3-D Magnetostatic Moment Method Dedicated to Arc Interruption Process Modeling

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The behavior of arcs in circuit breakers is affected by interactions of several physics. To simulate this, a 3-D modeling system has already been developed. It considers the arc fluid dynamics, the radiation, the plastic vaporization, the current flow within the electrodes and the plasma, and the magnetic field generated by currents. However, to consider the influence of ferromagnetic regions, the current simulation model has to be extended. The magnetic moment method (MoM) is an integral method where only active regions (i.e., ferromagnetic regions in our context) are meshed. Thus, it is particularly well adapted for breaking modeling with a few ferromagnetic regions in comparison with the whole air region. In this paper, circuit breaker principle is explained, the arc interruption modeling is described, a nonlinear MoM model based on a point matching approach is discussed, and finally an arc interruption process modeled with the introduction of ferromagnetic regions is presented.

Index Terms—Circuit breakers, magnetostatics, moment methods, nonlinear equations, plasma simulation.

I. INTRODUCTION

WHEN a circuit breaker detects a fault current, it opens the electrical circuit and an arc appears. The electrical network viewed from this circuit breaker can be modeled as follows:

$$L_s \frac{dI}{dt} + R_s I = U_s - U_{arc}$$

where $L_s$, $R_s$, and $U_s$ are, respectively, the inductance, the resistance, and the voltage of the system and $U_{arc}$ is the arc voltage.

If the arc voltage ($U_{arc}$) rises above the system voltage ($U_s$), the time derivative of current becomes negative and the current can decrease. This process is so-called the current limitation [1] and is shown in Fig. 1.

Because of the limitation, the maximum current is reduced and the zero current is obtained sooner.

To increase the arc voltage, it is necessary to lengthen and split the arc into series of arcs by metallic splitter plates. Thereby, the arc voltage is increased by multiple cathode and anode falls [2]. The arc displacement during a breaking process is shown in Fig. 2.

This displacement is due to hydrodynamic constraints and magnetic forces. To design a circuit breaker, it is important to have a detailed knowledge of the arc behavior.

The breaking process is fast (~millisecond) and energizing (>kilojoules). It makes the experimentation a difficult task and it is almost always an intrusive experimentation.

Simulations also help to obtain additional information, which is usually not accessible by the experimental investigations, and thus can reduce the time for the development of new circuit breakers.

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II. ARC INTERRUPTION MODELING

The core of the arc system modeling [3], [4] is based on the commercial software FLUENT. It uses a finite-volume method.
solver to compute the fluid dynamics. Others phenomena are considered because of user-defined functions. A nonexhaustive list of those phenomena is given as follows.

1) **Real gas model**: all thermodynamic data \((C_p, C_v, \text{enthalpy, entropy, etc.})\) are described as function of the temperature \((300–30,000 \text{ K})\) and the pressure \((0.5–16 \text{ bars})\) with a polynomial interpolation

\[
\text{Property} (T, P) = \sum_k a_k (T) \log (P)^k.
\]

Each thermodynamic property is described with a polynomial function in pressure \((P)\) where polynomial coefficients are described because of a cubic spline interpolation in temperature \((T)\).

2) **Radiation**: a model of net emissivity and a discrete ordinates radiation model are used.

3) **Arc root phenomena**: a modified Neumann condition is used to consider the loss of energy at the arc root.

4) **Electromagnetism**: the current density \((J)\) is solved because of the static current flow equation and the finite-volume solver. The magnetic field \((H_{\text{ext}})\) due to the current is computed with the well-known Biot and Savart equation

\[
H_{\text{ext}} (r') = \frac{1}{4\pi} \int_V J(r) \times \nabla_r G(r, r') d^3 r
\]

where \(J\) is the current density, \(G\) is the Green function for a Laplace’s equation, \(r\) is the integration variable, and \(r'\) is the position of the computation.

All these models are shown in a simplified diagram of the modeling process in Fig. 3.

During a breaking process, the current density of the arc leads to Ohmic heating (Joule losses > energy source) and magnetic forces (Lorentz forces > momentum source), which cause gas flow and energy transport within and outside the arc.

In the case of a breaking process at low current, ferromagnetic regions can lead to substantial increase in the Lorentz forces on the plasma.

This paper presents a method to model the ferromagnetic effects because of the magnetostatic moment method (MoM) in the Schneider electric’s arc system modeling.

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**Fig. 3.** Diagram of the arc interruption model. \(T\): temperature. \(P\): pressure. \(\sigma\): electrical conductivity.

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**Fig. 4.** Nonlinear flowchart resolution where \(A\) is the solving matrix, RHS is the right-hand side vector, and \(\varepsilon_1\) and \(\varepsilon_2\) are the tolerance parameters.

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### III. 3-D MoM

The MoM is an accurate and inexpensive numerical approach dedicated to the modeling of electromagnetic devices. In this method, only active regions (i.e., ferromagnetic parts of the device) are meshed. Thus, it is particularly well adapted for the modeling of simple radiated devices with only a few ferromagnetic parts in comparison of the total air region [5].

#### A. General Equation

For a magnetostatic problem consisted of some ferromagnetic regions and coils supplied by currents, the magnetic field can be expressed as the sum of the external field \((H_{\text{ext}})\) and the reduced magnetic field \((H_r)\), which derives from the magnetic scalar potential [6]

\[
H = H_{\text{ext}} + H_r = H_{\text{ext}} - \nabla \phi
\]

with for the magnetic scalar potential

\[
\phi (r') = \int_V \nabla_r G(r, r') \cdot M (r) d^3 r
\]

where \(M\) is the magnetization, \(r\) is the integration variable, and \(r'\) is the position of the computation.

This formulation strongly ensures Maxwell–Ampere and Maxwell–Thomson equations. Therefore, to determine the magnetic field \(H\), the magnetic law behavior must also be ensured.
Fig. 5. 3-D nonlinear test case with a ferromagnetic plate excited by an inductor supplied by a current ($I$). The magnetic field is represented in the plate.

Fig. 6. Magnetic field ($H$) computed by FLUX (red) and the MoM (blue) on a posttreatment line and the relative error (green).

For that, the residual is defined as follows:

$$R(M) = \int_{V} (M - m(H)) \cdot W(r') \, d^3r' \quad (6)$$

where $m(H)$ is the image of the magnetic field view from the nonlinear magnetic law and $W$ is a vectorial test function.

B. Discrete Approximation

Several formulations exist with different interpolate functions of $M$ and different test functions. In our example, functions are chosen to be constant by element. Therefore, the magnetization is uniform in each cell and residual is minimized at each cell centroid (point matching method). In this framework and for $p$ magnetic cells.

1) The magnetic field equation (4) becomes

$$H = H_{ext} + [K]M \quad (7)$$

where ($H$, $H_{ext}$, $M$) are 3p vector and $[K]$ is (3p)$^2$ matrix. The matrix $[K]$ is a matrix of view factors between the cells computed by reduction of singularities [7].

2) The residual equation (6) becomes

$$R(M) = m(H) - M. \quad (8)$$

Fig. 7. During a breaking process, arc is represented by three isovales of temperature and the magnetization in the ferromagnetic plate can be observed.
To solve nonlinear magnetostatics problem, the Newton–Raphson process is described as follows [8]:

\[
\left( \frac{\partial \mathbf{m}(\mathbf{H})}{\partial \mathbf{H}} \right) \mathbf{H}^{-1}[\mathbf{K}] - [\mathbf{I}] \Delta \mathbf{M}^n = -\mathbf{R} \left( \mathbf{M}^n - \mathbf{1} \right)
\]

where the first matrix is a diagonal bloc matrix linked with the derivative of magnetic law, \( \mathbf{AM} \) is the increment of the magnetization, \( \mathbf{R} \) is the residual vector, and \( n \) is the iteration number.

The flowchart of the nonlinear resolution is represented in Fig. 4. To reduce the CPU time, the tangent matrix is not updated at each iteration. In other words, a fixed point (with the previous tangent matrix) and a Newton–Raphson method are alternatively used. The presented approach has been validated on several test cases to prove its effectiveness.

A line search has been added to stabilize the convergence and two convergence checks are used to stop the process.

C. 3-D Nonlinear Test Case

A ferromagnetic plate excited by an inductor supplied by a \( I \) is studied first. The test case and the magnetic field in the ferromagnetic plate obtained using the MoM are shown in Fig. 5.

To control the accuracy of the solution, the magnetic field close to the ferromagnetic plate is computed on a posttreatment line and compared in Fig. 6 with the solution obtained from FLUX (commercial finite-element electromagnetic software).

It can be observed that the relative error is very low and it increases close to the ferromagnetic plate. This increase can be further reduced by increasing the number of Gauss integration points.

The method of MoM has been used successfully on several test cases.

IV. CIRCUIT BREAKER APPLICATION

In a circuit breaker model, two ferromagnetic plates are introduced close to the breaking chamber. During the breaking process simulation, the magnetization in plates and their influence on the magnetic field seen by the arc can be observed in Fig. 7.

The comparison of the global electrical data (\( I \) and \( U_{arc} \) arc voltage) in Fig. 8 shows the influence of ferromagnetic parts on the breaking process. With ferromagnetic parts, the arc displacement is improving and therefore the arc voltage increases faster. The arc voltage being greater quickly, the current limitation is more important. The arc voltage is slightly smaller and does not have a zero current sooner. However, this configuration is still interesting because the thermal stresses caused by the flowing energy (\( I^2t \)) are reduced.

V. CONCLUSION

The MoM method is suitable and well adapted for the breaking modeling. Compared with other approaches [9], it avoids the meshing of the whole air region and the use of two different meshes.

The use of the compression method is in progress (hierarchical matrix and adaptive cross approximation [10]) with which a further reduction in the computation time and the memory usage is expected.

The introduction of MoM in the arc system modeling allows considering ferromagnetic regions position and shape optimization. An improvement in the design of circuit breakers is expected.

REFERENCES
