TEC gradient magnitudes.

Figure 3 shows a case of equatorial plasma bubbles with relatively low TEC gradient magnitudes (less than 200 mm/km). The blue curve on the left panel is the equivalent vertical TEC obtained during this GPS satellite pass, whereas the green curve is the GPS satellite’s elevation angle (in

![Fig. 3—Equivalent vertical TEC measurements (left panel) from the station CUIB at Cuiaba on 19 January 2015, showing some depletions that indicate the presence of equatorial plasma bubbles. The associated TEC gradients (right panel) calculated from the data. This dataset is a representative example of depletions/bubbles with moderate TEC gradients.](image)
degrees) as viewed from the receiver station on the ground. The TEC is given in the unit of TECU (= $10^{16}$ electrons/m$^2$; primary LHS y-axis), and also in terms of the induced radio propagation delay (in meters at GPS L1 frequency; secondary RHS y-axis). Meanwhile, the right panel shows the corresponding TEC gradient values (derived using the single-station method) as the IPP traversed through the ionosphere during the GPS satellite pass. The TEC gradient is given in the unit of TECU/km (the primary LHS y-axis) and in mm/km at GPS L1 frequency (the secondary RHS y-axis). In this case, the IPP trajectory passed through a series of equatorial plasma bubbles — recognizable as a set of dips in the equivalent vertical TEC curve (labeled A—D). Well outside of these bubbles, the VTEC values are negligibly small (close to zero level) as the background TEC has a rather smooth spatial variation. Here, the VTEC values only show significant excursions away from the zero level during the time segment when bubbles/depletions are clearly present. Nonetheless, this case actually represents a ‘baseline’ example of relatively benign bubbles, since the VTEC values associated with the bubbles in this instance are rather low in magnitude. In fact, TEC gradients with magnitude up to 200 mm/km are quite common in the data that we examined for this study. The remaining examples below show the larger TEC gradient cases that are representative of our general observations.

Figure 4 shows a case of depletions/bubbles with relatively large TEC gradient magnitudes (reaching 400 mm/km). When the GPS IPP encountered these depletions (around 01:00 UTC, labeled A–C), the corresponding TEC gradient values were fluctuating constantly with little or no interruption. The continuous presence of large TEC gradient values throughout this bubble encounter indicates that there were significant level of plasma density irregularities inside these bubbles/depletions. Although the amplitude of these plasma density irregularities might look relatively small in terms of the absolute TEC (as shown in the left panel), the short scale length associated with these rapid and sharp variations in TEC would ultimately lead to large TEC gradient magnitudes (as shown in the right panel). If the scale length of these density irregularities is in the Fresnel scale (roughly 400 m for an ionospheric layer 350 km above Earth’s surface), they could cause L-band amplitude scintillations and potentially loss-of-lock in GPS signals as well [18, 19]. Since plasma density irregularities can be widespread spatially throughout the interior of equatorial plasma bubbles (as demonstrated here), it becomes clear that the associated threats might persist for a considerably long time period until the bubbles had drifted away or decayed naturally. On a side note, this data plot also shows another case of TEC depletion/bubble (02:00–03:00 UTC, labeled D), but with negligibly small TEC gradient magnitudes. This observation highlights the reality that

Fig. 4—Same format as Figure 3, from the station BRAZ at Brasilia on 23 January 2015. This example shows a case of depletions/bubbles (around 01:00 UTC) with large values of TEC gradients due to some irregularities inside them. On the other hand, it also shows another depletion (02:00–03:00 UTC, labeled D), but with negligibly small TEC gradient magnitudes.
even neighboring ionospheric plasma bubbles might have a very different set of physical characteristics.

Figure 5 shows another case of depletions/bubbles with considerably large TEC gradient magnitudes (reaching 600 mm/km in this case). Contrary to the previous example, the bubbles in this example had relatively fewer density irregularities associated with them. As the GPS IPP encountered these bubbles (labeled A–F), several sharp spikes in the associated VTEC values can be observed at an
and analyzed in this study (August 2014–March 2015). In compiling this empirical probability distribution function, we have considered only the data points recorded within the time intervals 00:00–11:20 UTC and 20:40–24:00 UTC everyday. In addition, we have excluded the TEC gradients that are less than 50 mm/km in magnitude as they are far too common. Hence, this probability distribution function extends from roughly 50 mm/km up to approximately 1000 mm/km where the extreme TEC gradient cases occurred.

As shown in Figure 7, the overall probability distribution function curve of the TEC gradient magnitudes exhibits a piecewise linear behavior for a certain range of TEC gradient magnitude values. Furthermore, there is a break at ~200 mm/km where the linear slope changes quite abruptly. This break marks a boundary between two separate regimes (i.e., $|\text{VTEC}| < 150 \text{ mm/km}$ and $|\text{VTEC}| > 200 \text{ mm/km}$). Note that since both the $x$-axis and $y$-axis are in logarithmic scale, the aforementioned linear curve implies that the probability distribution function in fact follows a form of power-law distribution. The linear slopes (obtained via least-square fit) indicate the exponent of this power-law behavior in each respective regime. The fit curves are shown on the graph as red lines, and the corresponding slopes (i.e., the exponent for the power-law behavior) in each segment are denoted accordingly. For the regime $|\text{VTEC}| < 150 \text{ mm/km}$, the exponent is $n_1 = -2.4 \pm 0.1$ (the fit was performed using data points between 50 and 150 mm/km). Meanwhile, for the regime $|\text{VTEC}| > 200 \text{ mm/km}$, the exponent is $n_2 = -4.0 \pm 0.1$ (the fit was performed using data points between 200 and 500 mm/km). Note also that there is a sharp final drop at $|\text{VTEC}| \approx 750 \text{ mm/km}$, beyond which the probability distribution function switched characteristics into a more-or-less flat distribution.

**OVERALL STATISTICS**

Figure 7 depicts the overall distribution function of the TEC gradient magnitudes (on a log–log plot) based on the GPS data that have been collected and analyzed in this study (August 2014–March 2015). In compiling this empirical probability distribution function, we have considered only the data points recorded within the time intervals 00:00–11:20 UTC and 20:40–24:00 UTC everyday. In addition, we have excluded the TEC gradients that are less than 50 mm/km in magnitude as they are far too common. Hence, this probability distribution function extends from roughly 50 mm/km up to approximately 1000 mm/km where the extreme TEC gradient cases occurred.

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**Fig. 7**—The overall probability density function of TEC gradient magnitudes over the Brazilian airspace, based on the GPS data that have been collected and analyzed in this study.
The presence of a break separating the two regimes in the probability distribution function suggests that there are different physical mechanisms operating in each regime. In the case of TEC gradients associated with plasma density irregularities inside equatorial bubbles, the different exponents \( n_1 \) and \( n_2 \) in the two regimes might imply that we have two separate sets of power spectral density (PSD) for the smaller-amplitude and larger-amplitude irregularities, respectively. Here, we will only consider plasma density irregularities in the form of sinusoidal wavelike fluctuations (or a superposition of several sinusoidal components), as opposed to other signal forms that have nonzero/nonstationary trend(s). For such wavelike irregularities, the spatial variation in ionospheric TEC (along the horizontal x-coordinate) may be expressed conceptually as \( \delta \text{TEC} \sim e^{ikx} \), which gives an x-derivative of \( \frac{\partial}{\partial x} [\delta \text{TEC}] \sim ik \cdot \delta \text{TEC} \). Hence, the corresponding TEC gradient magnitudes would be given by \( |\nabla \text{TEC}| \sim k (\delta \text{TEC}) \); where \( (\delta \text{TEC}) \) is the RMS value of the TEC fluctuations. In each respective regime, the power-law behavior for the distribution of TEC gradient magnitudes is actually not so surprising because the PSD (i.e., the k-spectrum) of plasma density irregularities inside the equatorial bubbles had been previously shown to follow a form of power-law distribution \([20–22]\). Therefore, if the quantity \( (\delta \text{TEC}) \) is assumed to be uniformly distributed between zero and a certain maximum possible value, then two separate sets of PSD (i.e., \( f \sim k^{-\alpha_1} \) for smaller \( (\delta \text{TEC}) \) cases and \( f \sim k^{-\alpha_2} \) for larger \( (\delta \text{TEC}) \) cases) can be expected to yield a probability distribution function for \( |\nabla \text{TEC}| \) in the form of:

\[
f( |\nabla \text{TEC}| ) = C_1 \cdot |\nabla \text{TEC}|^{-\alpha_1} + C_2 \cdot |\nabla \text{TEC}|^{-\alpha_2}
\]

where \( C_1 \) and \( C_2 \) are constants. The above mathematical form (i.e., superposition of two power-laws) broadly exhibits the general features of the empirical probability distribution function depicted in Figure 7. In the case of side boundary walls of equatorial bubbles, similar argument can also be made using the ‘inverse scale length’ (i.e., \( \frac{1}{\Delta \text{TEC}} \frac{\partial \Delta \text{TEC}}{\partial x} \)) at the boundary wall and the ‘TEC step/drop’ (i.e., \( \Delta \text{TEC} \) at the edge of the bubble) in place of \( k \) and \( (\delta \text{TEC}) \), respectively.

However, unresolved questions remain on the exact cause and the full implications of the observed features in the empirical probability distribution function for the TEC gradient magnitudes. Further studies are certainly necessary in order to produce a more complete understanding on the observed statistical patterns, especially in terms of the responsible physical mechanism(s) behind these patterns. Until then, the interpretation outlined in the previous paragraph shall be treated with a reasonable amount of precaution.

**CONCLUSIONS**

In this study, we have characterized the occurrence pattern of ionospheric TEC gradients above a representative section of the Brazilian airspace. We determined that there are two main sources of large ionospheric TEC gradients in this region as follows: (1) steep side walls of the equatorial plasma bubbles, and (2) plasma density irregularities inside the bubbles. Furthermore, the accumulated statistics revealed that the probability distribution function for the TEC gradient magnitudes follows a form of power-law distribution \( f \propto |\nabla \text{TEC}|^n \) (where \( n < 0 \)). For TEC gradient magnitudes below 150 mm/km, the exponent of this power-law distribution is \( n_1 = -2.4 \pm 0.1 \). Meanwhile, for TEC gradient magnitudes above 200 mm/km, the exponent is \( n_2 = -4.0 \pm 0.1 \). In between these two regimes, a break is clearly visible at \( |\nabla \text{TEC}| \approx 200 \text{ mm/km} \). This empirical probability distribution function extends up to \( |\nabla \text{TEC}| \approx 1000 \text{ mm/km} \), and the aforementioned power-law behavior finally breaks down at approximately \( |\nabla \text{TEC}| \approx 750 \text{ mm/km} \).

The overall statistics obtained from this study can be expected to provide some useful information on the relative likelihood between various levels of ionospheric gradient threat in the region. However, further investigations are definitely needed to determine a more detailed picture on the actual physical mechanisms that control the occurrence of these large/steep TEC gradients. A complete understanding on the subject matter would be valuable to the collective efforts in assessing, monitoring, and mitigating the threats posed by these ionospheric gradients.

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