# WAVEFORM DESIGN FOR RADAR EMBEDDED

# COMMUNICATION

by

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# Acknowledgements

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# Abstract

Embedding of communication signals in the backscatter of radar by means of a RF tag/transponder has the disadvantage of either being covert and transmitting at a very low data rate or transmitting at a very high data rate but at the expense of the transmission not being covert. However in the proposed *intra-pulse* embedded communication, communication signals are embedded in the backscatter of the radar such that the communication is not only covert but also has a relatively high data rate compared to previous approaches. In *intra-pulse* communication, communication signals are embedded in the backscatter of radar on a per pulse basis in contrast to *inter-pulse* communication where the communication signal is embedded in a series of pulses of radar.

In order to achieve covertness and high data rate simultaneously, the communication waveforms to be embedded in the backscatter of radar need to be specifically designed and a coherent interference canceller is required to detect the waveforms. Three different types of design approaches for the design of covert communication waveforms (to embed in the backscatter of radar) have been discussed. The issues related to the waveform designs and the simulation results are presented.

# CHAPTER 1

# INTRODUCTION

Communicating covertly has always been the primary issue for military communications. In this thesis, embedding of covert communication waveform in the backscatter of radar on a per pulse basis is discussed, where the communication is not only covert, it also has a relatively high data rate compared to previous *inter-pulse* techniques [1]. Three different ways to design the covert communication waveforms are discussed in detail in this thesis.

Radar is an electromagnetic system for the detection and location of objects [2]. In general radar is used to obtain reflections from an object within the illuminated region thereby providing information such as range, radial velocity, target images, etc. A "radar system" consists of a radar transmitter, a reflection object or the target and a receiver. The target can be an RF tag/transponder. When the radar illumination is incident on the RF tag/transponder, it reflects or retransmits the incident radar illumination by remodulating the incident radar signal. This is known as "backscatter communication."

The title of the thesis is "Waveform design for radar embedded communication." The notion of radar-embedded communications can be summarized in the following manner. A given radar, which may or may not be cooperative, illuminates a given area in order to extract desired information (*e.g.*

moving targets, images) from the resulting backscatter. The task of embedded communication is undertaken by a RF tag/transponder within the radar-illuminated area which operates upon the incident radar illumination and subsequently reflects/retransmits the altered backscatter towards some desired receiver, with the normal radar backscatter masking the presence of the embedded signal. Given that the desired receiver (which may or may not be the radar) possesses prior knowledge of the set of possible embedded signals, each of which represents a communication symbol or uniquely identifies a particular tag/transponder, the receiver can extract the communication information by coherently estimating the most likely embedded signal.

## 1.1 MOTIVATION OF THESIS

The motivation for the thesis is to embed covert communication signals or waveforms in the backscatter of the radar with relatively high data rate. Past approaches in radar embedded communication are covert but they have inherently very low data rates, as the remodulation at the RF tag/transponder is done over a sequences of pulses, so that the reflection from the RF tag/transponder looks like a Doppler shift. Other approaches [3] have overcome the problem of low data rate at the cost of not being covert. In this thesis a tradeoff between data rate and covertness is achieved. The embedded communication is not only covert but also has a data rate of bits-per-pulse [1].

In contrast to previous approaches [4,5] where the embedding of waveforms is done on an *inter-pulse* basis, in this thesis the embedding of waveforms is done in a single pulse (or *intra-pulse)*. The goal of this thesis is to design the waveforms to embed in the covert radar embed communication. Radar has a "dirty spectrum" or is spectrally sloppy and radar spectrum generally exhibits a bleeding effect into the surrounding spectrum. The waveforms are designed such that they are in the spectral bleeding region of the radar. There is a tradeoff between the communication waveforms being covert and the data rate. The communication is inherently covert, as the communication signals are hidden under the backscatter of radar. The amount of interference from the radar illumination (which is the backscatter of the radar), in the direction of communication may not be sufficient to mask the communication symbols/waveforms being transmitted. Hence, to maintain covertness an extra RF tag/transponder can be placed that reflects the radar signal in the direction of communication which, will lead to more interference from the radar signal hence, more masking effect for the communication symbols/waveforms, but the higher masking for communication waveforms results in higher symbol error rate which will be discussed in the later chapters of the thesis. The waveforms are designed such that they are partially correlated with the interference hence making it difficult for an eavesdropper to intercept.

Radar illumination is incident on the RF tag/transponder and the RF tag/transponder communicates by phase modulating the incident radar illumination. Hence the RF tag/transponder remodulates the radar illumination by modulating a

phase onto the radar. This can be done in numerous ways. Three different approaches will be discussed in this thesis.

## 1.2 ORGANIZATION OF THESIS

The chapters of the thesis are organized in the following way. Chapter 2 discusses the background and the related work done in radar-embedded communication. Chapter 3 discusses the design of covert communication waveform, three different design approaches are discussed. Chapter 4 discusses the simulation results for different waveform design approaches and the hardware constraint such as phase constraint and sampling offset are discussed. Chapter 5 presents the conclusions and future work.

# CHAPTER 2

# <u>BACKGROUND</u>

This thesis discusses the design of waveforms, that are embedded in the backscatter of radar illumination (on a per pulse basis) with the help of a RF tag/transponder (which can be an RFID tag [6]) such that the communication is covert. Backscatter communication has been in use (initially used for military communication) from the 1950's. The idea of using modulated reflectors for communication was first given by Stockman (1948) [7], the primary idea of Stockman was to modulate the reflected radiation from the target so as to receive more information about the target, however the modulation was achieved by using mechanical means. A lot of research work has followed on the idea of modulated reflectors. This chapter discusses a few of these approaches. This discussion is followed by spread spectrum CDMA which is similar to *intra-pulse* communication. First let us have a look at a few of the *inter-pulse* communication techniques.

In [5] a passive tag is used to inject signals into a radar data collection by imparting a phase modulation to the reflected radar pulses. The phase modulator imparts a pulse-to-pulse modulation sequence such that the reflections from the tag look like a Doppler signature. Though this process is inherently covert it provides a low data rate of the order of bits-per-CPI where CPI (coherent processing interval) is the stream of pulses onto which the phase modulation is applied.

In [4] a radar system is used in conjunction with a coherent transponder as a communications system. A coherent transponder ("RF tag") receives a stream of radar pulses, which are modified and transmitted back to the radar. There are many methods by which a coherent transponder can modify a sequence of radar pulses. One method of modifying a radar pulse is to pass it thorough a finite impulse response filter to convolve coded information onto the pulse. The information is time coded onto a sequence of pulses. Hence the data rate is bits-per-CPI.

The patent [9] talks about a radar system, where pulses from radar cause a tag (or transponder) to respond to the radar signal. The radar, along with its conventional pulse transmissions, sends a reference signal to the tag. The tag recovers the reference signal and uses it to shift the center frequency of the received radar pulse to a different frequency. This shift causes the frequencies of the tag response pulses to be disjoint from those of the transmit pulse. In this way, radar clutter can be eliminated from the tag responses. The radar predicts the center frequency of tag response pulses within a small Doppler offset. The radar can create synthetic-aperture-radar-like images and moving-target-indicator-radar-like maps containing the signature of the tag against a background of thermal noise and greatly attenuated radar clutter. The radar can geolocate the tag precisely and accurately (to within better than one meter of error). The tag can encode status and environmental data onto its response pulses, and the radar can receive and decode this information. As radar clutter is removed from the tag response the process is not very covert. Similar approaches have focused upon radar illumination consisting of numerous

pulses such as is encountered in SAR applications. Similar approaches that discuss the backscatter communication by phase modulating a series of pulses of the radar signal are [10], [11], [12], [13]. Hence the inherent data rate is of the order of bits-per-CPI.

In [3] an IMR (impedance modulated reflector) is uses as the transponder. It is stated that the signal from IMR is more difficult to intercept as the reflected signal is low power and can be made highly directive. Results given in [3] contradict the fact of low probability of intercept, the spectrum peaks are order of 800V/Hz which is very high and hence this method is not very covert. Communicating by using a frequency hopping scheme and the feasibility of remote video surveillance that is transmitting video images in the reflected signal of the IMR is discussed. Hence this method has a very high data rate and is not covert.

In the approaches mentioned above either the backscatter communication is covert and has a data rate of bits-per-CPI which is very low or has very high data rate to transmit video images in the backscatter but is compromised in terms of covertness of the communication. This thesis discusses the backscatter communication in between the above mentioned 'covert communication and low data rate' and ' high data rate communication and non-covert,' such that the communication is not only covert but also has high data rate compared to bits-per-CPI. For this purpose waveforms similar to spread spectrum codes are used as the communication waveforms to embed in the backscatter of the radar illumination. So

let us have a more detailed look at spread spectrum and CDMA–code division multiple access.

CDMA uses spread spectrum modulation for the codes. Spread spectrum theory was initially developed in the 1950's and used for covert military communication. The first public accessible publication of spread spectrum modulation was R C Dixon [14] in 1976.

Spread spectrum modulation produces a signal whose bandwidth occupancy is expanded to be much higher than the signal bandwidth. There are basically two types of spread spectrum modulation: direct sequence spread spectrum (DSSS) or pseudonoise spread spectrum and frequency hopping spread spectrum (FHSS). In general a spread spectrum system transmits information by combining the information signal with a noise like signal (generally known as the signature waveforms) of a much higher bandwidth to generate a wideband signal [16]. The main idea of spread spectrum is to spread a signal over a frequency band that is much larger than the original signal band and transmit it with low power per unit bandwidth. CDMA uses direct sequence spread spectrum modulation to generate the codes. DS-SS realizes the band spreading by modulating a low rate symbol with a high data rate code.

In CDMA systems there are multiple users and all users transmit simultaneously in the same frequency band. In DS-CDMA users are assigned different signature waveforms or codes to identify each other [15]. The waveforms produced by DSSS technique have very little cross correlation with each other [16].

Each transmitter sends its data stream by modulating the data into the "signature waveform" (each user is assigned a spreading code known as the signature waveform) as in a single-user digital communication system. In CDMA communication there are multiple users and there is interference from all the other users. Thus a CDMA system has different users communicating at the same time by transmitting their signature waveforms and modulating the bit sequence onto the signature waveform. CDMA when considered with the different waveforms being transmitted with the interference from the multiple users looks similar to the *intra-pulse* communication used in this thesis and the similarities are discussed in detail in Section 3.1. It will be shown in the next chapters that *intra-pulse* communication communicates covertly and with a relatively high data rate in the backscatter of radar. Like spread spectrum waveforms the covert communication waveforms designed also spread the power in the frequency band available making the waveforms low power and covert. In the next chapter a detailed discussion of waveform design is presented.

# CHAPTER 3

## Waveform Design for *Intra-pulse* Communication

In Chapter 2, the previous approaches of embedding communication signals into the backscatter of radar have been discussed. A few of the approaches are the *inter-pulse* (pulse to pulse) technique where a communication signal is relayed to an intended receiver by imparting a Doppler-like phase-shift to each of a successive series of incident radar pulses and impedance modulated reflectors used as transponders to communicate using a frequency hopping scheme. The current chapter deals with waveform design and analysis pertinent to the *intra-pulse* modulation technique of embedding communication symbols into the backscatter of illuminating waveform. In *intra-pulse* modulation, waveforms are embedded on a per pulse basis as opposed to *inter-pulse* modulation where a single waveform is embedded in a stream of radar pulses.

This chapter can be divided into roughly three parts. In the first part of the chapter design space and similarities with CDMA are discussed. In the second part the actual waveform design and the issues related to waveform design are discussed. In the third part the receiver design is discussed. In this chapter a new idea is discussed regarding embedding of communication symbols in the reflection of the illuminating waveform from the RF tag/transponder on a per pulse basis, so that it is not only covert but the rate of information being transmitted is also reasonably high.

The new approach relies on the waveform-level diversity that results from phase re-modulation of the incident radar waveform into one of the *K* different communication waveforms, each of which acts as a communication symbol representing some pre-determined bit sequence ( or as a unique identifier for one of the several back-scattering devices). Given knowledge of the possible embedded communication waveforms, an intended receiver recovers the embedded information by determining which of the possible communication waveforms is most likely to be present within a given radar pulse-repetition-interval (PRI).

The *intra-pulse* communication is covert as the waveforms are designed such that they are partially correlated with the radar scatter or interference. From the detection point of view, it is desired that the waveforms are designed to be as different from the interference as possible, this is shown in Figure 3.1. As the signal and interference are different from each other it is simple for the intended receiver to decode and an eavesdropper can easily intercept the communication.
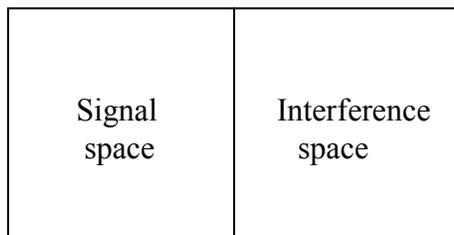
| Signal space | Interference space |
|---|---|

Figure 3.1: Separable signal and interference space

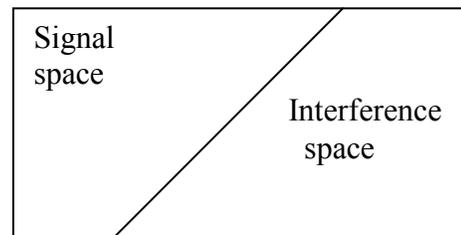| Signal space | Interference space |
|---|---|

Figure 3.2: Partially correlated signal and interference spaces

In the *intra-pulse* approach the waveforms are designed to be partially correlated with the interference, shown in Figure 3.2, thus making the communication covert. The intended receiver can decode the required waveform, as the receiver already has knowledge of the set of possible communication waveforms that may be transmitted. As the signal and the interference are partially correlated with each other, the eavesdropper will try to reduce the interference in an attempt to intercept the communication, resulting in the loss of signal.

The similarities between *intra-pulse* communication and CDMA are discussed in the following Section. Consider the backscatter object and the receiver shown in Figure 3.3. There are similarities in the properties of the communicating waveforms and the waveforms used in CDMA such as, the waveforms being spread over frequency and time and the receiver model is similar.

## 3.1 RELATION TO CDMA

Here a comparison of two communication paradigms is given in a conceptual way. In this comparison, consider the backscatter device (tag/transponder) and the receiver (shown in Figure 3.3). It is viewed as a communication system between the backscatter device and the receiver. The space between the backscattering device (RF tag/transponder) and the receiver is assumed to be a communication channel, rather than being a radar field.
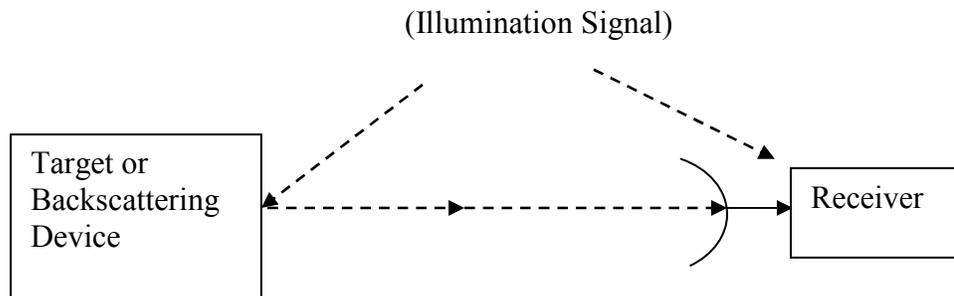
Figure 3.3: Backscatter of Radar System

Ideally the communicating waveforms would be orthogonal to each other, which would lead to the waveforms being easier to detect at the receiver. The communication waveforms are designed such that they occupy part of the design space time and frequency, which is similar to the CDMA spread spectrum waveforms occupying the available bandwidth.

At a single instant of time, CDMA has multiple users and multiple waveforms. Each user is assigned a signature waveform. All codes are present all the time and each code represents each user and each code is modulated for each individual user (shown in Figure 3.4). In contrast, for the general signaling scheme at a single instant of time only one waveform exists (though each waveform represents a communication symbol) and all the waveforms are used by one user (shown in Figure 3.5).

Consider the CDMA system given in Figure 3.4. Let $\mathbf{d}_1$, $\mathbf{d}_2$, .... $\mathbf{d}_n$ be the data being transmitted by user 1, user 2, .... user n respectively and $\mathbf{s}_1$, $\mathbf{s}_2$, .... $\mathbf{s}_n$ be

the signature waveforms of user 1, user 2, .... user n respectively. The signal available at the receiver is

$$\mathbf{y}(t) = \sum_{k=1}^{n} \mathbf{d}_k \mathbf{s}_k + noise. \qquad (3.1)$$

Consider the general signaling scheme given in Figure 3.5. Let $\mathbf{c}_1$, $\mathbf{c}_2$, .... $\mathbf{c}_k$ be the set of waveforms that could be transmitted. The signal available at the receiver is

$$\mathbf{r}(t) = c_j + noise + int \qquad (3.2)$$

where $j = 1, 2, ...k$.
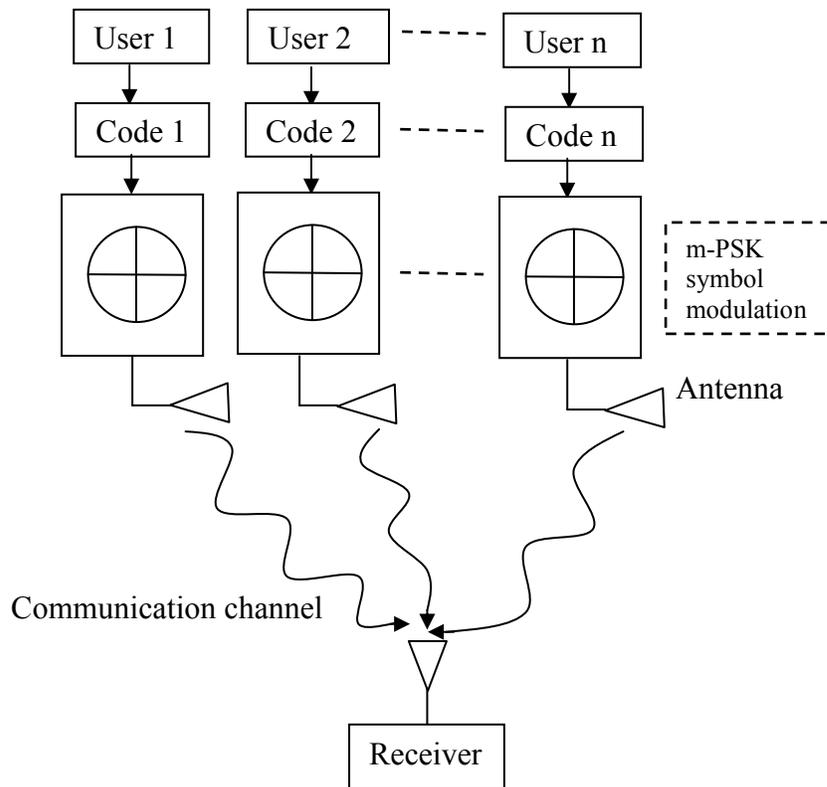


Figure 3.4: Information and coding for 1 symbol period for CDMA system.

In CDMA each spread spectrum code is used to represent each user and the data is modulated onto the code and transmitted, whereas in the general signaling scheme the waveforms are the modulation scheme. Consider multiple users in a CDMA system, as there are multiple users each user has interference from all the other users present, hence the amount of interference is significantly more than the noise present which is the case for the signaling scheme used. Though the signaling scheme has only a single user transmitting a single waveform at a time, there is a large amount of interference from the reflections of the radar from the surroundings.

Figure 3.5: Information and coding for one symbol period for *intra-pulse* system

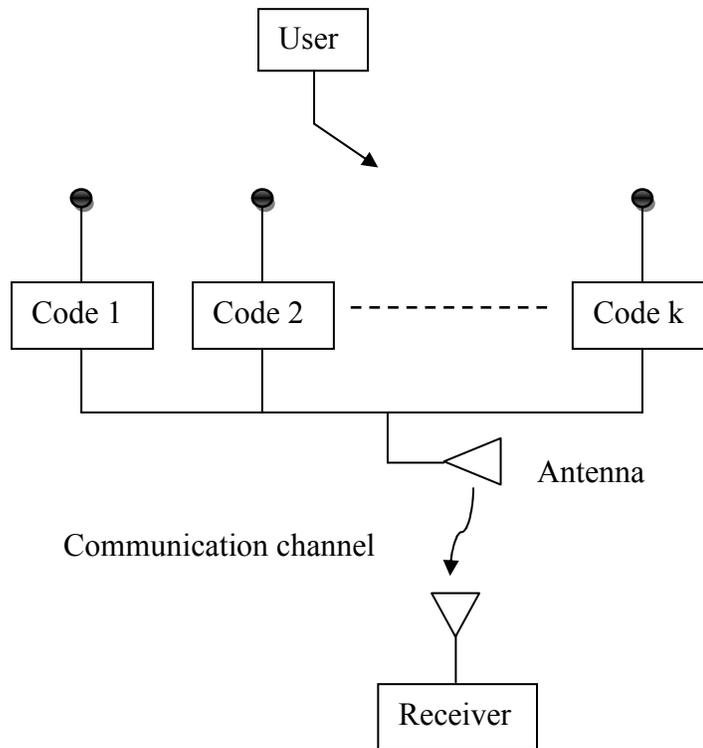CDMA uses one waveform for one user whereas in the signaling scheme multiple waveforms are used for one user (or backscatter device). Consider a CDMA system of $n$ users and an $m$-PSK modulation. The number of bits being transmitted is $n\log_2 m$, the number of bits being transmitted per user is $\log_2 m$. Consider the signaling scheme for $K$ codes or waveforms. The number of bits being transmitted is $\log_2 K$. The total throughput of the CDMA is more than the general signaling scheme used as CDMA is for $n$ users. But when considering the throughput for one single user, the data rate is more for *intra-pulse* communication.

CDMA information throughput for one user is equal to signaling scheme throughput when $m=K$. That is the number of waveforms used in intra-pulse communication is equal to the number of symbols on the unit circle for CDMA (or more number of symbols in the constellation diagram). So as the number of waveforms used increases there will be greater number of symbols on the unit circle and the complexity of decoding increases and the bit error becomes worse as the number of symbols increases the performance of the CDMA system degrades, this is due to the fact that as the number of symbols increases, the symbols are placed close to each other on the unit circle leading to higher decoding error. *Intra-pulse* communication is similar to the near-far effect of CDMA as the radar signal is of very high power (near) and the communication waveforms being transmitted are of very low power (far).

## 3.2 ILLUMINATING WAVEFORM

In the previous Sections the relation between CDMA system and *intra-pulse* system was discussed. In this Section the illuminating waveform that is used is described and the mathematical model is also given. In general the illuminating waveform can be any RF waveform.

In general, an illuminating waveform is used to either convey communication signals or to obtain reflections from objects within the illuminated environment, thereby providing information such as range, radial velocity, chemical composition, target images, etc. Here the illuminating waveform is exploited to convey information by embedding communication signals in the backscatter of the illumination.

The radar waveform (which may be continuous, binary, polyphase, etc.) can be sampled at Nyquist sampling rate resulting in $N$ samples which yield the vector

$$\mathbf{s} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \end{bmatrix}^T \qquad (3.3)$$

where $\mathbf{s} = e^{j\frac{\pi}{N}n^2}$ and $n = 0, 1, \ldots N\text{-}1$. The ambient radar backscatter consists of the reflections of the radar from the surroundings of the RF tag/transponder and the radar reflections due to multipath effects. The backscatter $\mathbf{S}$ (which is a N×2N-1 matrix) can be mathematically modeled by convolving the radar waveform with $\mathbf{x}$ given in Equation 3.4.

$$\mathbf{S} = \begin{bmatrix} s_{N-1} & s_{N-2} & \cdots & s_0 & 0 & \cdots & 0 \\ 0 & s_{N-1} & \cdots & s_2 & s_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & s_{N-1} & s_{N-2} & \cdots & s_0 \end{bmatrix} \mathbf{x} \qquad (3.4)$$

where $\mathbf{x}$ is of a vector of length 2N-1 and is modeled as resolution cell. The reflections from the surrounding objects of RF tag/transponder are collected over a sufficient interval of time and aggregated to model the resolution cell. The knowledge of average power of the surrounding backscatter is used to control the power of embedded waveforms.

One of the desired properties of the waveform is for them to be inherently dependent or similar to the scatter, as this would lead to higher masking from the radar backscatter. So, consider the eigenvalue decomposition of $\mathbf{S}$. The eigenvalue decomposition (EVD) of $\mathbf{SS}^H$ leads to $\mathbf{SS}^H = \mathbf{V}\,\boldsymbol{\Lambda}\,\mathbf{V}^T$, where the columns of $\mathbf{V}$ are the eigenvectors (form an orthonormal basis) and $\boldsymbol{\Lambda}$ is a diagonal matrix and has the associated eigenvalues.

The eigenvalues are plotted, it can be observed that the magnitude of the eigenvalues increases as shown in Figure 3.6. The higher the magnitude of an eigenvalue the higher is the correlation with the radar scatter. As the eigenvalue index increases the eigenvalues increase which means the eigenvectors are more correlated to the scatter. The eigenvectors form an *N*-dimensional space.
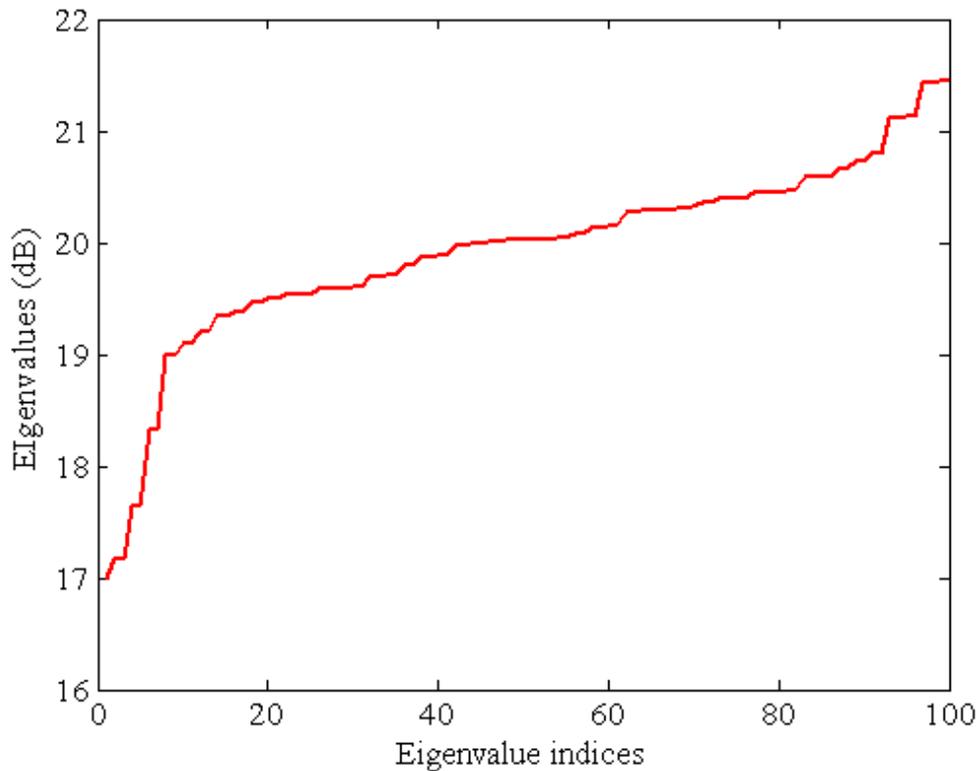
Figure 3.6: Eigenvalue plot of Linear Frequency Modulated waveform

In order to recover a given embedded communication waveform at an intended receiver, the set of communication waveforms must be sufficiently separable from one another in order to minimize the effects of interference (ideally orthogonal) and of course, if the communication waveforms are too separable from the scattering (e.g., frequency shifted out of band) then the natural masking supplied by the radar backscatter cannot be exploited. As such, a logical choice is to generate communication waveforms that reside in (or very near to) the passband of the incident radar illumination yet are "temporally coded" so as to possess a relatively

low cross-correlation with the radar waveform. Hence, there is a tradeoff in choosing the design space as the waveforms should be as different from illuminating waveform as possible so that they are easy to detect and yet they should be reasonably similar to the radar backscatter as they need to be covert (hiding under the illuminating waveform).

Radar waveforms are designed so as to fully occupy their passband thus leaving no spectral region within the passband to embed a communication waveform. However, it is well known that radar emissions are not strictly confined to their passband and exhibit a bleeding effect into the surrounding spectrum as depicted in Figure 3.7. This effect can be exploited to provide a covert region very near to the radar passband in which the embedded communication signal can reside. Furthermore by bandwidth expansion of the communication waveforms to encompass some of the surrounding spectrum of the radar illumination, sufficient design degrees-of-freedom become available.
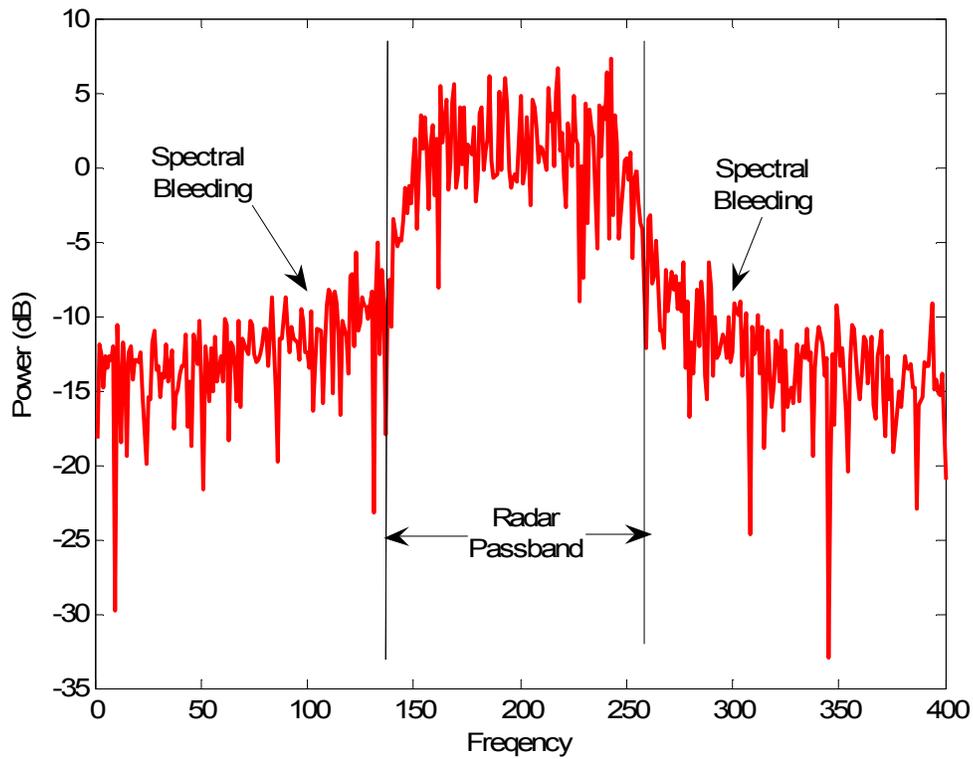
Figure 3.7 Radar spectral bleeding effect

## 3.3 BANDWIDTH EXTRAPOLATION

In the previous Section the illuminating waveform was described in detail. The design space for the communicating waveforms to be designed was discussed. The need for expansion of bandwidth was discussed. In this Section bandwidth extrapolation is discussed.

The nominally sampled radar waveform occupies all of the design space and leaves no design space for the actual waveform design. The RF tag/transponder has a sufficiently accurate representation of the radar waveform and the RF

tag/transponder oversamples the radar waveform and extrapolates the radar bandwidth. Thus a marginal expansion of the radar bandwidth by the covert communication device effectively provides the necessary design degrees-of-freedom to facilitate the embedding of good communication waveforms.

The bandwidth extrapolation is done by simply oversampling the radar waveform. Figure 3.8 shows the gain in design space by bandwidth extrapolation. The dominant space is relatively flat, while the eigenvalues associated with the non-dominant space have a considerably greater spread and thereby provide a space to design communication waveforms. It is the eigenvectors in **V** associated with the dominant and non-dominant spaces that shall be utilized to obtain suitable covert communication waveforms.

Consider a radar waveform **s**(t) having a bandwidth **B** that is illuminating a given area. The back-scattering device within the illuminated area can obtain a nominal discrete representation of the radar waveform by sampling the incident illumination at the Nyquist rate of $B$ complex samples/sec. The length of the nominally-sampled waveform is denoted as $N$. Of course, at the Nyquist sampling rate the radar waveform completely occupies the discrete spectrum. In order to accommodate the design of appropriate communication waveforms, the incident illumination is alternatively sampled at a rate of $mB$ complex samples/sec where $m$ is an over-sampling factor and dictates how much additional spectrum is to be utilized to embed a covert communication waveform. Thus the length of the oversampled waveform representation is $Nm$.
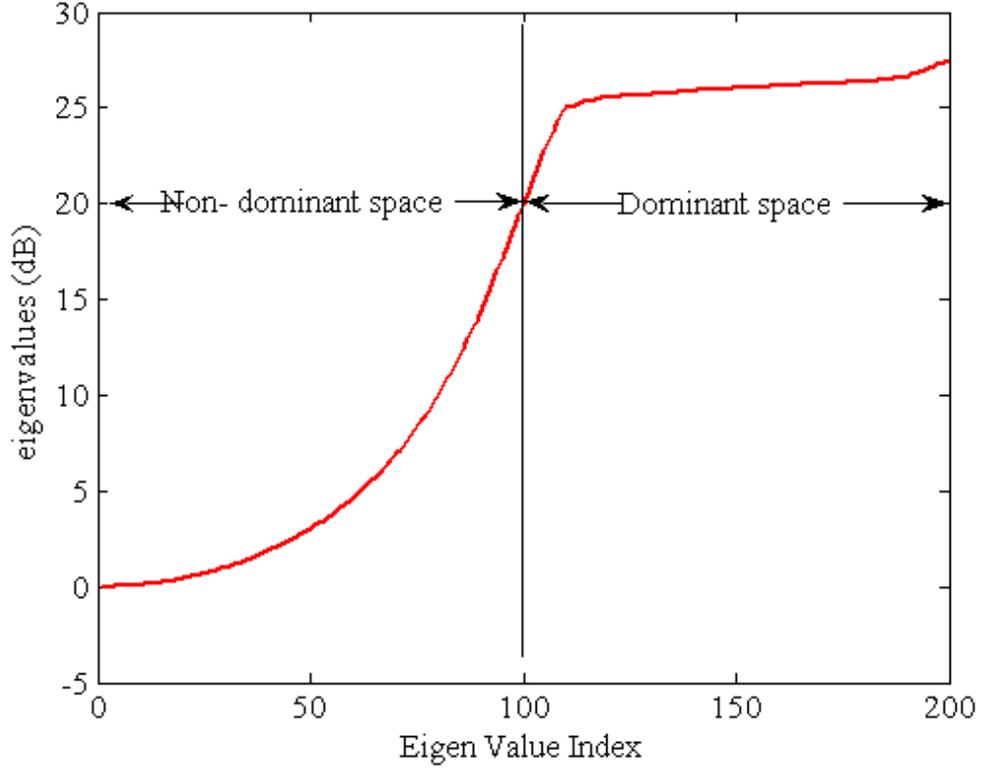
Figure 3.8: Eigenvalues plot when LFM is oversampled by 2

Let us denote the vector $\mathbf{s} = [s_0\ s_1\ \ldots\ s_{\mathrm{NM-1}}]^{\mathrm{T}}$ as the factor-of-M over-sampled discrete representation of the radar waveform. The $\mathbf{S}$ matrix which contains all the delay shifts of the radar becomes:

$$\mathbf{S} = \begin{bmatrix} s_{Nm-1} & s_{Nm-2} & \cdots & s_0 & 0 & \cdots & 0 \\ 0 & s_{Nm-1} & \cdots & s_2 & s_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & s_{Nm-1} & s_{Nm-2} & \cdots & s_0 \end{bmatrix} \quad (3.5)$$

Based on the radar waveform **s,** the design of appropriate communication waveforms is performed in the context of the ambient radar backscatter. The radar backscatter can be mathematically modeled as :

$$\mathbf{Sx} = \begin{bmatrix} s_{Nm-1} & s_{Nm-2} & \cdots & s_0 & 0 & \cdots & 0 \\ 0 & s_{Nm-1} & \cdots & s_2 & s_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & s_{Nm-1} & s_{Nm-2} & \cdots & s_0 \end{bmatrix} \mathbf{x} \quad (3.6)$$

The eigenvalue decomposition of the correlation of **S** is

$$\mathbf{S}^H \mathbf{S} = \mathbf{V} \boldsymbol{\Lambda} \mathbf{V}^H \qquad (3.7)$$

where $\mathbf{V} = [\mathbf{v}_0 \; \mathbf{v}_1 \; \ldots \ldots \; \mathbf{v}_{NM-1}]$ contains *Nm* eigenvectors, $\boldsymbol{\Lambda}$ is a diagonal matrix comprised of the associated eigenvalues (assumed to be in order of increasing magnitude), and $(\bullet)^H$ is the Hermitian operator. Let us now consider the actual waveform design approaches and the issues with each approach.

## 3.4    DESIGN OF WAVEFORMS

In the previous Sections the need for bandwidth extrapolation and the procedure to achieve bandwidth extrapolation was discussed. This Section discusses the design of waveform and the issues for each design. Why should the waveforms even be designed? Can random waveforms be used? If a random waveform is used

then it would be very difficult to detect at the receiver due to the very high interference from the scattering. Hence, the design of waveforms is necessary so that the transmitted waveforms are detectable at the receiver.

The waveforms occupy whole of the non-dominant space and are spread over the non-dominant space similar to the spread spectrum waveforms being spread in the available frequency and time (the first design approach Eigenvectors-as-Waveforms does not occupy whole of the non-dominant space). The waveforms are designed such that they occupy the non-dominant space. Why cannot the waveforms be inserted at a particular frequency? Waveforms are not designed such that they occupy a particular frequency as the waveforms would not be covert and when an eavesdropper tries to intercept the communication the waveform will have a peak at that particular frequency.

As stated earlier, the communication waveforms should be as different as possible from each other (ideally orthogonal) but similar to the scattering so that they are covert or masked under the illuminating waveform. Though the waveforms occupy most of the non-dominant space of the radar they can be separated from each other as they have very low cross-correlation with each other.

Embedding a communication signal into the radar backscatter such that it maintains covertness raises some pertinent technical issues that must be addressed. These issues are

1) The design of communication waveforms given the incident radar waveform,

2) The appropriate method for extraction of the embedded signal at the receiver, and

3) The trade-off between the covertness and the rate of communication errors at the receiver.

The ideal design property from the detection point of view is for the waveforms to be orthogonal to each other. The transmission also needs to be as covert as possible, hence it is desirable for the waveforms to be partially correlated with the scattering. The covertness of the waveforms can be achieved by masking the waveforms using the scattering. The higher the correlation of the waveforms with the scattering, higher is the interference, hence making the waveforms difficult to detect at the receiver. So there is a tradeoff in terms of maintaining the covertness of the waveforms and the accuracy of detection at the receiver.

The design of waveforms should be such that the waveforms are inherently dependent on the scattering, hence the eigenvectors found from the eigenvalue decomposition of **S** are used in the design of waveforms. Three types of design approaches are discussed:

1)      Eigenvectors as waveforms (EAW)

2)      Weighted-Combining (WC)

3)      Dominant-Projection (DP)


**3.4.1 EIGEN VECTORS as WAVEFORMS**

The desired property of the waveform from the detection point of view is for the waveforms to be orthogonal to each other. The Eigenvectors form an orthonormal basis set. In this design approach the Eigenvectors as used as the

Waveforms. As the Eigenvectors are orthogonal to each other the waveforms designed using this approach are also orthogonal to each other. The amount of correlation between the waveform and the scattering is proportional to the eigenvalue corresponding to the Eigenvector. Hence the amount of correlation of the waveform with the scattering is easy to choose as it is simply the eigenvalue associated with the eigenvector. The eigenvalues are chosen are such that the eigenvalue is comparatively less than the maximum eigenvalue or such that the Eigenvector lies in the non-dominant region. The waveforms are given by

$$\mathbf{c}_i = \mathbf{v}_i \quad \text{for } i=1,2, \ldots, L$$

where $L$ is the dimension of non-dominant space.

If an eavesdropper were attempting to intercept the communication, they would examine the spectrum present. So let us consider the spectrum, the spectrum consists of the backscatter of radar and the waveforms being transmitted that is the Eigenvectors in this case. Figure 3.9 shows the spectrum. The Eigenvectors have a huge peak in the spectrum. Hence the disadvantage of Eigenvectors as Waveforms is that it is easy to detect by an eavesdropper. Once the eavesdropper detects the presence of a communication then it is easy to decode as the eavesdropper has access to the radar waveform and can obtain the communication waveforms by the Eigenvalue decomposition of the scatter, assuming the over-sampling or the bandwidth extrapolation factor is known.
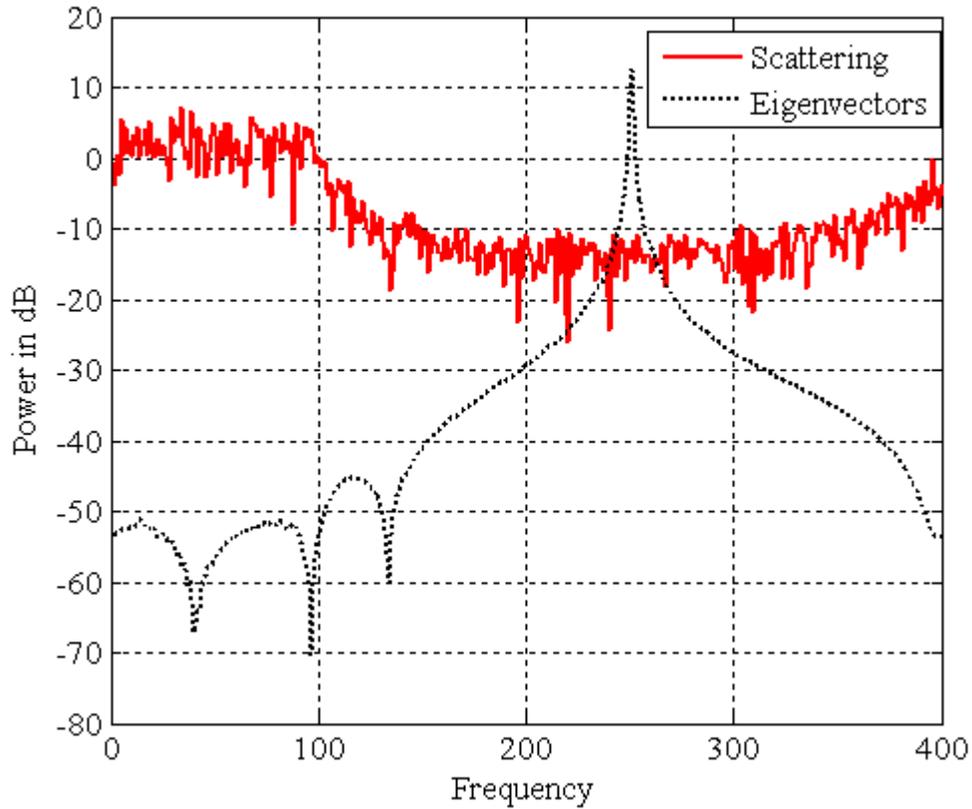
Figure 3.9:  Spectrum of Radar and Eigenvectors as waveforms

## 3.4.2 WEIGHTED-COMBINING (WC)

As seen before the Eigenvectors-as-Waveforms is not covert as the Eigenvectors have a peak in the spectrum. The Eigenvectors in the non-dominant region are combined together. The eigenvectors in the non-dominant space are given by

$$\widetilde{\mathbf{V}}_{ND} = [ \mathbf{v}_0\ \mathbf{v}_1 \ldots \mathbf{v}_{L\text{-}1}] \qquad (3.8)$$

where $L$ denotes the non-dominant space. A set of $K$ communicating waveforms can be formed by combining the $L$-dimensional non-dominant space as:

$$\mathbf{c}_k = \widetilde{\mathbf{V}}_{ND}\, \mathbf{b}_k \ \ \text{for} \ \ k=1,2,\ldots K \qquad (3.9)$$

where each $\mathbf{b}_k$ is a different $L \times 1$ weight vector known only to the tag/transponder and the intended receiver, $\mathbf{b}_k$ is considered to be a complex Gaussian. For modeling purpose the waveforms are modeled as unit norm.

Here all the Eigenvectors in the non-dominant region are combined together hence the resulting communication waveforms are partially correlated with the scattering, hence making the waveforms more covert. This can be seen from Figure 3.10.

The Weighted-Combining waveforms do not have a peak in the spectrum like the Eigenvectors-as-Waveforms. When an eavesdropper attempts to detect the communication, the Weighted-Combining waveform appears like noise. Hence the Weighted-Combining waveforms are more covert when compared with Eigenvectors-as-Waveforms. For modeling purpose the Weighted-Combining waveforms are designed such that they are unit norm. As the received power of the embedded communication signal is very low compared to the radar backscatter, there is inherent masking from the scatter leading to a lower probability of detection.
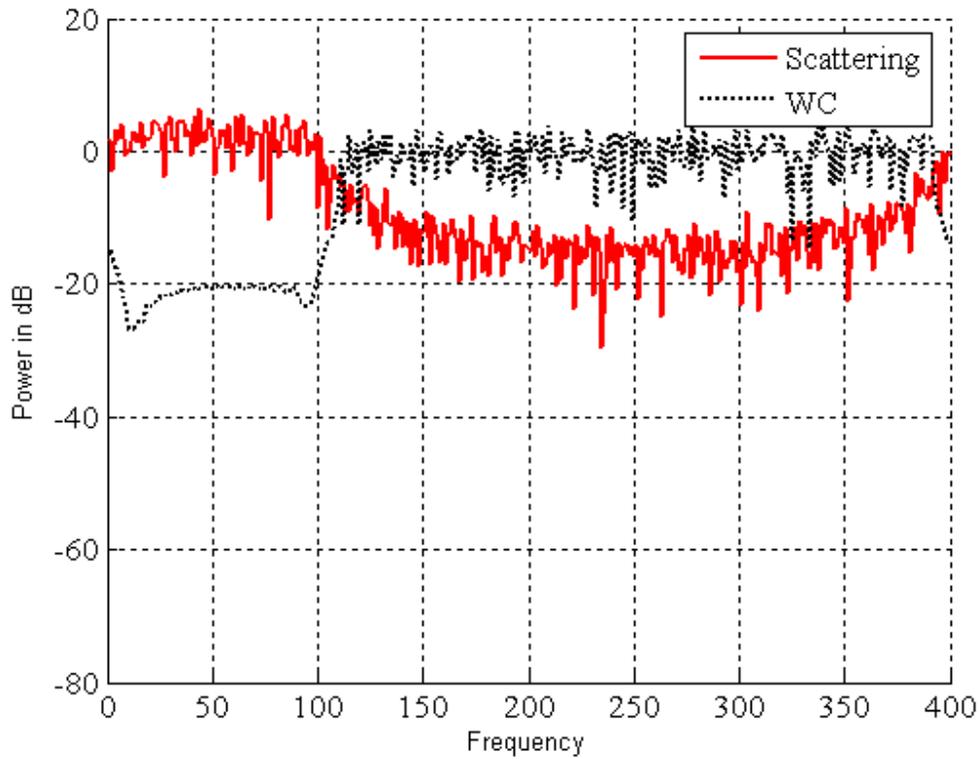
Figure 3.10: Spectrum of radar + Weighted-Combining

### 3.4.3   DOMINANT PROJECTION (DP)

The waveforms are designed such that they are not in the dominant space of

the radar, which leads to the waveforms being in the non-dominant space of the

radar but the waveforms need not occupy whole of the non-dominant space

available, whereas in Weighted-Combining the waveforms are designed so as to

occupy whole of the non-dominant space of the radar.  The Eigenvectors in the

dominant space are projected away from the dominant space and the waveforms now

designed lie in the remaining non-dominant space of the radar. The waveforms

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designed may not occupy whole of the remaining non-dominant space but they do lie in the non-dominant space.

The projection matrix **P** is given as follows:

$$P_1 = I - \sum_{i=L}^{Nm-1} V_{1,i} V_{1,i}^H \qquad (3.10)$$

where **I** is the identity matrix of size $(Nm\text{-}1) \times (Nm\text{-}1)$ and $L$ is the dimension of non-dominant space, $(.)^H$ is the Hermitian operator, **P** gives a matrix which when multiplied by any waveform will project the waveform away from the desired space. That is

$$c_1 = P_1 d_1 \qquad (3.11)$$

where $d_1$ is a random waveforms and $c_1$ is the waveform that is designed by projecting away from the dominant space. Now the second communication waveforms is designed by projecting away the dominant space and $c_1$ , so $c_1$ is appended to the **S** matrix and thus resulting in a new matrix $S_{P,1}$ given by

$$S_{P,1} = [S \mid c_1] \qquad (3.12)$$

The Eigenvalue decomposition of $S_{P,1}$ is performed:

$$S_{P,1} S^H_{P,1} = V_{P,1} \Lambda_{P,1} V^H_{P,1} \quad (3.13)$$

The dominant space is projected out and the projection matrix $P_2$ is calculated as follows

$$P_2 = I - \sum_{i=L-1}^{Nm-1} V_{2,i} V_{2,i}^H \qquad (3.14)$$

The waveform is designed by multiplying a random waveform $\mathbf{d}_2$ with the projection matrix $\mathbf{P}_2$ given as follows

$$\mathbf{c}_2 = \mathbf{P}_2\,\mathbf{d}_2 \qquad (3.15)$$

This is continued until the $k^{\text{'th}}$ waveform is obtained, where the $\mathbf{S}_{\mathrm{P},k\text{-}1}$ is the matrix got from $\mathbf{S}$ and appending all the waveforms designed that is

$$\mathbf{S}_{\mathrm{P},k\text{-}1} = [\ \mathbf{S}\,|\,\mathbf{c}_1\,|\,\mathbf{c}_2\,|\,....\,|\,\mathbf{c}_{k\text{-}1}\ ] \qquad (3.16)$$

The eigenvalue decomposition of $\mathbf{S}_{\mathrm{P},k\text{-}1}$ is performed

$$\mathbf{S}_{\mathrm{P},k\text{-}1}\,\mathbf{S}^{H}{}_{\mathrm{P},k\text{-}1} = \mathbf{V}_{\mathrm{P},k\text{-}1}\,\mathbf{\Lambda}_{\mathrm{P},k\text{-}1}\,\mathbf{V}^{H}{}_{\mathrm{P},k\text{-}1} \qquad (3.17)$$

The projection matrix $\mathbf{P}_k$ is calculated as follows

$$\mathbf{P}_k = \mathbf{I} - \sum_{i=L-k-1}^{NM-1} \mathbf{V}_{k,i}\mathbf{V}_{k,i}^{H} \qquad (3.18)$$

The waveform $\mathbf{c}_k$ is calculated as follows

$$\mathbf{c}_k = \mathbf{P}_k\,\mathbf{d}_k \qquad (3.19)$$

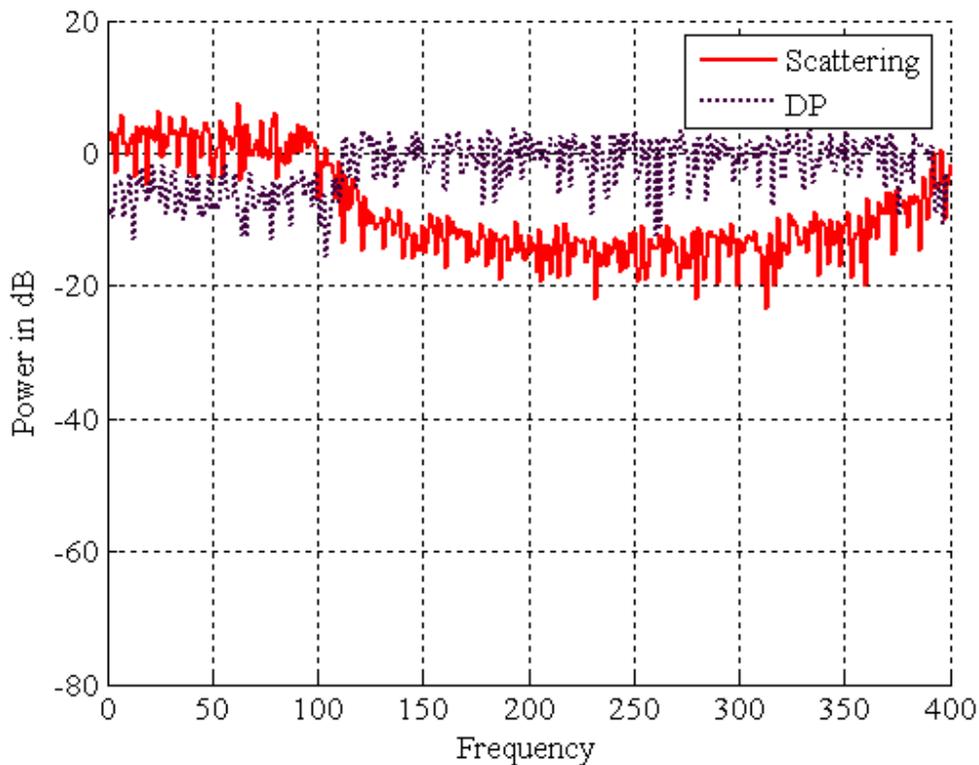where $\mathbf{d}_k$ is a random waveform.

Figure 3.11: Radar + DP

An eavesdropper attempting to intercept the communication would observe the spectrum. Let us consider the spectrum which consists of the scatter and the waveforms being transmitted. The spectrum is shown in Figure 3.11. For modeling purpose the waveforms are designed such that they are unit norm waveforms. The power of received embedded waveforms is comparatively lower than the power of the backscatter of the radar which results in the masking of the waveforms by the backscatter which in turn ensures a low probability of detection. From Figure 3.11 it can be seen that the Dominant-Projection waveforms appear to be like noise, hence making the waveforms covert.

It should be noted that all the design approaches are applicable to both "continuous" and "discrete" radar waveforms as long as the entire bandwidth is taken into account. In all the discussions above it is assumed that the backscatter device and the receiver have a significantly accurate representation of the radar waveform and thus an identical set of eigenvectors $\mathbf{V}$. This assumption is justified by the fact that radar illumination has generally very high power and undergoes one way path loss.

## 3.5    RECEIVER DESIGN

Given knowledge of the possible embedded communication waveforms, an intended receiver recovers the embedded information by determining which of the possible communication waveforms is most likely to be present within a given radar pulse-repetition-interval (PRI). As such the intended receiver selects the highest likelihood among a set of known possibilities while an eavesdropper (which presumably has no knowledge of the particular set of possible communication waveforms) is forced to ascertain if an embedded signal even exists among the masking interference, thereby maintaining the covert nature of the embedded signal.

Over the time period within which the embedded communication signal arrives at the intended receiver, the total received discretized signal of length $Nm$ (synchronization assumed) has the form

$$\mathbf{r} = \mathbf{c}_k + \mathbf{S}\,\tilde{\mathbf{x}} + \mathbf{v} \qquad (3.20)$$

where $\mathbf{c}_k$ for $k=1,2,\ldots,K$ is the $k^{\text{th}}$ communication waveform, $\tilde{\mathbf{x}}$ is some set of $2Nm$-1 samples of radar backscatter which are not necessarily the same as those in $\mathbf{x}$ from Equation 3.3.2 and $\mathbf{v}$ is a vector of $Nm$ additive noise samples. The receiver model can also be written as

$$\mathbf{r} = \mathbf{c}_k + \sum_{i=1}^{N} a_i \mathbf{S}_i + noise \qquad (3.21)$$

where $\mathbf{S}_i$ are the row vectors of $\mathbf{S}$, $\alpha$ is a complex Gaussian (the interference is modeled as a complex Gaussian for simulation purpose). The CDMA receiver model is given as

$$\mathbf{r} = \sum_{k=1}^{N} \mathbf{c}_k \mathbf{d}_k + noise \qquad (3.22)$$

where $N$ is the number of CDMA users present, $\mathbf{c}_k$ is the spread spectrum code used to identify each waveform and $\mathbf{d}_k$ is the data stream modulated onto the code. It can be seen from Equation 3.21 and 3.22 that the receiver models for CDMA and *intra-pulse* communication is similar. Hence a CDMA receiver may be used. In this Section two simple detectors are discussed the matched filter and the decorrelator receiver. It should be noted that there are many ways to design the receiver and the matched filer and the decorrelating receiver are considered as they are simple and already used is CDMA multiuser detection.

In the matched filter detection [18] , the receiver matches the received signal to one of the given set of waveforms that are possibly present. The waveform that has the maximum correlation with the received sequence is chosen as the waveform

received and the underlying bit sequence or the communication symbol is decoded. It can be expressed in mathematical terms as follows:

$$\hat{k} = \arg\left\{ \max_k \left\{ \left| \mathbf{c}_k^H \mathbf{r} \right| \right\} \right\} \quad (3.23)$$

where $\hat{k}$ is the index of the $k^{th}$ waveform. It is expected that the matched filter will not perform well due to the fact that the waveforms are correlated with the scatter and the received signal has interference from the radar signal.

Due to its relative simplicity and because it does not require knowledge of relative power levels, we consider the decorrelator receiver [19] . Given the set of communication waveforms and the matrix comprising the shifts of the radar waveform, the $Nm \times (2Nm+k-1)$ matrix $\mathbf{C}$ can be formed by appending the $K$ communication waveforms to $\mathbf{S}$ as

$$\mathbf{C} = [\ \mathbf{S}\ \mathbf{c}_1\ ...\mathbf{c}_k] \quad (3.24\ )$$

which models all the possible signal components (radar and communication) that may be present in the received signal $\mathbf{r}$. The $k^{th}$ decorrelating-filter can thus be obtained as

$$\mathbf{w}_k = \left( \mathbf{C}\mathbf{C}^H \right)^{-1} \mathbf{c}_k \quad \text{for } k=1,2,....K \quad (3.25)$$

It is expected that the decorrelator receiver performs better than matched filter due to the fact that $\mathbf{w}_k$ de-correlates the waveform from the interference. This is shown in results in Section 4.1. The disadvantage of using a decorrelator as the

receiver is at low SNR values the decorrelator enhances the noise. Hence a receiver may be designed so as to optimize the performance of the waveforms.

## 3.6 TWO COMMUNICATION SYMBOLS PER WAVEFORM

The design of waveforms and the issues related to the design of waveforms and the receiver design have been discussed in the previous Section. In all the above discussions it has been assumed that each communication waveform represents a unique communication symbol. In this Section each communication waveform is represented by two communication symbols.

When two symbols per waveform is considered most of the approach remains similar to one-symbol per waveform except for the mathematical set up of the delay shifts of radar waveform matrix $\mathbf{S}$ (size of matrix if $Nm/2 \times (2Nm\text{-}2)$) given as

$$
\begin{bmatrix}
s_{Nm-1} & s_{Nm-2} & \cdots & s_{Nm/2-1} & \cdots & s_0 & 0 & \cdots & s_{Nm-1} & s_{Nm-2} & \cdots & s_{Nm/2+1} \\
0 & s_{Nm-1} & \cdots & s_{Nm/2+1} & \cdots & s_1 & 0 & \cdots & 0 & s_{Nm-1} & \cdots & s_{Nm/2+2} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & s_{Nm-1} & \cdots & s_{Nm/2-1} & 0 & \cdots & 0 & 0 & \cdots & s_{Nm-1}
\end{bmatrix}
$$

-------- (3.26)

All the design approaches remain the same except for the fact that the eigenvectors have changed. The same procedure can be followed for 4 symbols per waveforms.

The decoding complexity increases, as there are $K$ different symbols, when transmitting 2 symbols at a time we have $K^2$ combinations that can be transmitted. Hence the receiver has to consider $K^2$ combination to detect a waveform. In contrast, one symbol per waveform has only $K$ symbols that can be transmitted, hence there are only $K$ combinations at the receiver. The decoding complexity for two symbols per waveforms is higher than one symbol per waveforms. The simulation results for different waveform design approaches are presented in the next chapter.

# CHAPTER 4

## Simulation Results

In chapter 3, three new approaches to design waveforms are given and the advantages and disadvantages of each design were discussed. In this chapter the performance of each design and the simulation results are discussed. The performance of different waveform design is compared in terms of Symbol Error rate (SER) curves. The performance of the matched filter is compared with the performance of the decorrelator. The performance of each design is discussed for different cases of phase constraint and sampling offset. The covertness of each design is discussed in Section 4.2 (measure of low probability of intercept). The performance of the different waveform designs are discussed when a Polyphase Barker code [20] (discrete waveform) is used as the illumination signal.

Simulations are performed considering the illumination waveform as the P3 code [20] (Nyquist sampled version of LFM) which is given mathematically as

$$s(n) = e^{j\frac{\pi}{N}n^2} \qquad (4.1)$$

Where $N$ is the length of waveform transmitted and $n = [0\ 1\ \ldots..N\text{-}1]$. The waveform is oversampled by a factor of $m = 2$.

Each code or waveform represents $x$ bits which are in turn represented by symbols. There can be any $2^x$ symbols where $x$ is the number of bits represented by

the symbols. Simulations are done with $K = 4$ communications symbols/waveforms unless otherwise specified, hence the date rate is 2 bits-per-pulse.

Monte Carlo simulations are performed for over 100,000 trials. That is 200,000 bits are randomly generated and converted to 100,000 symbols and 100,000 trials are run. For each trial one symbol is selected and a waveform is associated with each symbol. Since $K = 4$, there are 4 different symbols or waveforms. Then noise and interference are added to the waveform, depending on the interference and noise levels selected. The waveform is decoded at the receiver using the receiver design discussed in Section 3.5. The probability of the waveforms being detected at error at the receiver is calculated and plotted, known as the bit-error rate (BER) or symbol-error rate (SER) curves. In the results given symbol-error rate is discussed. SER is used as the parameter to compare the performance of different waveforms design. Though SER is not the only parameter considered, the covertness of the waveforms is also compared using the measure of LPI (low probability of intercept).

The noise level, that is SNR (signal to noise ratio), is the power of noise (modeled as white Gaussian) with respect to the signal power level. The SNR values are considered with respect to the received signal. The interference level that is SIR (Signal to interference ratio) is the power of interference with respect to the signal power level. The interference is the backscatter of the radar and it is modeled as given in Section 3.3. Signal power is the power of waveform (for modeling purposes all the waveforms are designed such that they are unit norm). Simulations are performed with respect to particular SNR and SIR values. The signal, interference

and noise power levels are scaled respectively, to achieve the required SIR and SNR values.

## 4.1 RESULTS FOR EAW, WC, DP

In this Section the SER curves for Eigenvectors-as-Waveforms, Weighted-Combining and Dominant-Projection are compared. The relative performance of the different waveform design is compared in terms of Symbol-Error-Rate curves. The SNR and SIR range is -20 to 10 dB and -40 to -30 dB respectively. The interference level considered is more than the noise level as interference is more predominant. Interference is the reflection of the illuminated radar waveform returned from the objects surrounding the RF tag/transponder or in other words radar clutter and as radar clutter is present when transmitting the waveforms, hence the interference is more dominant.

## 4.1.1 SER CURVES FOR EIGENVECTORS-as-WAVEFORMS

A Monte Carlo simulation is run for the SER curves for EAW (Eigenvectors-as-Waveforms). The Eigenvectors corresponding to the smallest possible Eigenvalues are chosen as the waveforms for the simulation. Very little non-dominant space of the radar is used for the design of these waveforms. The simulations are run for particular SIR values and changing the SNR values and the simulations are repeated for different SIR values and the probability of symbol error

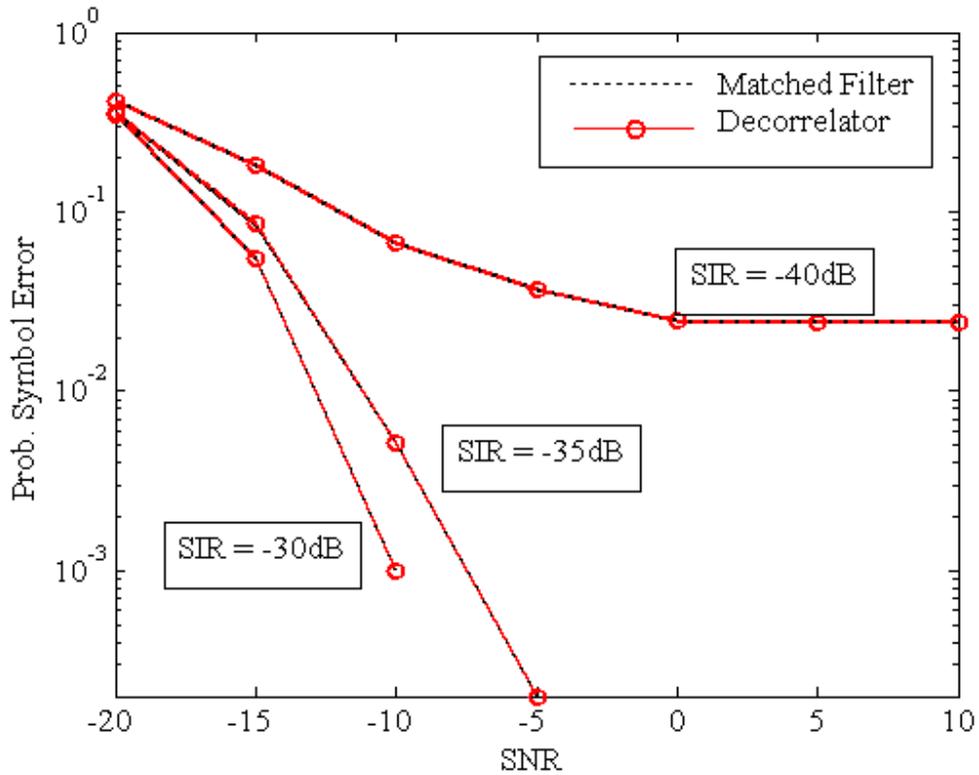is plotted. Figure 4.1 shows the SER curves for both the matched filter and the decorrelator.



Fig 4.1 Symbol-error-rate for Eigenvectors as waveforms

The SER curves for the matched filter and the decorrelator are identical to each other. The Eigenvectors have negligible amount of correlation with the scattering. Though the decorrelator de-correlates the waveforms from the interference, as the Eigenvectors chosen have inherently negligible correlation with the scattering, the performance of the matched filter and the decorrelating receiver is

identical. As the Eigenvectors chosen have negligible correlation with the scattering, this method is not very covert. The Eigenvectors corresponding to the smallest possible Eigenvalues have a peak in the spectrum, as shown in Section 3.4.1. The covertness is also discussed in Section 4.2 in terms of measure of LPI.

### 4.1.2 SER CURVES FOR WEIGHTED-COMBINING

A Monte Carlo simulation is performed for WC (weighted combining of Eigenvectors). The non-dominant space used to combine for the waveform design is $l$=90. It should be noted that almost half of the space is used to design the waveforms. The SER values are calculated and plotted for both the matched filter and the decorrelator.

The decorrelator performs better than the matched filter, due to the fact that the waveforms designed using Weighted-Combining approach are inherently correlated to the interference. Unlike the matched filter, the decorrelator acts as a coherent interference canceller and reduces the interference, hence the decorrelator performs better than the matched filter.

As the waveforms are inherently correlated to the interference the masking provided by the scattering increases and hence Weighted-Combining waveforms are more covert than Eigenvectors-as-Waveforms (EAW). When an eavesdropper tries to intercept the communication, the Weighted-Combining waveforms look like noise as shown in Section 3.4.2.
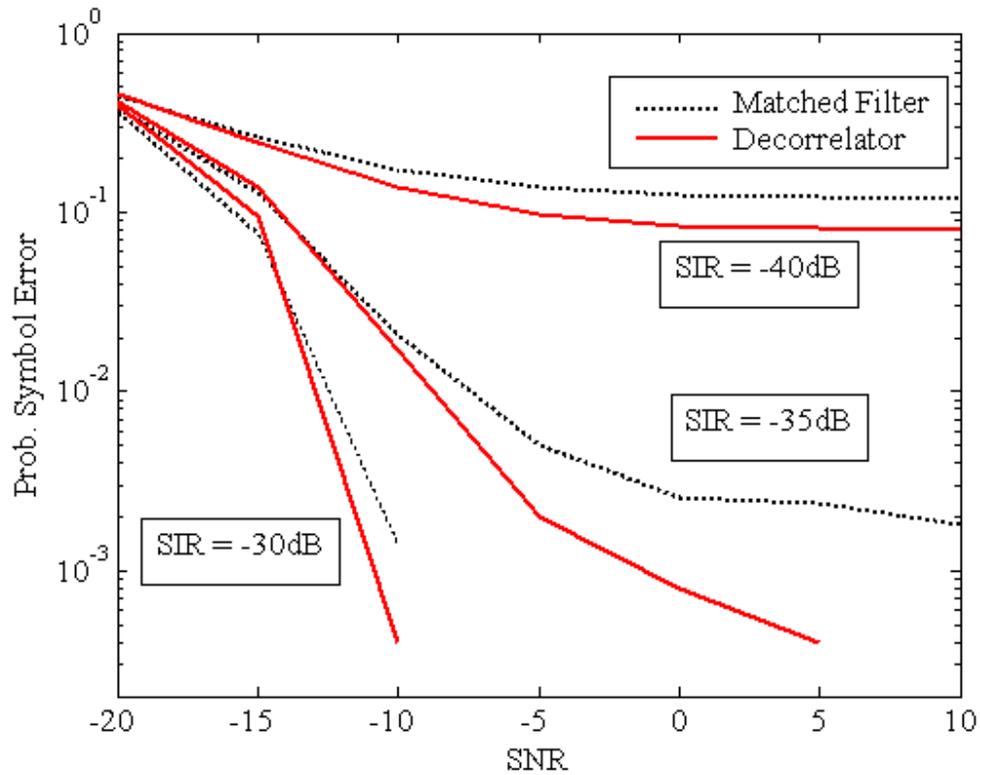
Figure 4.2 Symbol-error-rate for Weighted-combining

The Eigenvectors-as-Waveforms performance of the matched filter is better than the Weighted-Combining performance of the matched filter in terms of the SER curves. The Eigenvectors-as-Waveforms performance of the decorrelating receiver is also better than the Weighted-Combining performance of the decorrelating receiver. This is due to the fact that the waveforms are designed in Eigenvectors-as-Waveforms so as to minimize the interference. The improvement in performance of

Eigenvectors-as-Waveforms comes at the expense of maintaining the covertness of the waveforms which is discussed in detail in Section 4.2

### 4.1.3 SER CURVES FOR DOMINANT-PROJECTION

A Monte Carlo simulation is performed for DP (Dominant projection). The waveforms are designed by projection out $l$=110 of the dominant space. It should be noted that almost half of the non-dominant space is used to design the waveforms. The SER curves are calculated and plotted.

It can be observed from Figure 4.3 that the performance of the decorrelating receiver is better than the performance of matched filter. The waveforms designed using Dominant-Projection are such that the waveforms are partially correlated with the interference. As the decorrelating receiver minimizes the interference of the waveforms, the decorrelating receiver performs better than the matched filter. As the waveforms are inherently partially correlated with the interference, the waveforms are masked by the back-scatter of the radar, hence maintaining covertness. When an eavesdropper examines the spectrum of the radar, Dominant-Projection waveforms look like noise as shown in Section 3.4.3.
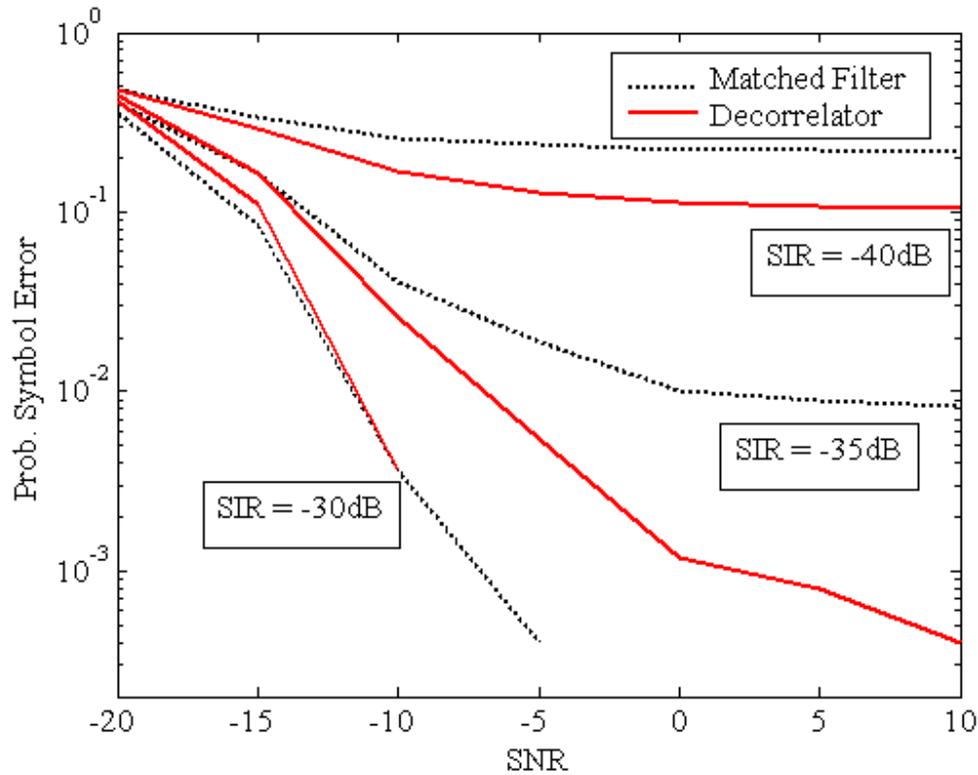
Figure 4.3 Symbol-error-rate for dominant-space projection

The decorrelator performance of both the Weighted-Combining and the Dominant-Projection is similar. This is due to the fact that the same amount of non-dominant space is used to design these waveforms. The Eigenvectors-as-Waveforms performance of the matched filter is better than the matched filter performance of Weighted-Combining and Dominant-Projection, this is due to the fact that the waveforms in EAW have negligible correlation with the interference. The improved performance of the Eigenvectors-as-Waveforms (in terms of SER curves) comes at

the cost of Eigenvectors-as-Waveforms being less covert, which is discussed in the next Section Measure of Low Probability of Intercept.

## 4.2 MEASURE OF LPI (Low Probability of Intercept)

In the previous Section the simulation results (SER curves) for Eigenvectors-as-Waveforms, Weighted-Combining and Dominant-Projection were discussed. In this Section the measure of Low Probability of Intercept (LPI) for each waveform design is discussed. It has been stated before that the waveform design is such that the waveforms are covert and an eavesdropper cannot detect them. In this Section the covertness of each design is discussed by presenting results.

In this Section a different measure for the covertness of the waveforms in discussed. The received signal present for the eavesdropper to collect is given as

$$\mathbf{r} = \mathbf{c}_k + \boldsymbol{\alpha}_n \sum_{i=1}^{i=N} \mathbf{S}_i + noise \qquad (4.2)$$

where $\mathbf{S}_i$ is the row vector of $\mathbf{S}$ and $\boldsymbol{\alpha}_n$ is the a Gaussian vector scaled according to the average power level of interference. The projection matrix $\mathbf{P}$ is calculated by projecting away the dominant space of the radar and is given as follows

$$\mathbf{P} = \mathbf{I} - \tilde{\mathbf{V}}\tilde{\mathbf{V}}^H \qquad (4.3)$$

where $\mathbf{I}$ is the identity matrix and $\tilde{\mathbf{V}}$ is the set of eigenvectors in the dominant space of the radar. The product of projection matrix and received signal is given by

$$\mathbf{z} = \mathbf{P} \times \mathbf{r} \qquad (4.4)$$

The normalized correlation between **z** and the possible set of waveforms that can be transmitted is calculated as follows:

$$corr = \frac{\mathbf{c}^H \mathbf{z}}{\sqrt{\mathbf{c}^H \mathbf{c}}\sqrt{\mathbf{z}^H \mathbf{z}}}$$  (4.5)

The normalized correlation gives an idea of the accuracy with which the eavesdropper can detect the transmitted waveforms.

### 4.2.1 MEASURE OF LPI FOR EAW

The measure of LPI is calculated for an over-sample factor of *m*=2 and *K*=4 that is there are four different waveforms. The Eigenvectors corresponding to the smallest possible eigenvalues are used as waveforms. The interference and noise levels are SIR=-35dB and SNR=-5dB, which corresponds to a symbol error probability of $10^{-4}$.

From Figure 4.4 it can be seen that as the dominant part of the space is projected out the normalized correlation increase due to the fact that as the dominant space is projected out the interference is being removed. The Eigenvectors-as-Waveforms are designed such that they are in the non-dominant space and have very negligible correlation with the interference, hence resulting in high normalized correlation between the received signal and **P**. When all the dominant space is projected out such that the only space left is the Eigenvectors-as-Waveforms, it can be seen that the correlation is unity (or 100%), thus the eavesdropper can actually extract the waveforms being embedded into the backscatter. Hence using Eigenvectors as

waveforms is not very covert, but it has the advantage of lower probability of symbol error. Hence there is a tradeoff between being covert and lower probability of symbol error.
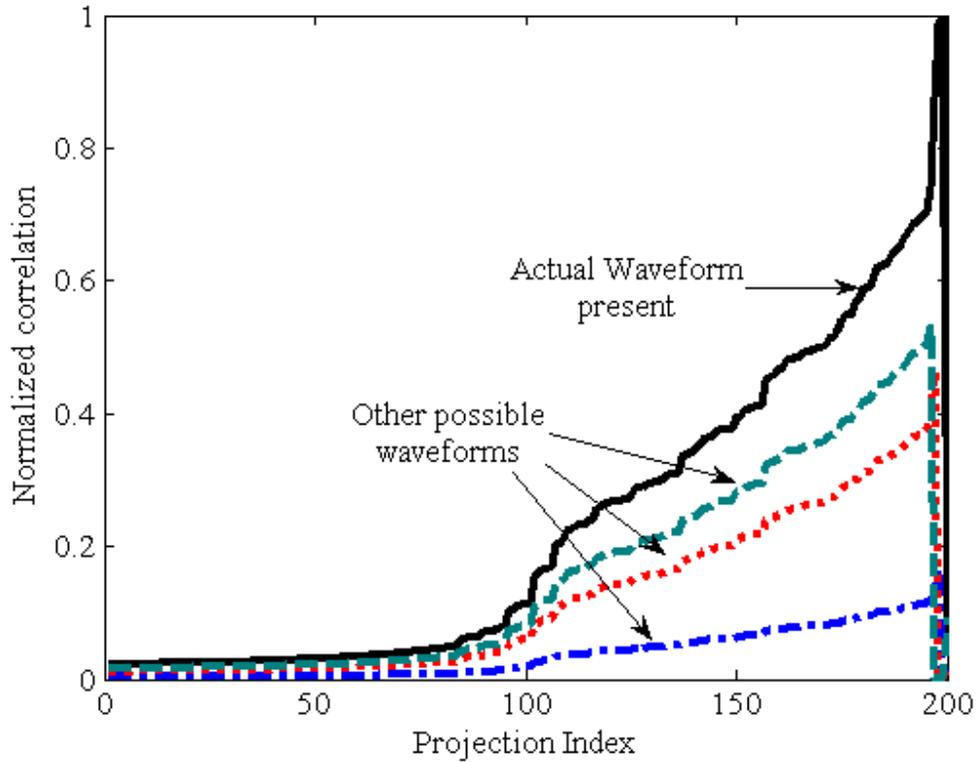


Figure 4.4 Measure of LPI for EAW

## 4.2.2 MEASURE OF LPI FOR WC

The measure of LPI is calculated for an over-sample factor of $m=2$ and the non-dominant space used to combine for the waveform design is $L=90$. The interference and noise levels are SIR=-35dB and SNR=-5dB that is for a symbol error probability of $10^{-3}$.
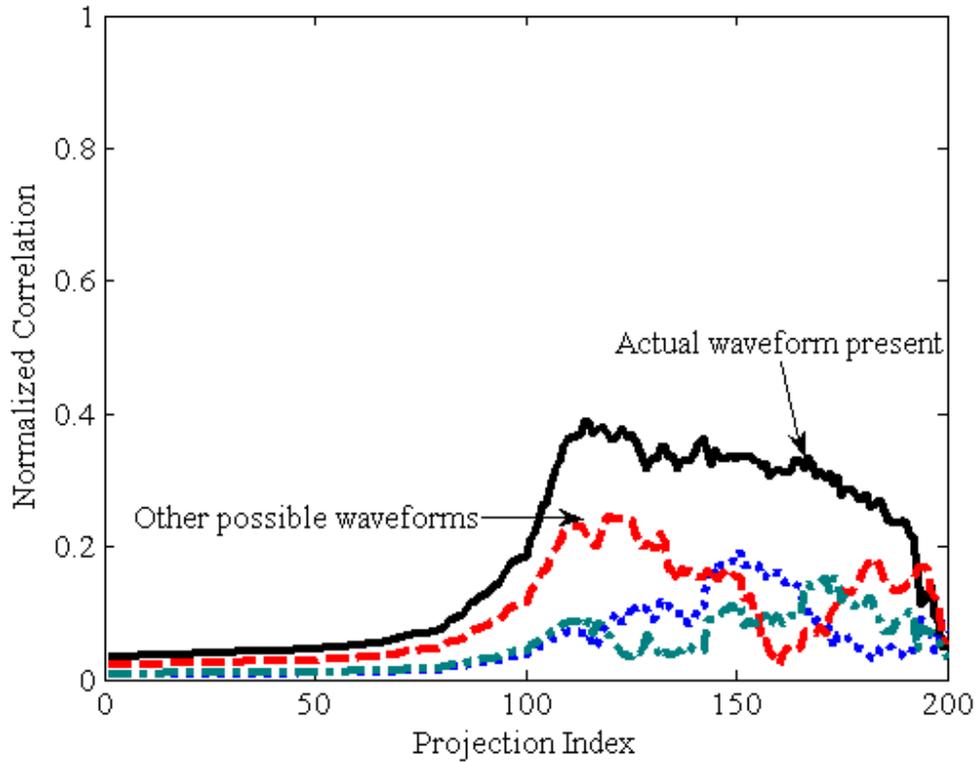
Figure 4.5 Measure of obfuscation for WC

From Figure 4.5 it can be seen that as the dominant part of the space is projected out the correlation between the **P** matrix and the information present with the eavesdropper increases. When the dominant space is projected out such that the non-dominant space is left is equal to $l$=90 the cross correlation reaches its highest peak of 40%. The normalized correlation is maximum when the eavesdropper projects out the dominant space such that the space left is same as the design space used to design the waveforms. It is assumed that the eavesdropper is synchronous to the transmission and has knowledge of the dimensionality of the communication

waveforms. The maximum correlation is still less than 100% which is the maximum correlation for Eigenvectors-as-Waveforms. Hence Weighted-Combining is more covert than Eigenvectors-as-Waveforms but Weighted-Combining as the disadvantage of having higher SER than Eigenvectors-as-Waveforms.

### 4.2.3 MEASURE OF LPI FOR DOMINANT-PROJECTION

The measure of LPI is calculated for an over-sample factor of $m$=2, $K$=4 that is there are four possible waveforms and the dominant space used to project out for the design of waveforms is $l$=110. The interference and noise levels are SIR=-35dB and SNR=-5dB or in other words for a symbol error probability of $10^{-3}$.

Figure 4.6 shows the measure of LPI for Dominant-Projection. It can be seen that as the dominant space is projected out the normalized correlation between the actually transmitted waveforms and the product of **P** and **r** increases. Unlike the other two approaches the maximum value of correlation is not maximum when the dominant space removed is $l$=110 but the maximum correlation is at $l$=150 and the maximum value of correlation is 30%. Hence not only is the maximum value of correlation less than the maximum value of correlation for Weighted-Combining (40%) and Eigenvectors-as-Waveforms (100%), the maximum value does not occur at $l$=110 which is the space used to design the waveforms. Hence Dominant-Projection is more covert than Weighted-Combining and Eigenvectors-as-Waveforms and the SER performance of Dominant-Projections is same as Weighted-Combining.
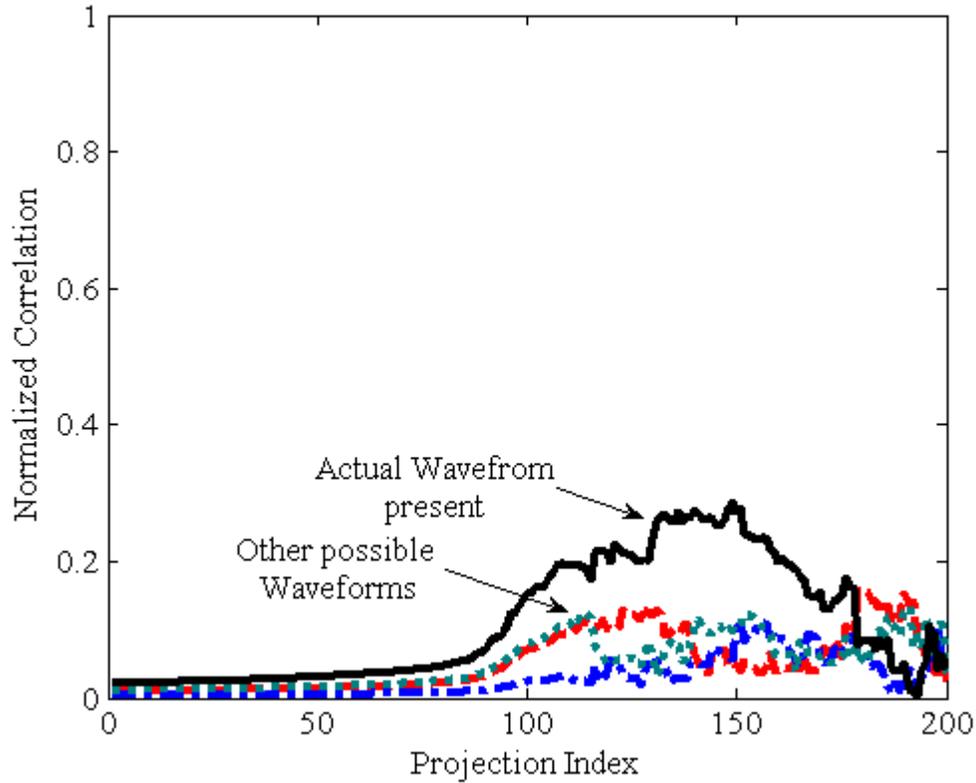
Figure 4.6 Measure of LPI for DP

## 4.3  PHASE CONSTRAINT

In the previous Sections the performance of different waveform design is compared with each other in terms of SER and LPI. In this Section the performance of the waveforms is compared in terms of phase constraint.

In general the back-scatter device can be a tag/transponder. When a passive RFID tag is used as the back scattering device, the tag introduces a phase constraint. In other words when the radar signal is incident on the tag, the tag remodulates the radar waveform into one of the set of communication waveforms. When transmitting

a communication waveform the tag does not have the whole $360°$ to transmit the communication waveform which is known as a phase constraint. If there is no phase constraint then the waveforms have phase from 0 to $\pi$ and 0 to $-\pi$ as shown in Figure 4.7(a). In the simulations performed the phase available is 0 to $\pi/2$ and 0 to $-\pi/2$ shown in Figure 4.7(b).

Phase constraint is considered, as this is the more practical case when a passive RFID tag is used as the backscattering device. When an active RFID tag or a transponder is used, it needs an external power source to remodulate the incident radar waveform and transmit the communication symbols/waveforms. When a passive RIFD tag is used, it uses power from the incident radar waveform to re-modulate the incident radar waveform and transmit the communication symbols/waveforms.
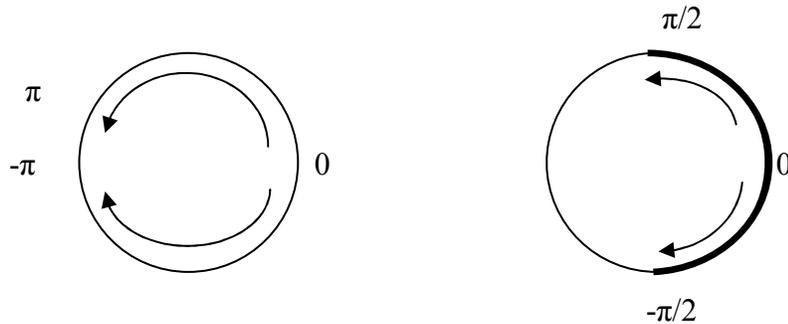
Figure 4.7(a) No phase constraint          Figure 4.7(b) Phase constraint of $0.5\pi$

The simulations are performed for a phase constraint of $0.5\pi$. The range of SIR and SNR values is -40 to -30 dB and -20 to 10dB respectively. Simulations are performed for 2,00,000 bits and for an over-sample factor of $m$=2 and $K = 4$ (the

possible waveforms are 4). The decorrelating receiver is used as the receiver to estimate the waveforms. The SER curves for Eigenvectors-as-Waveforms, Weighted-Combining and Dominant-Projection are compared for the cases of phase constraint and no phase constraint.

### 4.3.1 SER FOR EIGENVECTORS-as-WAVEFORMS

The eigenvectors corresponding to the smallest possible eigenvalues are used as waveforms. From Figure 4.8 it can be seen that due to a phase constraint of $0.5\pi$ the performance of the EAW degrades, but it is still a reasonable degrade considering the fact that only half of the phase is freedom available.As there is a phase constraint the waveforms are squeezed into the available phase hence making the waveforms less covert. As the phase reduces, the design degrees of freedom available for the waveform design reduces. Hence the SER curves shift to the right.
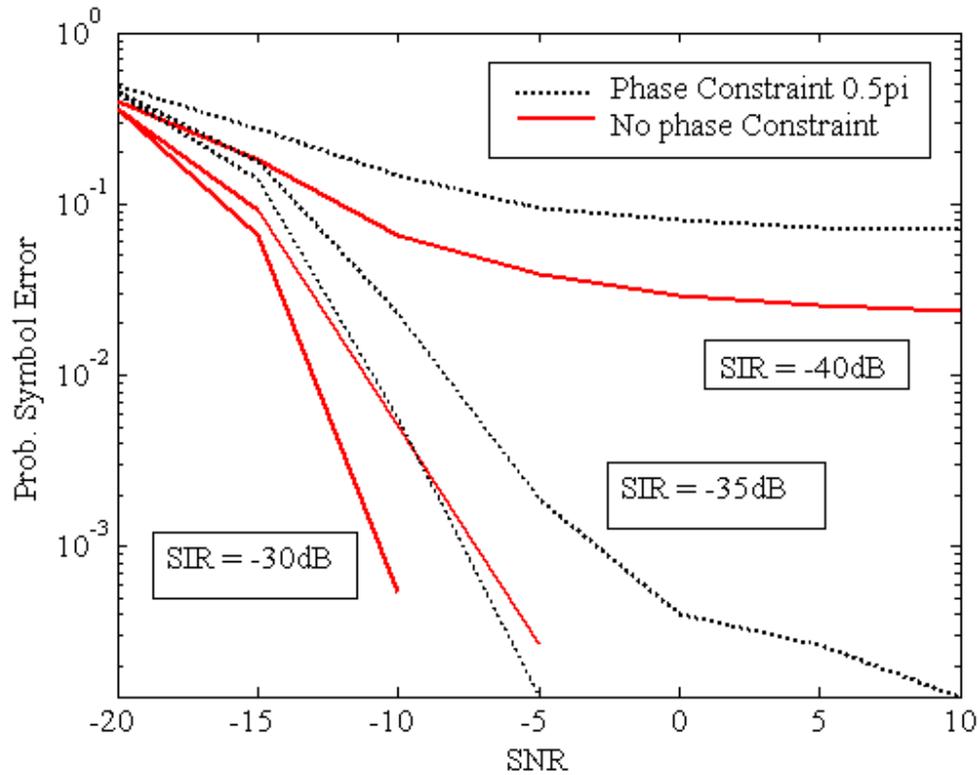
Fig 4.8 SER for Phase constraint of $0.5\pi$ for EAW

## 4.3.2 SER FOR WC

The waveforms used are designed by combining $L=90$ of the non-dominant space. From Figure 4.9 it can be observed that for a phase constraint of $0.5\pi$ the SER curves shift to the right when compared to no phase constraint. The degradation in performance is still reasonable considering the fact that the phase available is only half of the actual phase.
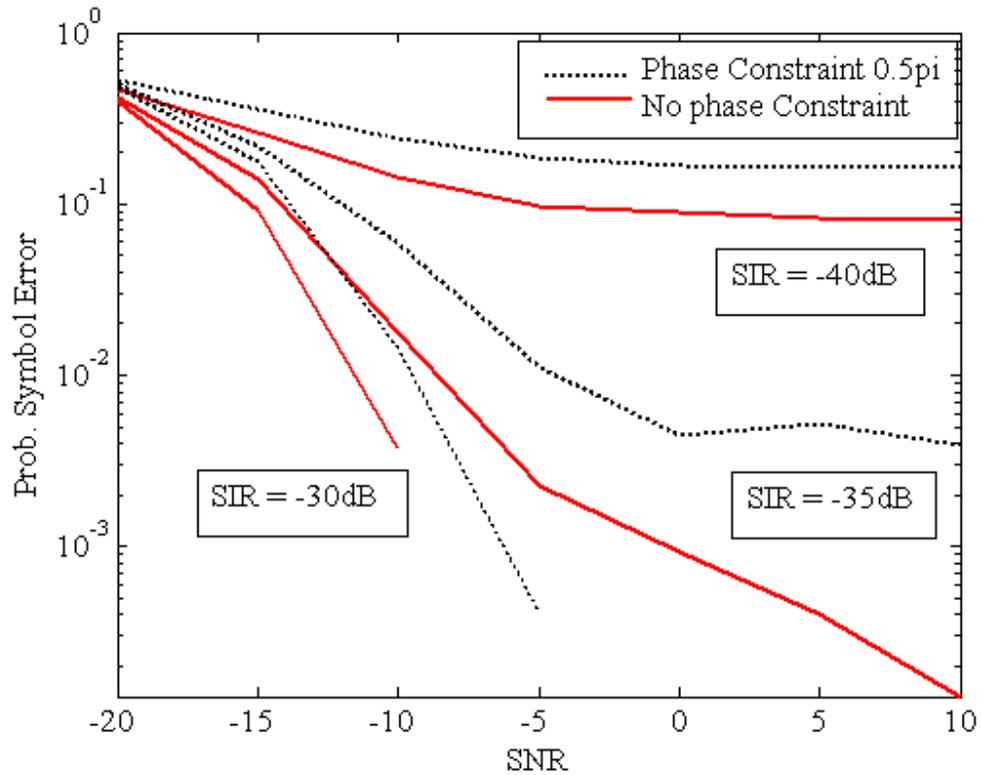
Fig: 4.9 SER for Phase constraint of $0.5\pi$ for WC

As the available phase reduces the probability of bit error increases. The degradation in the SER for Eigenvectors-as-Waveforms and Weighted-Combining is same. This is due to the fact that both the waveforms have less design degree of freedom and hence degradation is similar.

### 4.3.3 SER FOR DP

The waveforms used are designed by projecting away $L$=110 eigenvectors of the dominant space. From Figure 4.10 it can be observed that for a phase constraint

of $0.5\pi$ the SER curves shift to the right when compared to no phase constraint. The performance of the SER curves degrades for phase constraint of $0.5\pi$ when compared with the performance of no phase constraint. As the available phase is only half of the actual phase the degradation is performance is still tolerable.
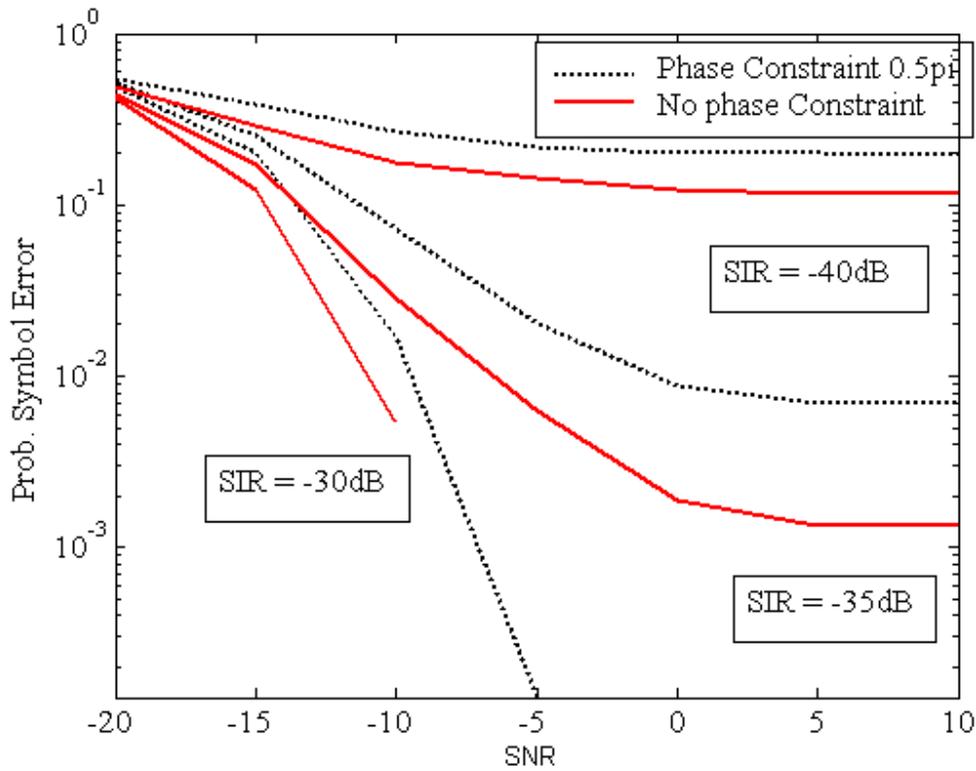


Fig 4.10 SER for Phase constraint of 0.5pi

The SER curve shifts to the right as the phase constraint increases this is due to the fact that the phase of the waveform is constrained by the phase constraint. The degradation in performance of Eigenvectors-as-Waveforms, Weighted-Combining

and Dominant-Projection is same due to the fact that as the phase constraint is same for all the designs and all the correlations of the waveforms are affected similarly for different designs.

## 4.4 TWO SYMBOLS PER WAVEFORM

In all the previous simulations performed, each waveform represented one symbol. Now consider two symbols per waveform that is each waveform represents 2 symbols and each symbol represents 2 bits. So the new data rate is 4 bits/pulse.

Monte Carlo simulations are performed for over 50,000 trials. For each trial, two symbols are selected and a waveform is associated with each symbol. Since $K=4$, there are 4 different symbols or waveforms. The waveforms are designed using WC method. Then noise and interference are added to the waveform depending on the interference and noise levels selected. The SIR and SNR values used for the simulation are -35dB and -20 to 10 dB respectively. The symbol-error rate (SER) curves are plotted. The radar waveform is oversampled by a factor of $m=4$ this is due to the fact that for an oversample factor of $m=2$ the degradation in the performance of two symbols per waveforms is very high.

Figure 4.11 shows the SER for two symbols per waveform and one symbol per waveform. The SER curve for two symbols per waveform shifts to the right i.e., the performance of two symbols per waveform degrades when compared with one symbol waveform. At a probability of symbol error of $10^{-3}$ there is a 5dB loss for two

symbols per waveform when compared with one symbol/waveform. Hence there is a tradeoff between higher data rate and low probability of symbol error.
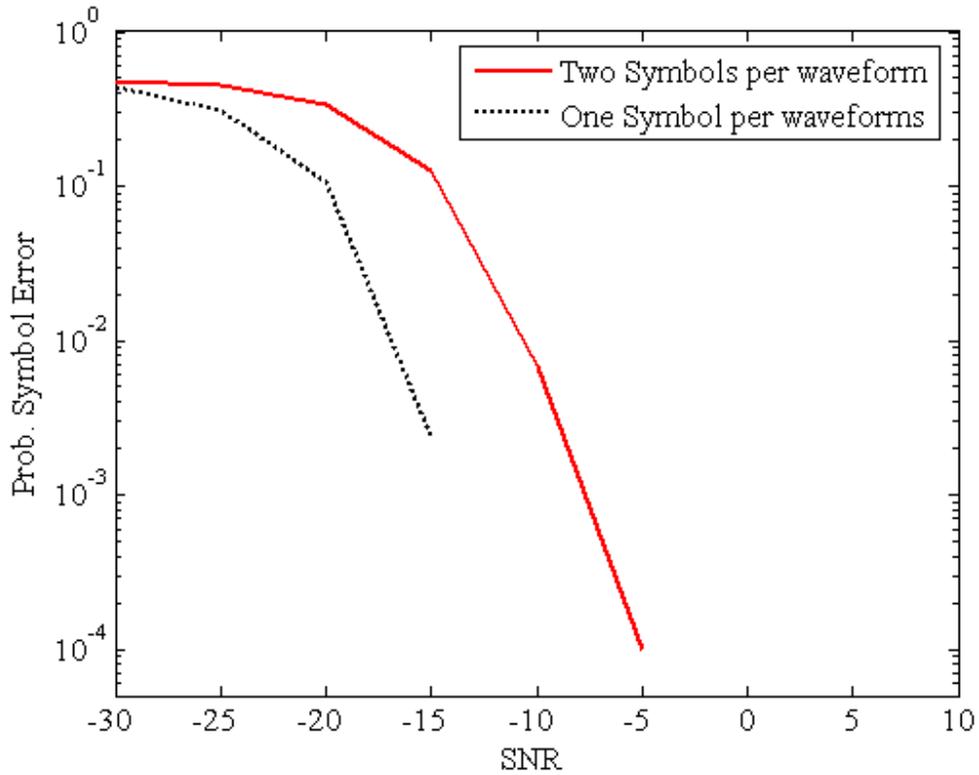


Fig 4.11 SER for Two symbols per waveform

The decoding complexity for two symbols per waveform increases when compared to one symbol per waveform. When decoding two symbols at a time, $K^2$ combinations have to be considered at the receiver, as compared to one symbol per waveform where there are $K$ symbols and hence $K$ combinations at the receiver. Say if symbol 1 is one of the two symbols present then there are 4 combinations that can

actually be present: (1,1), (1,2), (1,3), (1,4). Hence the probability of error increases as 50% of the waveform is the same.

## 4.5 COMPARISION BETWEEN DIFFERENT DATA RATES

In all the previous simulations $K=4$ so the data rate is 2 bits/ pulse. To increase the data rate either, more symbols can be transmitted per waveform interval or the number of communication waveforms can be increased, that is $K=8$, 16 etc. When $K=8$ the data rate is 3 bits/pulse and for $K=16$ the data rate is 4 bits/pulse. The performance for $K=4$, 8 and 16 is compared in terms of probability of bit error. The waveforms are designed using Weighted-Combining combining and $l=90$ and the oversample factor is $m=2$.

Monte Carlo simulations are performed for over 50,000 trials. For each trial one symbol is selected and a waveform is associated with each symbol. Then noise and interference are added depending on the interference and noise levels selected. The SIR and SNR values are -30dB and -20 to 10 dB respectively. The Symbol error rates are calculated and the SER curves are plotted.

Figure 4.12 shows the SER curves for $K=4$, 8, 16. As the data rate increases the SER curve shift to the right. There is approximately a 2dB loss as the data rate increases from 2 to 3 to 4 bits/pulse. Hence there is a tradeoff between higher data rate and low probability of bit error.
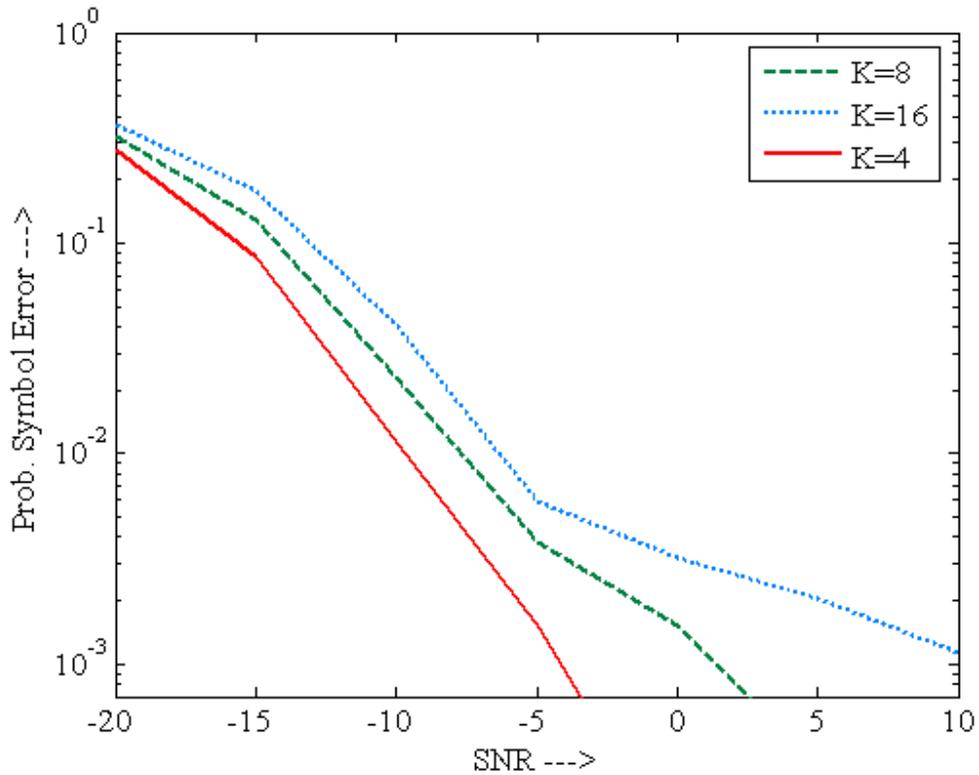
Figure 4.12 SER for *K*=4,8,16

## 4.6 SAMPLING OFFSET

A more common problem is sampling offset which is a triggering hardware problem. The radar waveform has been sampled at Nyquist rate. The sampling interval is given as $T = \dfrac{1}{2B}$. There is always a small amount of offset when a signal is given for the system to start. Hence there is a small amount of offset in the samples known as sampling offset. The amount of sampling offset considered for simulation purpose are 0.2,0.5 and 1 that is $\Delta T = \dfrac{0.2}{2B}, \Delta T = \dfrac{0.5}{2B}$ and

$\Delta T = \dfrac{1}{2B}$ that is the samples are offset by 0.2 of the sampling time for a sampling

offset of 0.2 and the samples are offset by half of the sample for a sampling offset of

0.5 and the samples are offset by one sample for an offset of 1. Though there are

ways to overcome sampling offset, there will always be a small sampling offset in

the hardware triggering so it is safe to make the system robust to sampling offset.

The performance of the waveform designs is compared in terms of SER curves.

Monte Carlo simulations are performed for an oversample factor is $m$=2 and

$K$=4. It can be observed that as the oversampling increases the robustness to

sampling offset decreases. The SIR and SNR ranges are -30 dB and -20 to 10 dB

respectively. The symbol-error rate (SER) curves are plotted for each waveform

design.

### 4.6.1 SAMPLING OFFSET for EAW

The eigenvectors corresponding to the smallest possible eigenvalues are used as

the waveforms. Figure 4.13 shows the SER curves for EAW with and without

sampling offset. Symbol error rate curves are given for a sampling offset of 0.2, 0.5

and 1. For a sampling offset of 0.2 the degradation in SER is reasonable but as the

sampling offset increases the performance of EAW degrades very rapidly which can

be seen in Figure 4.13. Hence, using the Eigenvectors-as-Waveforms design is not

very robust to sampling offset. This may be attributed to the fact that the EAW

waveforms are designed by choosing the eigenvectors corresponding to the smallest

possible eigenvalues. When there is an offset sample the eigenvectors change a little and the eigenvectors corresponding to the smallest possible eigenvalues are no longer the same. The decorrelating receiver is used to decode the waveforms.
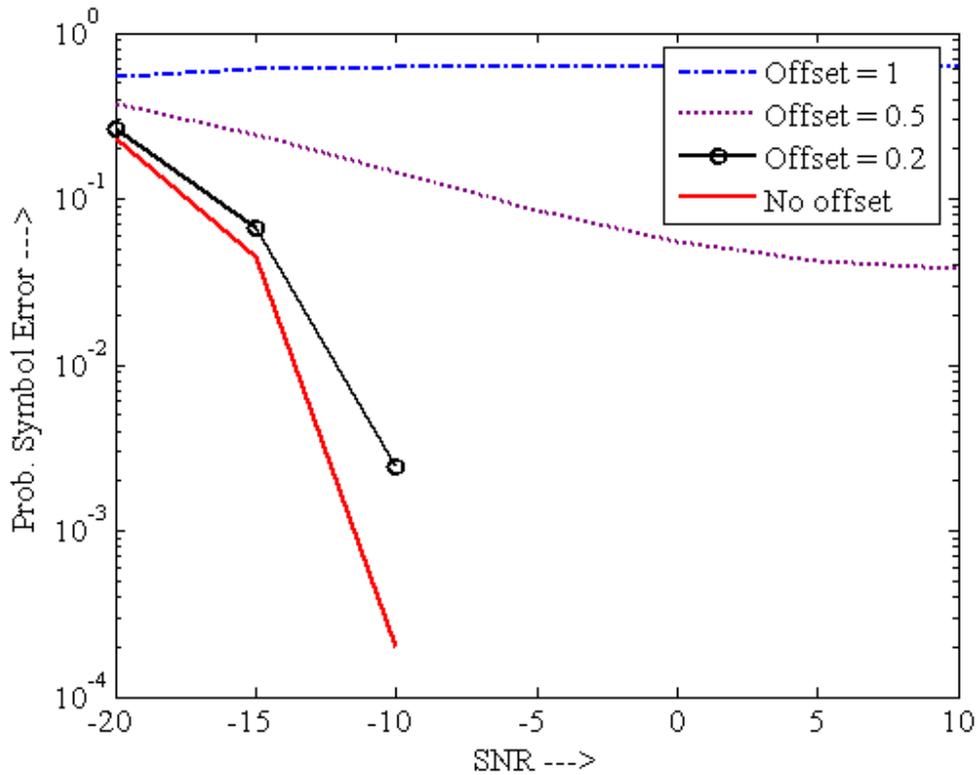


Figure 4.13: SER curve for sampling offset of 0.5 for EAW

**4.6.2 SAMPLING OFFSET for WC**

The waveforms are designed by combining $l$=90 of the non-dominant space. Figure 4.14 shows the SER curves for EAW with and without offset sampling. For a sampling offset of 0.2 the degradation in SER is reasonable. But as the sampling

offset increases, SER performance of Weighted-Combining degrades. The performance of Weighted-Combining is still better when compared to the SER performance of Eigenvectors-as-Waveforms. The waveforms in Weighted-Combining are designed by combining the eigenvectors in the non-dominant space.
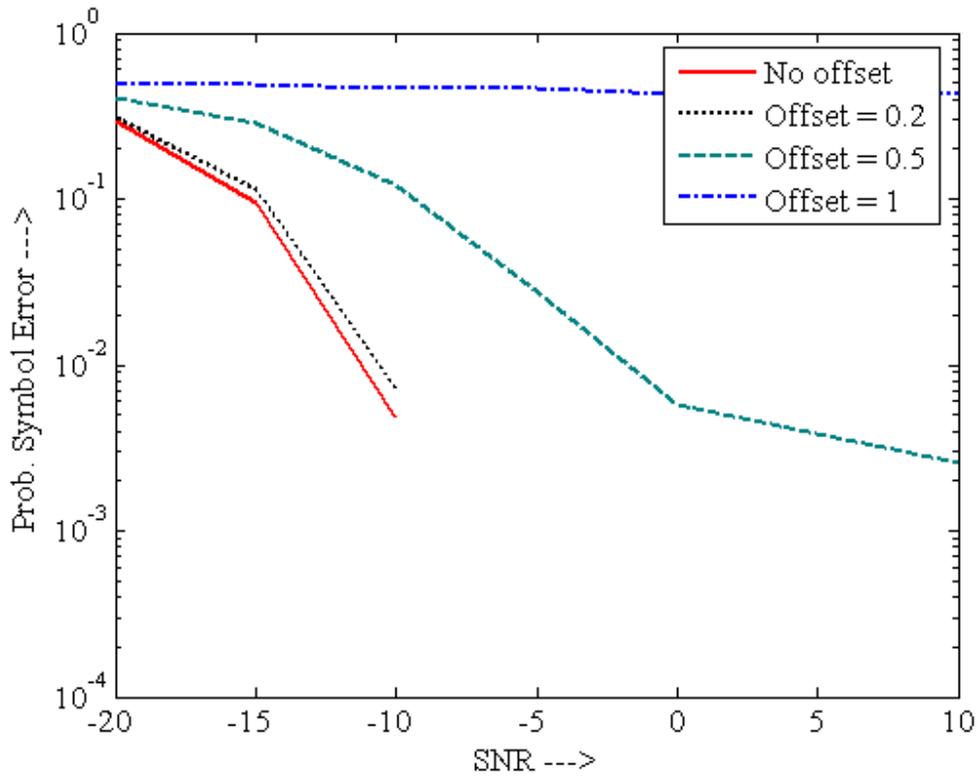


Figure 4.14 SER curve for sampling offset of 0.5 for WC

When there is sampling offset the eigenvectors tend to change and hence the order in which the eigenvectors are combined changes resulting in degradation of performance of Weighted-Combining. The degradation in performance of SER for Weighted-Combining is still better than Eigenvectors-as-Waveforms as the whole

non-dominant space is used to design the waveforms for Weighted-Combing. Using the design of Weighted-Combining as waveforms design is not very robust to sampling offset but it still performs better than Eigenvectors-as-Waveforms.

### 4.6.3 SAMPLING OFFSET for DP

The waveforms are designed by projecting out $l=110$ of the dominant space. Figure 4.15 shows the SER curves for Dominant-Projection with and without offset sampling. Dominant-Projection performance is same for any sampling offset. The waveforms are designed by projecting out the dominant space of the radar. The waveform design does not depend on the order of the eigenvectors (as in Weighted-Combing the order of combing the eigenvectors is important). As long as the dominant space of the radar is the same the waveform design of Dominant-Projection is not affected and sampling offset does not change the dominant-space of the radar. Hence the performance of SER is not affected much due to sampling offset. Using the Dominant-Projection for design of waveforms is very robust to sampling offset.
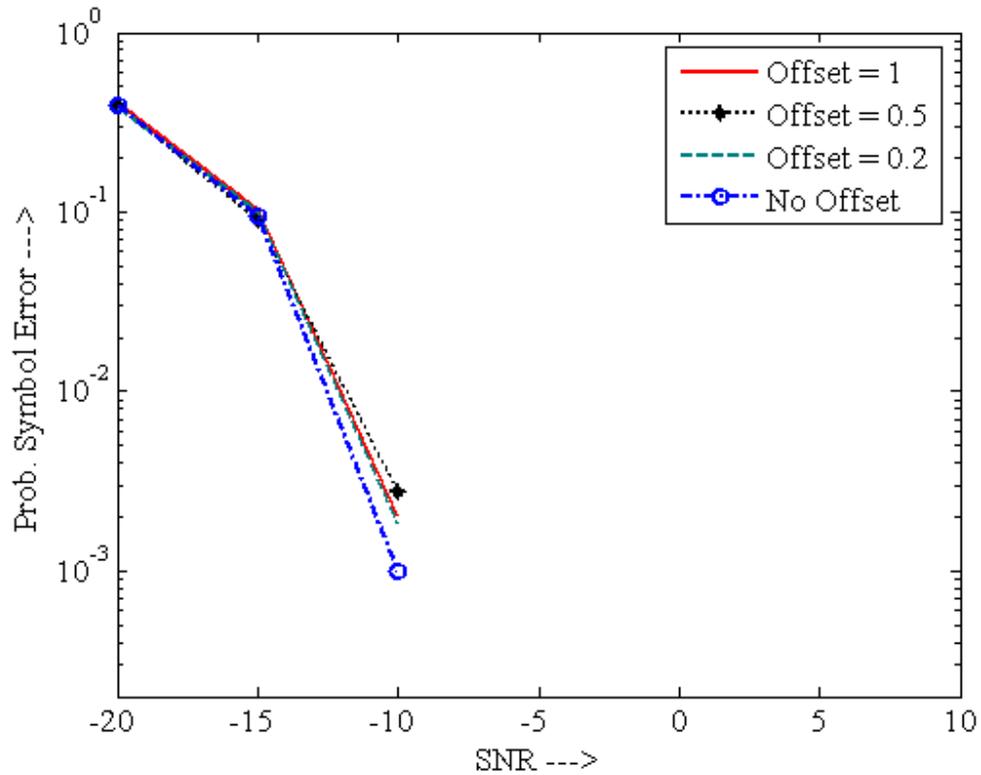
Figure 4.15 SER curve for Sampling offset of 0.5 for DP

## 4.7 OVER SAMPLING

It has been observed in Section 4.5 that as the data rate increases the probability of symbol error increases. Oversampling solves the problem of achieving both higher data rate and lower probability of symbol error. In this Section the performance of WC waveform for an oversample factor of $m$=2 and $m$=3 is compared. The WC waveforms are designed by combining $l$=90 of the non-dominant space for $m$=2 and $l$=135 for $m$=3 so that the non-dominant space combined is maintained at 45% and $K$=4. Monte Carlo simulations are performed for over

1,00,000 trials. The noise and interference are added to achieve SIR and SNR values of -35dB and -20 to 10 dB respectively.
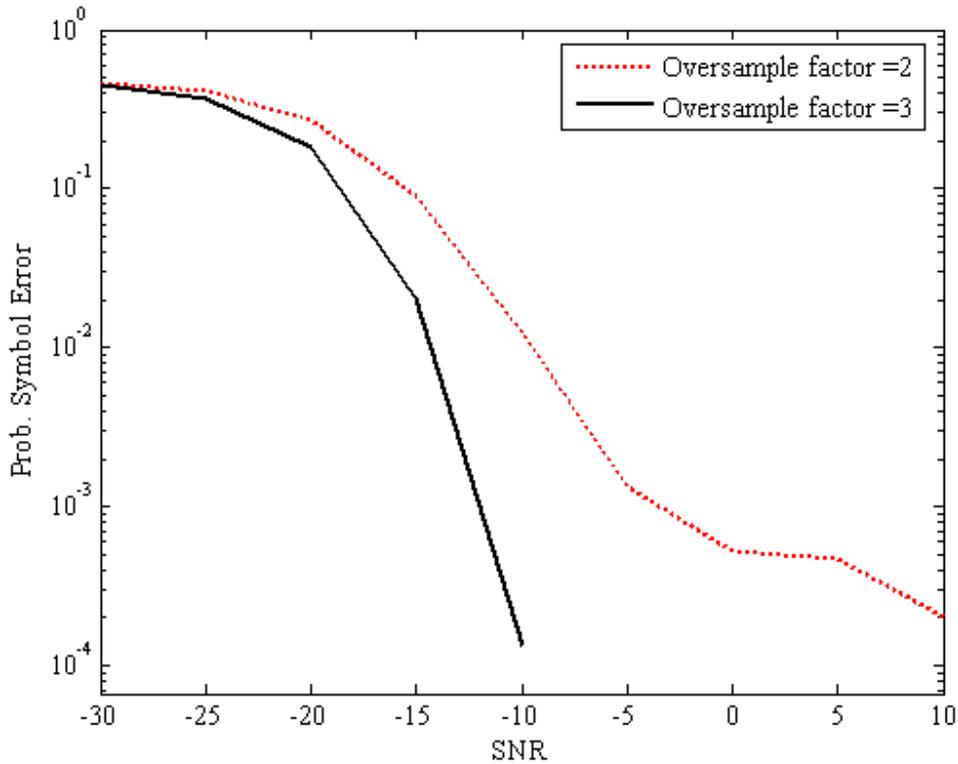


Figure 4.16: SER curve for Oversample factor of 2 and 3

Figure 4.7 shows the SER curve for an oversample factor of $m=2$ and $m=3$. The performance in terms of SER improves a lot from an oversample factor of $m=2$ to an oversample factor of $m=3$. There is almost a 5dB gain (at a symbol error probability of $10^{-3}$) by oversampling from 2 to 3. But as the oversampling increases the hardware complexity increases.

**4.8 DISCRETE RADAR WAVEFORM**

In the previous simulations the radar waveform transmitted is a continuous waveform. When a discrete radar waveform is used, the bandwidth increases due to the transition regions of the discrete radar waveform. The transition region is the sharp transitions from one discrete phase (or chip) to another. These transitions determine the actual bandwidth of the discrete waveform.
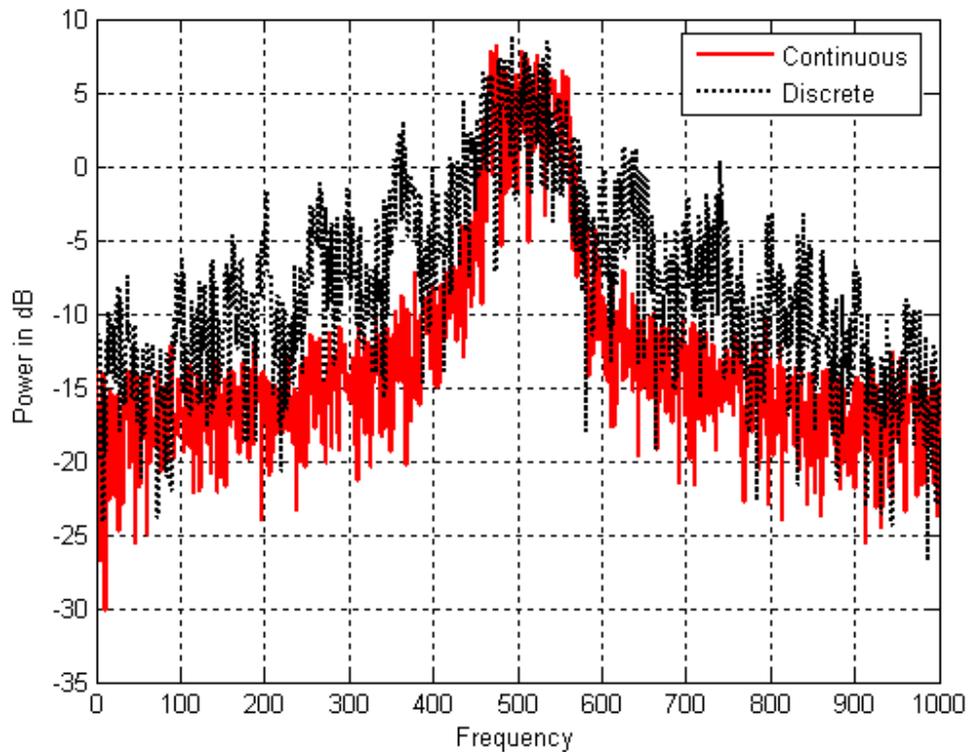


Figure 4.17: Spectrum of continuous and discrete radar P3 code

The continuous Linear FM (LFM) and discrete P3 code (Nyquist sampled LFM) have been oversampled by a factor of 10. The phase transitions for the

continuous radar waveform are smooth from one chip to another whereas a discrete radar waveform exhibits a very sharp phase transition from on chip to another, these sharp transitions define the bandwidth of the radar waveform. It can be seen from Figure 4.17 that the discrete waveform has more bandwidth compared to the continuous waveform.

## 4.9 BARKER CODE

Instead of considering the continuous LFM as the illuminating waveform, we shall now consider a discrete Polyphase Barker sequence [21]. A Polyphase Barker sequence of length $N$=40 [21] . The mathematical model for the barker sequence is

$$A_m = \exp\{2\pi j,\ m/M\} \text{ where } 0 \leq m \leq M\text{-}1$$

This sequence is also known as $M$ phase Barker sequence where $M$=90. The Barker sequence is generated from the table given in reference. The performance of the different waveform designs using the Polyphase Barker code as the illuminating waveform is discussed. Monte Carlo simulations are performed for an oversample factor of $m$=3, $K$=4 and for SIR and SNR value range are -40 to -30 and -20 to 10 respectively. The SER curves are plotted for Eigenvectors-as-Waveforms, Weighted-Combining and Dominant-Projection.

## 4.9. 1 SER FOR EAW

The eigenvectors corresponding to the smallest possible eigenvalues are used as the waveforms. Figure 4.18 shows the SER curve for EAW. The performance of

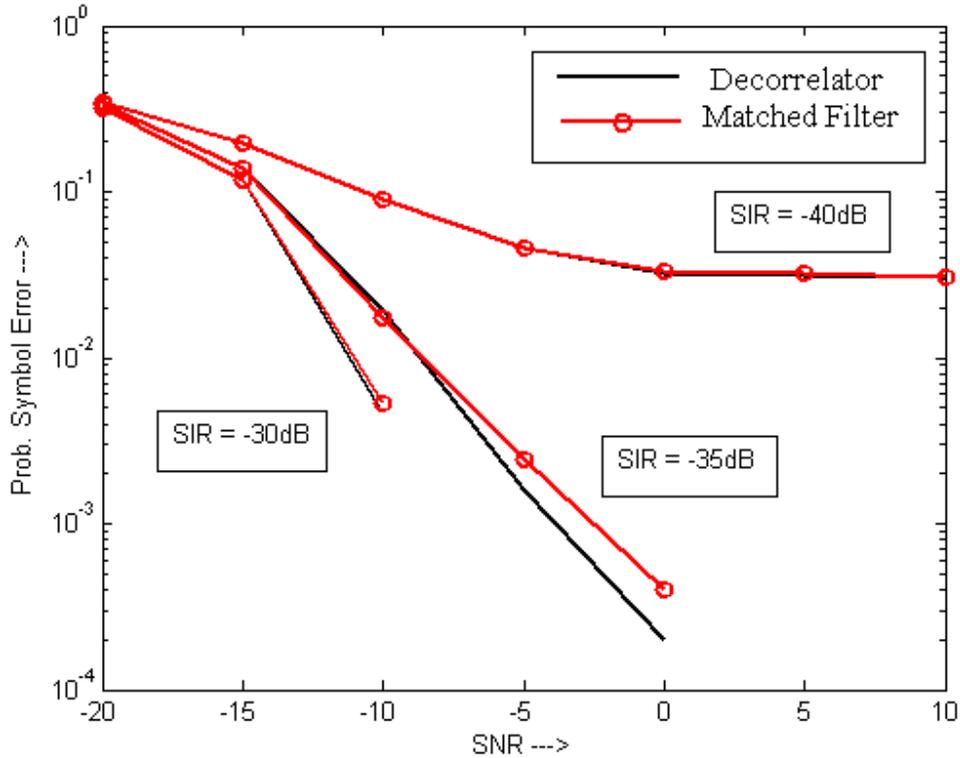decorrelator and the matched filter are identical. This is similar to the performance of using LFM waveform.



Fig 4.10.1 SER for EAW

## 4.9.2 SER FOR WC

The non-dominant space combined is $l$=40. The SER value is calculated and plotted for both the matched filter and the decorrelator. It can be seen from Figure 4.19 that the performance of decorrelator is better than the matched filter.   The EAW performs better than Weighted-Combining for both the receivers (the matched

filter and the decorrelator). The SER curves follow the same trend as the previous case of using the LFM waveform.
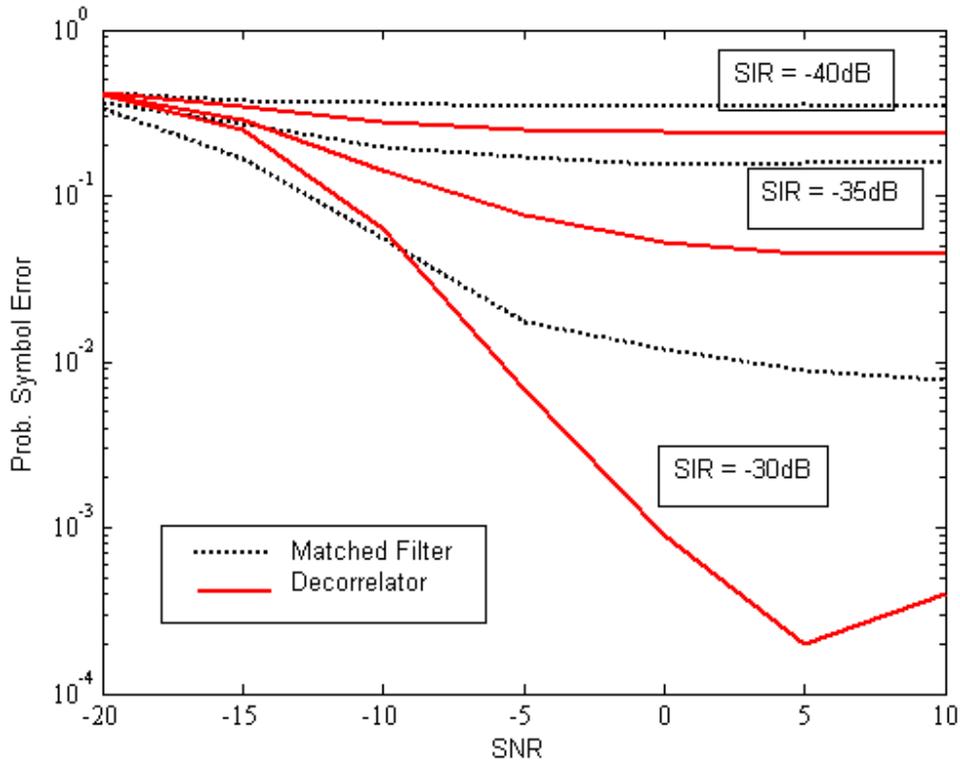


Fig 4.19 SER for WC

### 4.9.3 SER FOR DP

The dominant space projected out is $l=60$. The performance of the decorrelator is better than the matched filter. The performance of Dominant-Projection is similar to Weighted-Combining and the Eigenvector-as-Waveforms performs better than DP and WC for both the receivers (the matched filter and the decorrelator).
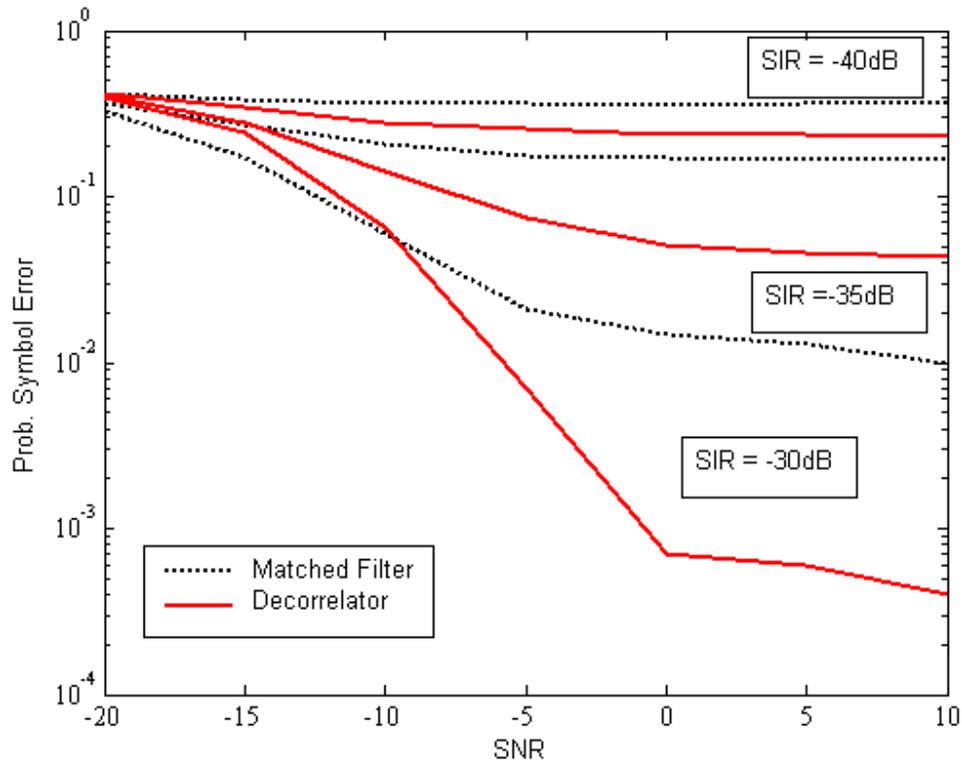
Fig: 4.20 SER for DP

The general performance of the waveform design for Eigenvectors-as-Waveforms, Weighted-Combining and Dominant-Projection is similar for both the discrete Barker code and the continuous LFM. Eigenvectors-as-Waveforms performs better than Dominant-Projection and Weighted-Combining and the performance of Weighted-Combining and Dominant-Projection is similar. Hence the design of waveforms is such that irrespective of the illuminating waveform used and the performance of the waveforms will still follow the same trend.

# CHAPTER 5

## <u>CONCLUSIONS and FUTURE WORK</u>

In this thesis the embedding of covert communication waveforms into the backscatter of radar and the problems pertinent to this issue have been discussed. The main aim is to communicate covertly and at a subsequently higher data rate (than the previous *inter-pulse* approach) in the backscatter of the radar. For this purpose, waveforms have been designed and the design strategies and issues have been discussed. Increasing the design space was one of the issues in the waveform design and it has been overcome by bandwidth extrapolation (or oversampling). The three different strategies of waveform design Eigenvectors-as-Waveforms (EAW), Weighted-Combining (WC) and Dominant-Projection (DP) have been discussed. One of the main issues in the waveform design process is the tradeoff between the waveforms being covert and bit-error-rate. Radar has a sloppy spectrum and it tends to bleed into the surrounding spectrum exhibiting a "bleeding" effect. In all the waveforms design approaches the waveforms are designed such that they are in the spectral "bleeding" region of the radar. The Weighted-Combining and Dominant-Projection are designed such that they occupy almost all of the non-dominant space of the radar whereas EAW occupies a very small non-dominant space. Hence, Weighted-Combining and Dominant-Projection are more covert when compared to Eigenvectors-as-Waveforms. Simulation results have been presented. The Symbol-error-rate curves for Eigenvectors-as-Waveforms, Weighted-Combining and

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Dominant-Projection have been compared for both the matched filter and the decorrelator. The measure of low probability of intercept (LPI) i.e., which is the measure of covertness is discussed. The continuous waveform (Linear frequency modulated radar) and the discrete waveform (Polyphase Barker code) are both used and the results are shown to prove that waveform design strategies apply for any illuminating waveform in general.

In terms of symbol error rate Eigenvectors-as-Waveforms performs best and the performance of Weighted-Combining, Dominant-Projection is similar, as the waveforms design in Eigenvectors-as-Waveforms is such that it minimizes the interference from the radar. In terms of covertness Weighted-Combining and Dominant-Projection are the best and more covert than EAW as the waveforms designed using Dominant-Projection and Weighted-Combining are partially correlated with the radar and hence have the natural radar masking whereas, the waveforms designed using Eigenvectors as waveforms are such that it minimizes the interference from the radar and hence reducing the masking effect of the radar. Also, Weighted Combining and Dominant Projection waveforms are designed such that they occupy most of the non-dominant space of the radar where as EAW occupy very less non-dominant space of the radar. In terms of the hardware constraint offset sampling Dominant Projection performs best followed by Weighted Combining and EAW.

**FUTURE WORK**

1. All the results and discussions presented are given for a simple case of having a single user or a single backscattering device. It can be extended to multi-users. But in multi-users there is the issue of mutual interference from one user to another and the means to differentiate one user from the other.

2. The design degrees of freedom for waveform design may be increased by considering polarization and time coded convolution.

3. An optimum set of waveforms may be designed such that they can be used for any given illuminating waveform.

4. The receiver design may be optimized such the probability of symbol error can be reduced or to achieve higher data rates.

5. Error Correction schemes may be designed so as to achieve higher data rates.

6. *Inter-pulse* modulation schemes may be used over the given *intra-pulse* modulation scheme thus increasing the data rate and covertness of the communication waveforms.

7. The waveform design may be optimized so as to minimize synchronization issues.

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