A Study on the Axial Leakage Magnetic Flux in a Spoke Type Permanent Magnet Synchronous Motor

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Abstract—A spoke type permanent magnet synchronous motor (Spoke PMSM) generates axial leakage magnetic flux due to its flux-concentrating rotor structure and therefore degrades motor performance. Although a three dimensional finite element method (3D-FEM) can be used to estimate the axial leakage magnetic flux, it requires a lot of analyzing time. In this study, a calibration coefficient is proposed to estimate the axial leakage magnetic flux using a two dimensional finite element method (2D-FEM) only instead of applying 3D-FEM. This calibration coefficient is expressed by the major geometric parameters of a motor and can be calculated using the material information of the permanent magnet and the bridge of rotor core in 2D-FEM. The validity of the proposed equation is verified by 3D-FEM and tests.

Keywords—Spoke PMSM; Axial leakage magnetic flux; Calibration coefficient;

I. INTRODUCTION

Recently studies on applying spoke type permanent magnet synchronous motor (Spoke PMSM) to home appliances and vehicles have been actively conducted [1-5]. It is based on the fact that the Spoke PMSM has a flux-concentrating rotor structure and that represents higher efficiency and power density than other permanent magnet (PM) motor having non-flux concentration structure such as surface permanent magnet synchronous motor (SPMSM). However, Spoke PMSM has a problem that an axial leakage magnetic flux which does not exist in the non-magnetic flux concentration motor occurs due to the difference of magnetic potential between rotor cores. Such an axial leakage magnetic flux reduces the effective magnetic flux and degrades the performance characteristics of the motor. There have been few studies on this in the existing paper, and recently one paper discusses some of these[6]. However, this paper is confined to the axial flux leakage through the shaft, and its influence is not significant. The leakage magnetic flux discussed in this paper deal with the flux generated through axial air between adjacent rotor cores. This axial leakage magnetic flux decreases effective flux and therefore degrades motor performance. In order to accurately predict performance of Spoke PMSM, it is essential to use 3D-FEM, which can consider the axial leakage magnetic flux. However, the 3D-FEM is not used in a design process because it requires a lot of analyzing time compared to that of the 2D-FEM. The 3D-FEM is mainly used as a verification purpose. As a result, it is very difficult to consider the axial leakage magnetic flux in an initial design process. In this study, a calibration coefficient that can consider the axial leakage magnetic flux in a 2D-FEM process is proposed. The calibration coefficient is derived based on a magnetic equivalent circuit (MEC) and can be determined as the geometric parameters of a motor such as number of poles, stack length, rotor diameter, and air gap length. The magnetic equivalent circuit is designed to consider a fringing effect generated from the axial leakage magnetic flux through combining it with conformal mapping. For verifying the validity of this study, the conventional 2D-FEM, calibration coefficient applied 2D-FEM, and 3D-FEM were compared. Then, a motor was fabricated to compare the results obtained from both the test and the analysis method.

II. AXIAL LEAKAGE MAGNETIC FLUX

Fig. 1 (a) shows the rotor structures of Spoke PMSM. There are significant potential differences between rotor cores because the rotor core of the Spoke PMSM is separated by PMs and is only connected by saturated core bridge. Both magnetized PMs and saturated core bridge have similar permeability to air[7]. As a result, the magnetic potential difference in the rotor core of the Spoke PMSM, which has a flux-concentrating structure, produces a leakage magnetic flux through the axial air of the motor between adjacent rotor cores as shown in Fig. 1 (b).

![Fig. 1. Axial leakage magnetic flux and its cause in Spoke PMSM](image)

III. CALIBRATION COEFFICIENT

The magnetic flux that can be considered in the 2D-FEM analysis is the x- and y-direction component. However, the
Calculate the linkage flux per poles ($\Phi_{2D,MEC}$) of 2D-MEC, which has been neglected axial leakage magnetic flux in Spoke PMSM

Calculate the linkage flux per poles ($\Phi_{3D,MEC}$) of 3D-MEC which has been considered for axial leakage magnetic flux in Spoke PMSM

Calculate the calibration coefficient ($K_{axial}$) by dividing the $\Phi_{3D,MEC}$ by the $\Phi_{2D,MEC}$

Calibrate both the residual magnetic flux density of PM ($B_r$) and magnetic flux density of the rotor bridge ($B_b$) by multiplying $K_{axial}$

Axial leakage magnetic flux fundamentally includes the z-direction component, it cannot be considered with a general 2D-FEM. In order to solve this problem, we propose the process presented in Fig. 2. In this process, Step 1 to Step 3 represents a process of calculating the calibration coefficient. The process applies an analytic method using a MEC modified by conformal mapping. Step 4 shows a method of applying the determined calibration coefficient to 2D-FEM. The calibration coefficient is applied to both the residual magnetic flux density of the permanent magnet and the magnetic flux density of the rotor bridge. The performance degradation caused by the axial leakage magnetic flux is reflected to 2D-FEM through this process and the results are similar to that of 3D-FEM.

A. Magnetic flux per pole by using 2D-MEC without considering axial leakage magnetic flux (Step 1)

Fig. 3 (a) shows the Spoke PMSM that includes a stator without slots for the convenience of calculations. The stator that includes general slots can be replaced by a stator without slots using the Carter’s coefficient. The configuration of the MEC for the analysis model presented in Fig. 3 (a) is illustrated in Fig. 3 (b). Here, the air gap permeance is presented by Eq. (1).

$$P_g = \frac{1}{R_g} = \frac{1}{\int_R^R \frac{P}{\mu_0 L_s 2(1-\alpha) \pi} \, dx} = \frac{\mu_0 L_s 2(1-\alpha) \pi}{P \ln \left( 1 + \frac{g}{R} \right)}$$

Where, $P$ is number of poles, $R$ is rotor radius, $\alpha$ is the thickness of the permanent magnet determined by angles, $g$ is air gap length, $L_s$ is stack length, and $\mu_0$ is vacuum permeability.

Eq. (2) shows the permeance of PM.

$$P_m = \int_{R-L_m}^R \frac{\mu_0 \mu_r P L_s}{\alpha \pi x} \, dx = \frac{\mu_0 \mu_r P L_s}{\alpha \pi} \ln \left( \frac{R}{R - L_m} \right)$$

Where, $L_m$ is PM length, and $\mu_r$ is the relative permeability of PM.

Eq. (3) shows the value modeled by the power source of the permanent magnet.

$$\phi_m = B_r A_m = B_r L_m L_s$$

Where, $B_r$ is the residual magnetic flux density of PM.

Eq. (4) shows the modeling value of the leakage magnetic flux in the bridge inside the rotor presented in Fig. 3 (a).

$$\phi_b = B_b A_b = B_b W_b L_s$$

Where, $B_b$ is the magnetic flux density in the bridge, and $W_b$ is bridge thickness.

Using the equations from Eq. (1) to (4) and the circuit in Fig. 3 (b), the linkage flux per pole of Spoke PMSM, in which the axial leakage magnetic flux path is neglected, can be expressed as Eq. (5).

$$\Phi_{2D,MEC} = \frac{(2\phi_m - \phi_b) P_g}{P_g + 2P_m}$$

B. Magnetic flux per pole by using 3D-MEC with considering axial leakage magnetic flux (Step 2)

As shown in Fig. 4, the difference between 3D- and 2D-circuits is the presence of an axial leakage magnetic flux path. The axial leakage path formed between adjacent rotor cores can be assumed to be semicircular as shown in Fig. 5 (a) and the axial leakage permeance is obtained by using the semicircular path as shown in Eq. (6).
In practical case, however, it is not possible to avoid the fringing effect presented in Fig. 5 (b) because it is a curvilinear path, not a straight line, with large air gap. However, it is almost impossible to consider the actual path of the ellipse shown in Fig. 5 (b) during integration for the permeance calculation of Eq. (6). In order to solve the problem caused by fringing effect, this paper used a method of changing the elliptic path to the straight path using conformal mapping. Using the transformation equation, Eq. (7), the conventional semicircular path presented in Fig. 6 (a) can be transformed to square path as illustrated in Fig. 6 (b) [8].

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\[
Z = \ln W
\]  

(7)

Here, the idealized case without the fringing effect calculated using the square path shown in Fig. 6 (b) can be expressed as Eq. (8).

\[
P_{\text{ideal}} = \frac{2\mu_0 L_s}{\pi} \ln \left(1 + \frac{c}{m}\right)
\]  

(8)

Where, \(m\) is the width of PM as shown in Fig. 6 (a), and \(c\) is the width of rotor core as shown in Fig. 6 (a).

The actual path considering the fringing effect is presented in Fig. 7 (a), (b). For considering the trapezoid type flux path presented in Fig. 7 (b), two additional variables are added. The first variable, \(k_1\), shows decreases in the lower section of the flux path in a circular shape. The second variable, \(k_2\), represents increases in the upper section of the flux path in a circular shape. \(k_1\) and \(k_2\), due to the symmetry as shown in Fig. 7 (b), it is reasonable to assume that have the same value \(k\). The axial leakage permeance that introduce these two variables are presented as Eq. (9) respectively.

\[
P_{\text{real}} = \frac{1}{\mu_0 L_s} \int_{0}^{\frac{\pi}{2}} \frac{2 \ln (k^2)}{v + \ln \left(1 + \frac{c}{m}\right)} \, dv
\]

(9)

Using the ratio of Eq. (9) and Eq. (8), the correct factor for the fringing effect occurred in the axial leakage flux path can be derived as Eq. (10)

\[
k_{\text{frg}} = \frac{P_{\text{real}}}{P_{\text{ideal}}} = \frac{\ln (k^2)}{\ln \left(1 + \frac{c}{m}\right) \ln \left(1 + \frac{c}{m}\right)}
\]  

(10)
C. Calibration coefficient for 2D-FEM for considering of axial leakage magnetic flux (Step 3)

If the ratio of the linkage flux per pole calculated in Step 1 and Step 2 is the same as that of the calculations using 2D-FEM and 3D-FEM, it can be expressed as Eq. (13).

\[
\Phi_{3D, MEC} : \Phi_{2D, MEC} = \Phi_{3D, FEM} : \Phi_{3D, FEM}
\] (13)

Then, Eq. (13) can be denoted as Eq. (14).

\[
\Phi_{3D, FEM} = \frac{\Phi_{3D, MEC}}{\Phi_{2D, MEC}}
\] (14)

As the ratio of the linkage flux per pole calculated using the MEC presented in Eq. (14) is assumed as the calibration coefficient of the axial leakage magnetic flux, it can be presented as Eq. (15) and it is always smaller than 1.

\[
k_{axial} = \frac{\Phi_{3D, MEC}}{\Phi_{2D, MEC}}
\] (15)

Where \( \Phi_{3D, MEC} \) is the flux per pole by 3D-MEC, \( \Phi_{2D, MEC} \) is the flux per pole by 2D-MEC.

D. Applying the calibration coefficient to 2D-FEM (Step 4)

By substituting Eq. (5) to Eq. (15) for applying the calibration coefficient to practical 2D-FEM, it can be denoted as Eq. (16).

\[
\Phi_{3D, MEC} = k_{axial} \Phi_{2D, MEC} = k_{axial} \frac{(2\phi_m - \phi_h) P_g}{P_g + 2P_m}
\]

\[
\Phi_{3D, MEC} = \frac{(2B_r A_m - B_b A_h) P_g}{P_g + 2P_m}
\] (16)

As the ratio of the linkage flux per pole calculated using the 2D-FEM the performance degradation due to the axial leakage magnetic flux and it is operated by multiplying the calibration coefficient to both the residual magnetic flux density of PM and the flux magnetic density of the rotor bridge.

IV. VERIFICATION

A. Verification by 3D-FEM in variation of parameters

For verifying the validity of the proposed calibration coefficient, case studies under the condition noted in Table 1 were performed. In order to accurately calculate the axial leakage magnetic flux, an air dummy that fully covers the axial leakage path along the stack direction at both upper and lower
sections of the rotor core is required in the implementation of the 3D-FEM as shown in Fig. 10 (a). If the axial leakage path is a semi circle as illustrated in Fig. 10 (b), the height of the air dummy can be expressed as Eq. (17).

\[
h = R \sin \left( \frac{\pi}{P} \right)
\]

(17)

when, \( h \) is the height of the air dummy, \( R \) is the radius of rotor, \( P \) is the number of poles.

![Air dummy for axial leakage](image)

(a) Air dummy for axial leakage

![Air dummy height when path is circle](image)

(b) Air dummy height when path is circle

Fig. 10. 3D-FEM model for considering the axial leakage magnetic flux

![3D FEM results according to dummy height at different poles](image)

(a) 3D FEM results according to dummy height at different poles

![3D FEM results according to dummy height at different radius of rotor](image)

(b) 3D FEM results according to dummy height at different radius of rotor

Fig. 11. Comparison of 3D-FEM results according to air dummy height

However, the height of the air dummy is to be determined as a larger value than Eq. (17) due to the fringing effect. Fig. 11 shows the change in axial leakage magnetic flux according to varies in the height of the air dummy. As a result, the height of air dummy is to be determined by about 2.2 times than Eq. (17) in order to accurately calculate the axial leakage magnetic flux within 0.5%. In this paper, the height of the air dummy in all 3D-FEM was determined by 2.2 times than Eq. (17).

### Table I. 3D-FEM Analysis Models for Verification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sym.</th>
<th>Unit</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Poles</td>
<td>( P )</td>
<td>-</td>
<td>10</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>Radius of Rotor</td>
<td>( R )</td>
<td>mm</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Air gap length</td>
<td>( g )</td>
<td>mm</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Stack length</td>
<td>( L_s )</td>
<td>mm</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Magnet width ratio</td>
<td>( \alpha )</td>
<td>-</td>
<td>0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Magnet length</td>
<td>( L_m )</td>
<td>mm</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Bridge width</td>
<td>( W_b )</td>
<td>mm</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Residual magnetic flux density</td>
<td>( B_r )</td>
<td>T</td>
<td>0.41</td>
<td>0.41</td>
</tr>
</tbody>
</table>

![Fig. 12. 3D-FEM analysis model for considering the axial leakage flux](image)

Fig. 12. 3D-FEM analysis model for considering the axial leakage flux

![Fig. 13. Comparison the analysis results of conventional 2D-FEM, 3D-FEM and proposed 2D-FEM with the calibration coefficient](image)

Fig. 13. Comparison the analysis results of conventional 2D-FEM, 3D-FEM and proposed 2D-FEM with the calibration coefficient.

The analysis model used in the 3D-FEM of the case study in Table 1 and its mesh information are shown in Fig. 12. The
3D-FEM results and the conventional 2D-FEM and the proposed 2D-FEM results for the analysis models in Table 1 are shown in Fig. 13. As a result, the conventional 2D-FEM showed large analysis errors compared to 3D-FEM but the results of the proposed 2D-FEM with the calibration coefficient were consistent with 3D-FEM.

B. Verification by test of fabricated motor

The fabricated motors for test are shown in Fig. 14 and its specifications are shown in Table 2. Fig. 14 (a) and Fig 14 (b) show the rotor assembly and stator assembly.

![Fig. 14. Fabricated motor for verification](image)

**TABLE II. SPECIFICATIONS OF THE FABRICATED MOTOR**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sym.</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Poles</td>
<td>P</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Number of slots</td>
<td>N_s</td>
<td>mm</td>
<td>12</td>
</tr>
<tr>
<td>Air gap length</td>
<td>g</td>
<td>mm</td>
<td>0.6</td>
</tr>
<tr>
<td>Stack length</td>
<td>L_s</td>
<td>mm</td>
<td>50</td>
</tr>
<tr>
<td>Radius of rotor</td>
<td>R</td>
<td>mm</td>
<td>40</td>
</tr>
<tr>
<td>Magnet length</td>
<td>L_m</td>
<td>mm</td>
<td>20</td>
</tr>
<tr>
<td>Bridge width</td>
<td>W_b</td>
<td>mm</td>
<td>1</td>
</tr>
<tr>
<td>Residual magnetic flux density</td>
<td>B_r</td>
<td>T</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Fig. 15. Measured no-load electromagnetic force at 600rpm

Test results are presented in Fig. 15. Similar to the previous case studies, the proposed 2D-FEM were consistent with experimental results. However, the conventional 2D-FEM is calculated to be 8% larger than the experimental value, because the reduction of the linkage flux by the axial leakage is not possible to be considered in the conventional 2D-FEM. Although similar results to the proposed 2D-FEM can be obtained with a general 3D-FEM, the analysis time of the proposed 2D-FEM is considerably faster, about 1/70 of the 3D-FEM.

V. CONCLUSION

This paper proposed the calibration coefficient that can consider the axial leakage magnetic flux generated from the Spoke PMSM in 2D-FEM was proposed. The proposed calibration coefficient was presented as the geometric parameters and showed the same results as 3D-FEM. The validity of the proposed method was confirmed by comparison with 3D-FEM for various models. Finally, it was verified by experiments of actually fabricated motor. The proposed methods can be used not only SFMSM but also any motor having the axial leakage magnetic flux such as interior permanent magnet synchronous motor(IPMSM).

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