A Multivariate Markov Regime-Switching High-Frequency-Based Volatility Model for Optimal Futures Hedging

Yu-Sheng Lai, Her-Jiun Sheu, and Hsiang-Tai Lee

This study proposes a multivariate Markov regime-switching high-frequency-based volatility (MRS-HEAVY) model for modeling the covariance structure of spot and futures returns, and estimating the associated hedge ratios. S&P 500 equity index data are used in estimations, and the results reveal that the MRS-HEAVY model has a shorter response time than that of the Markov regime-switching GARCH model; this difference is more pronounced in the high-volatility regime than in the low-volatility regime. Out-of-sample hedging exercises illustrate that the MRS-HEAVY exhibits superior hedging performance in terms of both variance reductions and utility gains; it is robust even when transaction costs are considered. © 2017 Wiley Periodicals, Inc. Jrl Fut Mark

1. INTRODUCTION

Generalized autoregressive conditional heteroskedasticity (GARCH) models and their variants are widely applied for volatility modeling. Models within the GARCH framework use daily (squared) returns to determine volatility levels (Bollerslev, 1986; Engle, 1982). However, squared returns offer limited information on ex-post return variation (Andersen & Bollerslev, 1998). Therefore, standard GARCH models might perform poorly in situations where volatility levels change rapidly (Andersen, Bollerslev, Diebold, & Labys, 2003).

Recent studies have documented that incorporating realized variances into standard GARCH (known as GARCH-X models) increases explanatory power and improves predictive accuracy. Using HF data, a number of realized measures of volatility (e.g., the commonly used realized variance estimator of Andersen, Bollerslev, Diebold, & Labys, 2001 and Barndorff-Nielsen and Shephard, 2002) have been proposed for true asset return variation; Andersen, Bollerslev, Christoffersen, and Diebold (2006) and McAleer and Medeiros (2008) have recently reviewed these methods.

Yu-Sheng Lai is Associate Professor at Department of Banking and Finance, National Chi Nan University, 1, University Rd., Puli, Nantou Hsien, Taiwan. Her-Jiun Sheu is Professor at Department of Finance, Ming Chuan University, 250, Zhong Shan N. Rd., Sec. 5, Shihlin District, Taipei, Taiwan. Hsiang-Tai Lee is Professor at Department of Banking and Finance, National Chi Nan University, 1, University Rd., Puli, Nantou Hsien, Taiwan. We are grateful to professor Bob Webb (editor) and an anonymous referee for the valuable suggestions to improve the paper. We thank participants at the 30th International French Finance Association Conference at Lyon, France, and the International Banking, Economics and Finance Association (IBEFA) Conference at San Francisco, USA, for their helpful comments. We also acknowledge financial support from the Ministry of Science and Technology of Taiwan under grant number 99-2410-H-260-028.

Received October 2015; Accepted December 2016

© 2017 Wiley Periodicals, Inc. Jrl Fut Mark

DOI: 10.1002/fut.21842
ability for the dynamic features of volatility (Engle, 2002; Koopman, Jungbacker, & Hol, 2005). High-frequency-based volatility (HEAVY) models can adjust more quickly to changes in volatility than standard GARCH models can (Hansen, Huang, & Shek, 2012; Shephard & Sheppard, 2010).

Precisely forecasting the covariance structure of spot and futures is essential to optimal futures hedging, because the optimal hedge ratio is computed as the conditional covariance of spot and futures returns over the conditional variance of futures returns. HEAVY-type models have produced precise forecasts of the conditional covariance matrix. On this basis, Lai and Sheu (2010) and Sheu and Lai (2014) calculated the hedge ratio using high-frequency (HF) data and discovered that the gains on hedging are substantial, compared with models that include daily prices only. This result illustrates the importance of applying HEAVY models in futures hedging.

Another strand in the literature focuses on forecasting the covariance structure of spot and futures returns with regime-switching GARCH models. Lee and Yoder (2007a) and Alizadeh, Nomikos, and Pouliasis (2008) have applied a regime-switching BEKK GARCH; Lee and Yoder (2007b) suggested a regime-switching varying correlation GARCH; Lee (2010) adopted a regime-switching dynamic conditional correlation GARCH; and Sheu and Lee (2014) proposed a multi-chain Markov regime-switching GARCH for calculating optimal hedge ratios by forecasting the conditional second moments. A common finding has been that incorporating regime switching into multivariate GARCH models enhances the effectiveness of futures hedging.

If the joint distribution of spot and futures returns and hence the hedge ratio is state dependent, a more flexible regime switching model that can swiftly adapt to market changes in a highly volatile regime has potential to improve the forecasts of the optimal hedge ratio and thus the effectiveness of futures hedging. This is the first study to apply a multivariate high-frequency-based regime-switching model in modeling dynamic spot and futures covariance structures. This study fills this gap in the literature by proposing a multivariate Markov regime-switching high-frequency-based volatility (MRS-HEAVY) model for futures hedging. The contribution of this paper to this field of research is twofold. First, the proposed MRS-HEAVY model possesses both HF and regime-switching properties. These dual properties allow us to investigate whether the response time of a HEAVY model is shorter than that of a GARCH model in the high volatility regime, as well as whether MRS-GARCH models can efficiently track sudden changes in the covariance structure of spot and futures markets under market turmoil. Second, the proposed MRS-HEAVY model is applied in S&P 500 spot and futures contracts as well as out-of-sample hedging exercises. The results demonstrate that the MRS-HEAVY model exhibits superior hedging performance in terms of both variance reduction and utility gains.

The remainder of the paper is organized as follows. The specifications of the MRS-HEAVY model are presented in Section 2. As reported in Section 3, a simulation study is conducted to examine the performance of the proposed model. The optimal hedge ratio and measurements of hedging performance are described in Section 4, and Section 5 presents the data description and empirical results. Section 6 provides the conclusions of the study.

2. MRS-HEAVY MODEL

The multivariate HEAVY models introduced by Noureldin et al. (2012) are in a new class of models that use HF data to describe the dynamic features of return volatility. Compared with standard multivariate GARCH models, which utilize low-frequency (LF) data, HEAVY
models can adjust quickly to variations in volatility levels and consequently produce more accurate forecasts of the conditional covariance matrices of asset returns than other models can. In this section, a general description of HEAVY models is provided, following which the proposed MRS-HEAVY model is described with notations for its application in bivariate futures hedging.

Let \( R_t = [r_{st}, r_{ft}]' \) denote a \( 2 \times 1 \) vector consisting of spot and futures returns. Assuming that the return vector can be decomposed into a vector of mean returns \( m_t \equiv E[R_t | \mathcal{F}_{t-1}] \) and a vector of innovations \( u_t \equiv H_{t}^{1/2} z_t \). Then,

\[
R_t = m_t + u_t
\]

where \( \mathcal{F}_{t-1} \) is the information set up to time \( t - 1 \), \( H_t \equiv E \left[ (R_t - m_t)^2 | \mathcal{F}_{t-1} \right] \) denotes the covariance matrix for the returns, and the random vector \( z_t \) satisfies \( E[z_t] = 0 \) and \( var[z_t] = I_k \) (where \( I_k \) represents an identity matrix of order \( k \)).

Noureldin, Shephard, and Sheppard (2012) adopted a BEKK-type structure of the following form to parameterize the conditional covariance matrix in a HEAVY system:

\[
H_t = C'C + B'H_{t-1}B + A'V_{t-1}A
\]

where \( C \) is a lower-triangular \( 2 \times 2 \) parameter matrix, and \( B \) and \( A \) are \( 2 \times 2 \) parameter matrices. This parameterization guarantees that \( H_t \) is positive definite for all \( t \) under mild restrictions (Engle & Kroner, 1995). The notation \( V_t \) represents any \( 2 \times 2 \) realized measure of volatility (e.g., realized covariance matrix) at time \( t \). Usually, a realized covariance matrix estimator on day \( t \) is defined as \( V_t = \sum_{j=1}^{n} r_{j,t} r_{j,t}' \), where \( r_{j,t} \) denotes the \( j \)th uniform space vector of returns. Let 6.5 trading hours equate \( n = 78 \) for 5-minute returns; \( r_{j,t} \) is the vector of returns for the \( j \)th minute on that day.\(^2\) Noureldin et al. (2012) exhibited demonstrated that realized (co-)volatility estimators that they are useful in modeling and forecasting the conditional covariance matrices because their noise-to-signal ratio is much lower than that of daily squared (cross-product) returns when measuring unobserved covariance (Andersen et al., 2003; Barndorff-Nielsen & Shephard, 2004). In multivariate GARCH, \( V_{t-1} \) is generally defined as the outer product of daily return innovations, \( P_{t-1} = u_{t-1} u_{t-1}' \). Noureldin et al. (2012) indicated that \( V_t \) values are differentiated by their conditioning information set. HEAVY conditions on \( \mathcal{F}_{t-1}^{HF} \) are influenced by past realized volatility measures, whereas GARCH conditions on \( \mathcal{F}_{t-1}^{LF} \) are influenced by past daily returns. We use the terms HEAVY and GARCH to refer to the structures incorporating HF and LF data, respectively, in Equation (2).

The multivariate HEAVY models introduced by Noureldin et al. (2012) consist of two equations: HEAVY-P and HEAVY-V. Equation (2) is represented as “HEAVY-P” in the study by Noureldin et al. (2012), which described the dynamics of the conditional covariance matrix of daily returns. The financial applications of Fleming, Kirby, and Ostdiek (2003) and Sheu and Lai (2014) demonstrate the using of a HEAVY-P-like structure in forecasting \( H_t \). When forecasts beyond 1 day are required, the HEAVY-V equation should be included by modeling \( V_t \) itself, because in Equation (2), \( V_{t-1} \) drives the covariance matrix dynamics. This paper focuses on one-step-ahead forecasts for \( H_t \), because forecasted gains on \( H_t \) are usually more accurate at a shorter horizon (e.g., 1 day).

Based on the multivariate HEAVY framework of Noureldin et al. (2012), a MRS-HEAVY model is proposed for capturing the state-dependent covariance dynamics of spot and futures returns, in which all system parameters in HEAVY are

\(^2\)Noise-robust realized estimators (e.g., the realized kernels of Barndorff-Nielsen et al., 2011) can be used for correcting the biasness in estimations when the microstructure noise issue becomes excessive.
conditional on market regimes. The specifications of the MRS-HEAVY model are
detailed below.

Assume the spot and futures return vector is

\[ \mathbf{R}_t = \mathbf{m}_{s,t} + \mathbf{u}_{t,s} \]  

(3)

where \( s_t = \{1, 2\} \) is the unobserved state-dependent variable assumed to follow a two-state,
first-order Markov process with transition probabilities given by

\[
\begin{align*}
Pr(s_t = 1 | s_{t-1} = 1) &= P \\
Pr(s_t = 2 | s_{t-1} = 2) &= Q
\end{align*}
\]

(4)

\[ \mathbf{m}_s = [\mu_{s,s}, \mu_{f,s}]' \] is a vector of state-dependent conditional means, and \( \mathbf{u}_{t,s} = [u_{s,t,s}, u_{f,t,s}]' \) is a vector of state-dependent return innovations.

The state-dependent conditional covariance matrix is defined as

\[
\mathbf{H}_{t,s} = \mathbf{C}_s'C_s + \mathbf{B}_s'H_{s-1}B_s + \mathbf{A}_s'V_{t-1}A_s
\]

(5)

where \( \mathbf{C}_s, \mathbf{B}_s, \) and \( \mathbf{A}_s \) are state-dependent parameter matrices. Nested within the proposed
MRS-HEAVY model is multivariate HEAVY, which has a state-independent covariance
dynamics, as specified in Equation (2). The specifications of the time-varying state-
dependent covariance matrix \( \mathbf{H}_{t,s} \) of a bivariate diagonal MRS-HEAVY with a diagonal BEKK
are given by

\[
\begin{pmatrix}
  h_{s,s,t} & h_{s,f,t} \\
  h_{f,s,t} & h_{f,f,t}
\end{pmatrix}
= 
\begin{pmatrix}
  c_{s,s} & 0 \\
  c_{s,f} & c_{f,s}
\end{pmatrix}
\begin{pmatrix}
  c_{s,s} & 0 \\
  c_{f,s} & c_{f,f}
\end{pmatrix}'
+ 
\begin{pmatrix}
  b_{s,s} & 0 \\
  0 & b_{f,s}
\end{pmatrix}
\begin{pmatrix}
  h_{s,s,t-1} & h_{s,f,t-1} \\
  h_{f,s,t-1} & h_{f,f,t-1}
\end{pmatrix}
\begin{pmatrix}
  b_{s,s} & 0 \\
  0 & b_{f,s}
\end{pmatrix}
+ 
\begin{pmatrix}
  a_{s,s} & 0 \\
  0 & a_{f,s}
\end{pmatrix}
\begin{pmatrix}
  a_{s,s} & 0 \\
  0 & a_{f,f}
\end{pmatrix}'
\]

(6)

where \( h_{s,t} \) and \( h_{f,t} \) are the conditional variances of spot and futures returns, respectively, and
\( h_{sf,t} \) is the conditional covariance of spot and futures returns.

Gray (1996) and Lee and Yoder (2007a) have indicated that GARCH is intractable
under regime switching, because the conditional (co-)variance depends on the entire history
of the process. This path-dependency problem has been heavily explored in the literature. We
performed the following recombining procedure\(^3\) (Gray, 1996; Lee & Yoder, 2007a) to solve

\(^3\)Based on Equation (6), the state-dependent volatility of spot return can be rewritten as

\[ h_{s,s,t} = \gamma_{s,s} + \alpha_{s,s} \gamma_{s,s,t-1} + \beta_{s,s} h_{s,s,t-1} \]

where \( \gamma_{s,s} = \sigma_{s,s}^2 \), \( \alpha_{s,s} = \sigma_{s,s}^2 \), \( \beta_{s,s} = \sigma_{s,s}^2 \), and \( \gamma_{s,s,t-1} \) is the spot realized
volatility. Similarly \( h_{f,f,t} = \gamma_{f,f} + \alpha_{f,f} \gamma_{f,f,t-1} + \beta_{f,f} h_{f,f,t-1} \), where \( \gamma_{f,f} = \sigma_{f,f}^2 \), \( \alpha_{f,f} = \sigma_{f,f}^2 \), \( \beta_{f,f} = \sigma_{f,f}^2 \), and \( \gamma_{f,f,t-1} \) is the future realized
volatility. These volatility dynamics are identical in form to the aforementioned univariate regime-switching GARCH models (Gray, 1996). Therefore, Gray’s recombining procedure for univariate regime-switching GARCH can be applied directly as shown
in Equation (7). Moreover, the recombining procedure in Lee and Yoder (2007a) can be applied directly to our MRS-
HEAVY model because the structure of the covariance matrix presented therein is identical to that in the proposed
model; the cross-product term for spot and futures residuals in multivariate regime-switching GARCH is replaced
with realized covariance in the MRS-HEAVY model. After the, recombining procedure is performed, we will be able
to capture the high- and low-volatility regime dynamics can be determined and thus solve the path-dependency
problem encountered in MRS-HEAVY can be solved.
the path-dependency problem in the MRS-HEAVY model:

\[ h_{i,t} = p_{t,1} \left( \mu_{i,1}^2 + h_{i,t-1} \right) + \left( 1 - p_{t,1} \right) \left( \mu_{i,2}^2 + h_{i,t-2} \right) - \left[ p_{t,1} \mu_{i,1} + \left( 1 - p_{t,1} \right) \mu_{i,2} \right]^2 \]

for \( i = \{s,f\} \) \hspace{1cm} (7)

\[ h_{sf,t} = p_{t,1} \left[ \mu_{s,1} \mu_{f,1} + h_{sf,t-1} \right] + \left( 1 - p_{t,1} \right) \left[ \mu_{s,2} \mu_{f,2} + h_{sf,t-2} \right] - \left[ p_{t,1} \mu_{s,1} + \left( 1 - p_{t,1} \right) \mu_{s,2} \right] \left[ p_{t,1} \mu_{f,1} + \left( 1 - p_{t,1} \right) \mu_{f,2} \right] \]

where \( p_{t,1} \) is the probability of being included in regime 1, calculated as

\[ p_{t,1} = \Pr(s_t = 1 | \mathcal{F}_{t-1}^{HF}) = \frac{f_{t-1,1} p_{t-1,1}}{f_{t-1,1} p_{t-1,1} + f_{t-1,2} (1 - p_{t-1,1})} \]

\[ + (1 - Q) \frac{f_{t-1,2} p_{t-1,1}}{f_{t-1,1} p_{t-1,1} + f_{t-1,2} (1 - p_{t-1,1})} \]

(9)

and \( f_{t,s} \) is the state variable dependent likelihood function

\[ f_{t,s} = f\left( \mathbf{R}_t | \mathcal{F}_{t-1}^{HF} \right) = (2\pi)^{-1} | \mathbf{H}_{s,t} |^{-1/2} \exp\left\{ -\frac{1}{2} \mathbf{u}_{t,s}^\prime \mathbf{H}_{s,t}^{-1} \mathbf{u}_{t,s} \right\} \]

The recombining procedure for spot and futures innovations is also necessary in conventional MRS-GARCH models. Given \( \mathbf{P}_{t-1} \), the procedure can be expressed as

\[ u_{i,t} = r_{i,t} - \left[ p_{t,1} \mu_{i,1} + \left( 1 - p_{t,1} \right) \mu_{i,2} \right] \]

(11)

After the recombining procedure, the conditional variances and covariance in Equation (6) depend only on the current regime, not on the entire history of the GARCH process. Additionally, the MRS-HEAVY model becomes tractable.

Noureldin et al. (2012) suggested that the unknown parameters \( \vartheta_H \) in HEAVY models can be estimated by maximizing the log-likelihood function:

\[ \hat{\vartheta}_H = \arg \max_{\vartheta_H} \sum_{i=1}^T l_{H,i}(\vartheta_H) \]

(12)

where \( l_{H,i}(\vartheta_H) = c_H - \frac{1}{2} \left( \log | \mathbf{H}_{i} | + \text{tr} \left( \mathbf{H}_{i}^{-1} \mathbf{P}_{i} \right) \right) \), \( c_H \) is the constant with respect to \( \vartheta_H \) and \( T \) is the total number of observations. Similarly, the unknown parameters \( \vartheta_{H,s} \) in the MRS-HEAVY model can be estimated by maximizing the log-likelihood function:

\[ \hat{\vartheta}_{H,s} = \arg \max_{\vartheta_{H,s}} \sum_{i=1}^T \log \left[ p_{t,1} f_{i,1} + \left( 1 - p_{t,1} \right) f_{i,2} \right] \]

(13)

The inference for HEAVY (GARCH) models is based on quasi-maximization likelihood estimators with respect to \( \mathcal{F}_{t-1}^{HF} \). The covariance stationarity condition presented in Noureldin et al. (2012) is imposed to guarantee the conditional covariance matrix is positive definite.
3. SIMULATION STUDY

A simulation study is conducted to examine the finite sample performance of the proposed method. Specifically, we use the bivariate factor stochastic volatility model considered in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2011) to simulate HF prices for each day $t \in [0, 1]$. This model is given by

$$
\begin{align*}
\frac{dY^{(i)}}{dt} &= \mu^{(i)} dt + \frac{dV^{(i)}}{dt} + dF^{(i)} \\
\frac{dV^{(i)}}{dt} &= \rho^{(i)} \sigma^{(i)} dB^{(i)} \\
F^{(i)} &= \sqrt{1 - (\rho^{(i)})^2} \sigma^{(i)} dW
\end{align*}
$$

(14)

for $i = 1, 2$, where $B^{(i)}$ and $W$ are independent Brownian motions, $V^{(i)}$ is an idiosyncratic component, and $F^{(i)}$ is a common factor. Hence, each price process $Y^{(i)}$ follows a diffusive stochastic volatility model with constant drift $\mu^{(i)}$ and random volatility $\sigma^{(i)}$. In this model, the underlying volatility term is represented as $\sigma^{(i)} = \exp(\beta^{(i)}_0 + \beta^{(i)}_1 g^{(i)})$ with $d\mathcal{G}^{(i)} = \alpha^{(i)} \mathcal{G}^{(i)} dt + dB^{(i)}$, implying that there is perfect statistical leverage between $V^{(i)}$ and $\mathcal{G}^{(i)}$. Additionally, the leverage between $Y^{(i)}$ and $\mathcal{G}^{(i)}$ is $\rho^{(i)}$, whereas the correlation between $Y^{(1)}(t)$ and $Y^{(2)}(t)$ is $\sqrt{1 - (\rho^{(1)})^2} \sqrt{1 - (\rho^{(2)})^2}$. The configuration of the parameters in Barndorff-Nielsen et al. (2011) for both processes is based on $(\mu^{(i)}, \beta^{(i)}_0, \beta^{(i)}_1, \alpha^{(i)}, \rho^{(i)}) = (0.03, -5/16, 1/8, -1/40, -0.3)$. Therefore, the volatility process is normalized; $E\left(\int_0^1 \sigma^{(i)}(u) du\right) = 1$, and the variance of $\sigma^2$ equals $\exp(-2\beta^2_0/\alpha) - 1$, which approximates 2.5.

We simulate 100 paths of the bivariate factor model for $T = 500, 1000, 1500$ days. Each day $t$ is assumed to have 6.5 hours, so that the intervals [0, 1] can be discretized into 23,400 equally spaced subintervals. Subsequently, the HF prices for both assets are simulated using an Euler scheme, based on the aforementioned configuration in Equation (14). Finally, the competing models are estimated through the maximum likelihood estimators of Equations (12) and (13) with the use of the simulated data.

The simulation results reveal that the MRS-HEAVY model usually obtains the highest log-likelihood scores in the estimations, followed in order by the HEAVY, MRS-GARCH, and GARCH models. For $T = 1000$, the average log-likelihood scores obtained using the MRS-HEAVY, HEAVY, MRS-GARCH, and GARCH models are 7725.20, 7700.41, 7469.19, and 7400.72, respectively. We conduct a standard likelihood ratio test on the nested models (MRS-HEAVY vs. HEAVY) and Vuong’s (1989) likelihood ratio test on the non-nested models (MRS-HEAVY vs. MRS-GARCH) with the estimated log-likelihood scores, to examine the significance of the improvement achieved using the MRS-HEAVY model. The results are plotted in Figure 1, in which the left (right) panel illustrates the histograms of the testing statistics for the nested (non-nested) models. The analysis of the nested models reveals that 62%, 92%, and 94% of the test statistics are higher than the critical value of 24.7250 (1% significance level, 11 degrees of freedom) for $T = 500, 1000$, and 1500, respectively. A test statistic more extreme higher than the critical value indicates the superiority of the MRS-HEAVY model. The test statistics for the non-nested models are larger than 2.3263, demonstrating that the MRS-HEAVY model outperforms the MRS-GARCH at a 1% significance level. In summary, our simulation results clearly illustrate that the performance of the proposed MRS-HEAVY model is generally higher than that of the HEAVY and MRS-GARCH models, regarding the processing of the data.
4. OPTIMAL HEDGE RATIO AND MEASUREMENTS OF HEDGING PERFORMANCE

The purpose of hedging is to minimize the risk of return on hedged portfolios, which are constructed by holding on 1 U of spot and shorting $\beta$ units of futures contracts. Let $r_{p,t+1} = r_{s,t+1} - \beta r_{f,t+1}$ be the realized return on a hedged portfolio from time $t$ to $t + 1$, conditional on a pre-determined hedge ratio $\beta$ at time $t$. When deriving the optimal hedge ratio, a hedger is assumed to have a quadratic utility function with a degree of risk aversion $\gamma > 0$ given by

$$U(\cdot) = E(r_{p,t+1}; \beta_t) - \gamma \text{var}(r_{p,t+1}; \beta_t)$$  \hspace{1cm} (15)

The optimal hedge ratio

$$\beta_{t|t-1}^* = \frac{h_{f,t|t-1}}{h_{f,s|t-1}}$$  \hspace{1cm} (16)

is obtained by maximizing the utility function with respect to $\beta$, where $h_{f,t}$ and $h_{f,s}$ are, respectively, the variance on futures returns and the covariance of spot and futures returns. Kroner and Sultan (1993) discovered that utility-maximizing and variance-minimizing hedge ratios are equivalent when the futures price follows a martingale.

Noureldin et al. (2012) demonstrated that HEAVY models have a relatively short response time compared with that of standard GARCH models. HEAVY models’ forecasts incorporate new information quickly, particularly in situations where the level of volatility or correlation abruptly changes. A state-dependent HEAVY model enables the observation of different response times under various volatility regimes, potentially a highly efficient method of tracking sudden changes in the covariance matrix during market turmoil. Consequently, the optimal hedge ratio can be estimated with high accuracy by implementing a hedging strategy that is based on the MRS-HEAVY model, because it exhibits both a short response time and impromptu structural changes in the covariance dynamic.

The hedging performance of MRS-HEAVY is evaluated according to the objective utility function of Equation (15), following Kroner and Sultan (1993). A mean-variance expected

---

**FIGURE 1**

Histograms of the Likelihood Ratio Statistics in Comparing the Nested (Left) and the Non-Nested (Right) Models Using the Simulated Data [Color figure can be viewed at wileyonlinelibrary.com]
utility maximization hedger will consider a model that delivers the highest expected utility scores. We assume that the anticipated return in Equation (15) is zero and \( \gamma = 1, 4, 20 \) to assess the performance gains across hedgers with different levels of risk aversion (Kroner & Sultan, 1993; Lai & Sheu, 2010; Sheu & Lai, 2014; Sheu & Lee, 2014). Under the assumption of martingale futures prices, utility-maximizing hedging and variance-minimizing hedging are generally equivalent (Brooks, Henry, & Persand, 2002). Therefore, the performance of hedging models can be compared using the variance of the hedged portfolios. The extreme risk aversion attitudes \( \gamma = 1 \) and \( \gamma = 20 \) can be interpreted as the evaluations of the utility-maximization and variance-minimization perspectives, respectively. Hedgers will select a model that most closely reflects their goals and attitudes toward risk.

In addition, Fleming, Kirby, and Ostdiek (2001) proposed the concept of economic value (EV), and Fleming et al. (2003) used realized volatility to measure volatility-timing. The EV for the improvement experienced when switching from alternate models to the MRS-HEAVY model is reported in this study for determining economic variability. It was computed according to the following equation:

\[
\hat{E}_n U(r^b_{t+1}; \gamma) = \hat{E}_n U(r^a_{t+1} - \text{EV}; \gamma)
\]

where \( \hat{E}_n \) denotes a sample average operator, and \( r^b_{t+1} \) and \( r^a_{t+1} \), respectively, represent the hedged portfolio returns obtained through a benchmark model and that obtained through the proposed MRS-HEAVY model. Lai and Sheu (2010) considered this measure to be the EV of a future hedge using realized volatility. The EV measures the average annualized basis point fees that a hedger who is endowed with the mean-variance utility function would be willing to pay to switch from a benchmark model to the MRS-HEAVY model.

5. DATA DESCRIPTION AND EMPIRICAL RESULTS

The proposed MRS-HEAVY model is applied to nearby S&P 500 futures contracts that were traded on the Chicago Mercantile Exchange, as well as their underlying equity index. We assume that hedgers possess spot holdings on the S&P 500 index and intend to hedge their spot position with S&P 500 futures. The data cover the period of January 2, 2002, to December 30, 2011, and consisted of a total of 2519 trading days. Both volatile periods (e.g., 2007–2008) and less volatile periods (e.g., 2004–2006) are included to allow for the exploration of potential regime shifts in the covariance dynamics. The MRS-HEAVY model as well as its competing models (HEAVY, MRS-GARCH, and GARCH) are estimated using in-sample data from January 2002 to December 2010. The remaining 1-year data are reserved for out-of-sample analyses.

We obtain the time series of the realized covariance matrix and price returns of each day and then estimate the models. The daily realized covariance matrix is constructed with fictitious returns obtained through the following procedure. First, tick-by-tick price records were received from Tick Data Inc. for each asset and each day. Only those prices that occurred between 8:30 a.m. and 3:00 p.m. are included in the database, to facilitate computation of the realized covariance matrix.4 Next, data for nearby futures contracts are applied. The current nearby contract is rolled over to the active nearby contract that is next available when the current contract nears its expiration date and the trading volume of the next available contract exceeds that of the current nearby contract. This should present the problems of thin trading and the expiration effect. Subsequently, the time for each day is

---

4The trading hours for the spot (futures) are 8:30 a.m. to 3:00 p.m. (3:15 p.m.) central time.
partitioned into several equally 5-minute grids. Moderate frequency selection is employed for balancing the bias and variance in the realized (co-)volatility calculations that are caused by microstructure noise (Andersen, Bollerslev, Diebold, & Labys, 2000). Usually, a 6.5-hour market suggests that 78 5-minute continuously compounded returns exist in a day.\(^5\) Finally, the formula \(V_t = \sum_{j=1}^{n} r_{j,t} r_{j,t}'\) is used to construct the realized covariance matrix according to the trading days. For daily return (open-to-close) series, they are defined as the logarithmic difference between the closing (3:00 p.m.) and opening (8:30 a.m.) prices of a day. This measure presents the use of noisy overnight returns, which might diminish the performance difference between models using \(\frac{P_t}{C_0} - \frac{C_0}{P_1}\) and those using \(V_t = \sum_{j=1}^{n} r_{j,t} r_{j,t}'\), and it matches the constructed realized covariance matrix.

Table I presents a summary of the descriptive statistics for returns (Panel A) and volatility measures (Panel B). Excess kurtosis is a stylized fact of asset returns, in which the empirical distribution has fatter tails than a normal distribution does. Panel B of Table I compares the different volatility measurements using LF and HF data. Therein, the annualized (co-)volatility by the realized measures tends to be approximately 17%, although the volatility level derived using daily returns is slightly higher than this level. The standard deviations of daily squared (cross-product) returns are much higher than those of the 5-minute realized variance (covariance) quantities. In addition, the lag-1 ACF statistic demonstrates that the degree of serial correlation for realized variance (covariance) is much higher than that of squared (cross-product) returns. GARCH models can successfully explain volatility clustering and excess kurtosis in the empirical distribution of returns. The different

\[ r_{s,t} \]
\[ r_{f,t} \]

\[ \text{Panel A. Daily Returns} \]

<table>
<thead>
<tr>
<th>Mean</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{s,t})</td>
<td>0.0016</td>
<td>1.2933</td>
<td>-0.1489</td>
<td>11.1342</td>
<td>-9.4571</td>
</tr>
<tr>
<td>(r_{f,t})</td>
<td>-0.0101</td>
<td>1.1562</td>
<td>-0.0142</td>
<td>10.3298</td>
<td>-8.0193</td>
</tr>
</tbody>
</table>

\[ \text{Panel B. Annualized Volatilities} \]

<table>
<thead>
<tr>
<th>(P_{t-1} = u_{t-1} u'_{t-1})</th>
<th>(V_t = \sum_{j=1}^{n} r_{j,t} r_{j,t}')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AVol)</td>
<td>SD</td>
</tr>
<tr>
<td>(h_{s,t})</td>
<td>20.53</td>
</tr>
<tr>
<td>(h_{f,t})</td>
<td>18.35</td>
</tr>
<tr>
<td>(h_{sdf,t})</td>
<td>18.39</td>
</tr>
</tbody>
</table>

**Note.** Study period is from January 2002 to December 2011 (2519 daily observations). Returns are calculated as 100 times the logarithm difference between the daily closing and opening prices (prices set at 8:30 a.m. and 3:00 p.m., respectively). Similarly, variance and covariance measure the price variations between 8:30 a.m. and 3:00 p.m., which are consistent with the open-to-close returns over the same interval. We use 5-minute returns for calculating the realized measures of volatility. SD represents standard deviation of daily returns (as a percentage), squared (cross-product) returns, or realized (co-)variances, annualized quantities not represented to scale. \(AVol\) corresponding to the square root of the mean of 252 times either the squared (cross-product) returns, or realized (co-)variances over the sample period. ACF represents the serial correlation of the quantities at lag 1.

\(^5\)The markets may close early on certain days such as the day after Thanksgiving and the days around Christmas. In these cases the records of time and price consider 8:30 a.m. to 12:15 p.m. (225 minutes) only.
volatility measurements of the spot asset using LF and HF data are plotted in the graphs portrayed in Figure 2.\(^6\) Evidently, the daily squared return is an extremely noisy proxy for measuring the volatility of the S&P 500 index, although it tracks the overall level of the realized variance. This preliminary analysis might provide a first insight into the relatively low efficiency of standard GARCH models in forecasting, as compared to HEAVY models.

Table II presents the estimated parameters of the MRS-HEAVY model and its competing models. Starting values based on the preliminary unconditional statistics are used in conjunction with various rational values to ensure that the estimation procedure converges to a global maximum. The last column of Table II demonstrates that the MRS-HEAVY model obtained the highest log-likelihood function, followed in order by the HEAVY, MRS-GARCH, and GARCH models.

Shephard and Sheppard (2010) and Noureldin et al. (2012) have illustrated that HEAVY models have a shorter response time than standard GARCH models do. The reason for this is that HEAVY models put more weight on informative realized shocks, whereas GARCH models are conditioned with LF data. As Table II displays, the HEAVY model estimated the weights on the realized shocks for spot and futures as 0.4504 and 0.4352, respectively, whereas the weights given by GARCH are much smaller, with values of 0.3014 and 0.2576 for spot and futures, respectively.

The multivariate HEAVY model nests within the MRS-HEAVY model. A likelihood ratio test was conducted on the models to examine the significance of this regime switching. The test statistic is 158.88, and with 11 degrees of freedom, the null hypothesis is soundly rejected at all conventional significance levels. Hence, the HEAVY model with regime shifts is superior to the single-regime HEAVY model in modeling the joint distribution of spot and futures. Figure 3 displays the regime and smoothed probabilities of being in the high-volatility regime produced by the MRS-HEAVY model. Therein, the high-volatility states are short and unstable. The average expected durations of the high- and low-volatility regimes are approximately 14 and 41 days, respectively (Table II). Additionally, the volatilities and

\(^6\)To conserve space, the results for futures volatility and co-volatility are not reported, but are available upon request.
correlations estimated using the MRS-HEAVY and HEAVY models have relatively similar patterns when the spot and futures markets are in the lower volatility regime, but diverge significantly when the market is in turmoil. This result indicates the importance of modeling the covariance dynamic of spot and futures returns with the MRS-HEAVY model to capture the regime-switching covariance dynamics.

The association between the degree of persistence in volatility and the market regime can be measured by \( b_{i_s,i_f}^2 + a_{i_s,i_f}^2 \). Based on the estimates of the MRS-HEAVY model, the persistence in spot volatility for the low- and high-volatility regimes is approximately 0.962 and 1.000, respectively, whereas the corresponding values for the futures are approximately 0.955 and 0.964. Return volatility in the low-volatility regime is less persistent than that in the high-volatility regime.\(^7\) This relationship is further illustrated by the state-dependent volatility in the spot market, as interpreted by the MRS-HEAVY model in the top panel of Figure 4.\(^8\) The conditional correlations (implied by the BEKK representation) are presented

\(^7\)This result is consistent with the findings in the extant literature such as those of Alizadeh et al. (2008) in the energy market, in which a high variance state is associated with high persistence and vice versa.

\(^8\)The state-dependent volatility in the futures market are similar to those in the spot market and are available upon request.

---

### TABLE II

<table>
<thead>
<tr>
<th>Mean equation</th>
<th>GARCH</th>
<th>MRS-GARCH</th>
<th>HEAVY</th>
<th>MRS-HEAVY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{s_s=1} )</td>
<td>0.0004 (0.0003)</td>
<td>0.0009* (0.0004)</td>
<td>0.0001* (0.0001)</td>
<td>0.0007** (0.0002)</td>
</tr>
<tr>
<td>( \mu_{t_s=1} )</td>
<td>0.0001 (0.0003)</td>
<td>0.0002 (0.0003)</td>
<td>(-0.0001** (0.0000))</td>
<td>0.0003 (0.0002)</td>
</tr>
<tr>
<td>( \mu_{s_s=H} )</td>
<td>(-0.0000 (0.0001))</td>
<td>(-0.0002* (0.0013))</td>
<td>(-0.0023* (0.0011))</td>
<td>(-0.0023* (0.0011))</td>
</tr>
<tr>
<td>( \mu_{t,s=H} )</td>
<td>0.0008 (0.0009)</td>
<td>(-0.0000 (0.0001))</td>
<td>(-0.0002* (0.0013))</td>
<td>(-0.0023* (0.0011))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Covariance matrix equation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{s,f} )</td>
<td>0.0012** (0.0002)</td>
<td>0.0001 (0.0002)</td>
<td>0.0018** (0.0002)</td>
<td>0.0000 (0.0000)</td>
</tr>
<tr>
<td>( c_{f,s} )</td>
<td>0.0010** (0.0001)</td>
<td>0.0016** (0.0001)</td>
<td>0.0018** (0.0002)</td>
<td>0.0011** (0.0004)</td>
</tr>
<tr>
<td>( c_{s,f} )</td>
<td>0.0002* (0.0001)</td>
<td>0.0012** (0.0001)</td>
<td>0.00000** (0.0000)</td>
<td>0.0012** (0.0003)</td>
</tr>
<tr>
<td>( b_{s,s} )</td>
<td>0.9495** (0.0057)</td>
<td>0.9532** (0.0012)</td>
<td>0.8778** (0.0130)</td>
<td>0.9537** (0.0309)</td>
</tr>
<tr>
<td>( b_{f,s} )</td>
<td>0.9615** (0.0042)</td>
<td>0.9671** (0.0011)</td>
<td>0.8714** (0.0156)</td>
<td>0.9537** (0.0290)</td>
</tr>
<tr>
<td>( a_{s,s} )</td>
<td>0.3014** (0.0193)</td>
<td>0.2546** (0.0095)</td>
<td>0.4504** (0.0259)</td>
<td>0.2284** (0.0484)</td>
</tr>
<tr>
<td>( a_{f,s} )</td>
<td>0.2576** (0.0196)</td>
<td>0.2082** (0.0111)</td>
<td>0.4352** (0.0283)</td>
<td>0.2140** (0.0764)</td>
</tr>
<tr>
<td>( c_{s,f} )</td>
<td>0.0000 (0.0001)</td>
<td>0.0005 (0.0002)</td>
<td>0.0005 (0.0005)</td>
<td>0.0016 (0.0007)</td>
</tr>
<tr>
<td>( c_{f,s} )</td>
<td>0.0024 (0.0025)</td>
<td>0.0005 (0.0002)</td>
<td>0.0050** (0.0010)</td>
<td>0.0005 (0.0007)</td>
</tr>
<tr>
<td>( b_{s,s} )</td>
<td>0.9138** (0.0574)</td>
<td>0.8671** (0.1129)</td>
<td>0.7788** (0.0390)</td>
<td>0.5347** (0.0571)</td>
</tr>
<tr>
<td>( b_{f,s} )</td>
<td>0.8671** (0.1129)</td>
<td>0.4063** (0.1059)</td>
<td>0.5347** (0.0571)</td>
<td>0.5983** (0.0583)</td>
</tr>
<tr>
<td>( a_{s,s} )</td>
<td>0.4063** (0.1059)</td>
<td>0.4747* (0.2213)</td>
<td>0.4747* (0.2213)</td>
<td>0.4747* (0.2213)</td>
</tr>
<tr>
<td>Transition probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>0.9581** (0.0465)</td>
<td>0.9756** (0.0090)</td>
<td>0.9756** (0.0090)</td>
<td>0.9756** (0.0090)</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.5429 (0.5387)</td>
<td>0.9290 (0.0423)</td>
<td>0.9290 (0.0423)</td>
<td>0.9290 (0.0423)</td>
</tr>
</tbody>
</table>

Log-L \(16,860.57\) \(16,931.64\) \(16,992.48\) \(17,071.92\)

Note. The sample period is from January 2002 to December 2010 (2267 daily observations). Log-L denotes log-likelihood function. Figures in parentheses are estimated standard errors. * and ** indicate significance at the 5% and 1% levels, respectively.
in the bottom panel of Figure 4 and are equal to 0.910 and 0.915 on average for the low- and high-volatility regimes, respectively. As demonstrated, the correlations in the high-volatility regime are more volatile than those in the low-volatility regime. Moreover, the standard deviations of the correlations are equal to 0.035 and 0.040 for the low- and high-volatility regimes, respectively.

Considering that HEAVY models have a relatively short response time to that of standard GARCH models, the MRS-HEAVY further enables us to investigate whether the response times differ significantly under the low- and high-volatility regimes. The MRS-GARCH and the MRS-HEAVY models are compared, through Vuong’s (1989) non-nested likelihood ratio test.
The test statistic is 6.10, indicating that the improvement associated with the MRS-HEAVY model is highly significant. Table II displays the MRS-HEAVY estimations for the weights placed on realized shocks for spot and futures in the high-volatility regime, which are, respectively, equal to 0.5347 and 0.5983 and are much higher than those estimated by MRS-GARCH 0.4063 and 0.4747 for spot and futures, respectively. By contrast, these weights are similar and relatively small for both the MRS-HEAVY and MRS-GARCH models (ranging from 0.208 to 0.255) in the low-volatility regime. As mentioned, Shephard and Sheppard (2010) and Noureldin et al. (2012) have indicated that HEAVY models have a relatively short response time compared to that of standard GARCH models. Our empirical evidence further suggests that models with HF data can respond more quickly than those with LF data only when the market is in the high volatility regime. The top (bottom) panels in Figure 5 display the spot volatilities in the high (low) volatility regime that were estimated with both the MRS-HEAVY and MRS-GARCH models. In addition, correlation estimates based on the MRS-HEAVY and MRS-GARCH models can diverge significantly when the market is in turmoil. Consequently, the shorter response time of the MRS-HEAVY model is profound only in the high-volatility regime.

Table III presents the in- (Panel A) and out-of-sample (Panel B) hedging performances of the MRS-HEAVY, HEAVY, MRS-GARCH, and GARCH hedging strategies. The in-sample results reveal that the proposed MRS-HEAVY model generates the lowest hedge portfolio return variance at 0.2824, an 82.70% reduction in the variance of the unhedged position. The improvements achieved using MRS-HEAVY hedging over HEAVY, MRS-GARCH, and GARCH hedging were equal to 0.12%, 1.21%, and 1.24%, respectively. Regarding economic performance, we assume the expected return in Equation (15) for the portfolios equals zero as in Kroner and Sultan (1993), provided that none of the hedging strategies generates portfolio returns that are significantly different from zero. The EV is the average annualized basis point fees that a hedger with a coefficient of risk aversion $\gamma$ would be willing to pay to switch from one model to the MRS-HEAVY model. When $\gamma = 4$, the fees for switching from the HEAVY, MRS-GARCH, and GARCH models are approximately 1.84,
### Table III
Hedging Performance of MRS-HEAVY Against the Hedging Strategies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel A. In-Sample (2267 Obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unhedged</td>
<td>0.2606</td>
<td>1.6323</td>
<td>–</td>
<td>–1.6323</td>
<td>340.02</td>
<td>–6.5292</td>
<td>1358.32</td>
<td>–32.6461</td>
</tr>
<tr>
<td>GARCH</td>
<td>1.0252</td>
<td>0.3027</td>
<td>81.46</td>
<td>–0.3027</td>
<td>5.09</td>
<td>–1.2106</td>
<td>20.36</td>
<td>–6.0532</td>
</tr>
<tr>
<td>MRS-GARCH</td>
<td>1.0712</td>
<td>0.3022</td>
<td>81.49</td>
<td>–0.3022</td>
<td>4.98</td>
<td>–1.2089</td>
<td>19.94</td>
<td>–6.0447</td>
</tr>
<tr>
<td>HEAVY</td>
<td>1.0948</td>
<td>0.2843</td>
<td>82.58</td>
<td>–0.2843</td>
<td>0.46</td>
<td>–1.3171</td>
<td>1.84</td>
<td>–5.6856</td>
</tr>
<tr>
<td>MRS-HEAVY</td>
<td>1.2053</td>
<td>0.2824</td>
<td>82.70</td>
<td>–0.2824</td>
<td>–1.1297</td>
<td>–5.6486</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

|                  |          |                               |         |            |         |            |         |            |
| **Panel B. Out-of-Sample (252 Obs.)** |          |                               |         |            |         |            |         |            |
| Unhedged         | –0.7436  | 2.0436                        | –       | –2.0436    | 340.32  | –8.1743    | 1359.32 | –40.8714   | 6496.00   |
| GARCH            | 0.6711   | 0.7052                        | 65.49   | –0.7050    | 4.37    | –2.8208    | 17.49   | –14.1041   | 87.68     |
| MRS-GARCH        | 0.7995   | 0.7000                        | 65.75   | –0.7000    | 3.06    | –2.7999    | 12.25   | –13.9993   | 61.40     |
| HEAVY            | **1.0499** | 0.6934                       | 66.07   | –0.6934    | 1.43    | –2.7737    | 5.72    | –13.8684   | 28.67     |
| MRS-HEAVY        | 0.9566   | **0.6877**                    | 66.49   | **–0.6877** | –       | **–2.7510** | –       | **–13.7550** | –        |

**Note.** The in-sample period is from January 2002 to December 2010, whereas the out-of-sample period is from January 2011 to December 2011. Portfolio returns are expressed as basis points of the price. Variance denotes the variance of the hedged portfolio expressed as basis points of the price. Percentage variance reduction is calculated as the difference of variance of unhedged cash position from estimated variance of hedged portfolio over variance of unhedged cash position multiplied by 100. Utility is the average daily utility for an investor with a mean-variance utility function and a coefficient of risk aversion $\gamma$, expressed as basis points of the price. Economic value (EV) measures the average annualized basis point fees that an investor with a coefficient of risk aversion $\gamma$ would be willing to pay to switch from one model to MRS-HEAVY. Figures in bold signify the model with the highest performance for each criterion.
19.94, and 20.36 basis points per annum, respectively. These values demonstrate that the EV enlarges as risk-aversion levels increase.

The in-sample performance of the model reflects its historical performance. Because hedging decisions are made ex ante, hedgers are more concerned with how effectively they can manage their spot portfolio exposures in the future. The out-of-sample hedging performances are also determined to further demonstrate the benefits of using the MRS-HEAVY model in futures hedging. Panel B of Table III indicates that MRS-HEAVY is the superior model in terms of variance reduction. Compared to the HEAVY, MRS-GARCH, and GARCH hedges, the gain in percentage variance reduction is 0.42%, 0.74%, and 1.00%, respectively. In the case of $\gamma = 4$, the switching fees for HEAVY, MRS-GARCH, and GARCH are approximately 5.72, 12.25, and 17.49 basis points per annum, respectively. The EV increases with risk aversion. This result implies that the more risk averse hedgers are the more willing they will be to pay high prices to switch from conventional hedging models to the proposed MRS-HEAVY model.

The 1-day-ahead hedge ratio forecasts generated by the MRS-HEAVY, MRS-GARCH, HEAVY, and GARCH models for the out-of-sample period are presented in Figure 6. The hedge ratios estimated using the MRS-HEAVY model fluctuates between 0.806 and 1.074 with a mean value of 0.998 and a standard deviation of 0.040, and the hedge ratios estimated using the MRS-GARCH model fluctuates between 0.533 and 1.572 with a mean value of 1.048 and a standard deviation of 0.156 (top panel, Figure 6). The bottom panel displays the hedge ratios estimated using the HEAVY and GARCH models. The HEAVY estimates of hedge ratios fluctuate between 0.673 and 1.175 with a mean value of 0.989 and a standard deviation of 0.062, whereas the GARCH estimates of hedge ratios fluctuate between 0.761 and 1.604 with a mean value of 1.059 and a standard deviation of 0.181. The hedge ratios conditioned on $\mathcal{F}_t$ are generally less volatile than those conditioned on $\mathcal{F}_{t+1}$.

Let the round trip transaction cost $c$ equal three basis points for holding one unit of spot asset. Without loss of generality, the net portfolio return on the net transaction costs is given by $r_{p,t+1} = r_{p,t+1} - c_{t+1}$, where $c_{t+1} = c |\beta^*_{t+1} - \beta^*_t|$ represents the reduced returns caused by daily transaction costs. Given the data and estimation outputs, the total rebalancing costs
incurred in the out-of-sample period according to MRS-HEAVY, HEAVY, MRS-GARCH, and GARCH are approximately 11.47, 22.79, 31.20, and 35.15 basis points, respectively. The reduced returns attributable to daily transaction costs were lowest in the MRS-HEAVY model, indicating that its superiority is robust even considering the transaction costs.

6. CONCLUSION

Implementing a hedging strategy with a model that can swiftly adapt to market changes is crucial for optimal futures hedging, particularly in a highly volatile regime, because hedging decisions based on estimated optimal hedge ratios can be rapidly adjusted to the current market conditions. This study proposes the MRS-HEAVY for effectively tracking sudden changes in the covariance structure of spot and futures returns during market turmoil. The contribution of this study to the literature on optimal futures hedging is twofold. First, the proposed MRS-HEAVY model features fluctuating (co-)volatility, state dependency, and HF data, rendering it more flexible and informative than other methods of modeling the dynamic covariance process. Recent studies (Noureldin et al., 2012; Shephard & Sheppard, 2010) have demonstrated that state-independent HEAVY models have a relatively short response time compared to that of standard GARCH models. The proposed MRS-HEAVY allows for further investigation into whether the response time of a HEAVY model is shorter than that of a GARCH model in the high-volatility regime. To the best of our knowledge, this study is the first to model the dynamic spot and futures covariance structures by using a multivariate high-frequency-based regime-switching model. The empirical results of this study on S&P 500 spot and futures contracts indicate that the response time of the MRS-HEAVY model is significantly shorter than that of MRS-GARCH models only in the high-volatility regime.

Second, the usefulness of the MRS-HEAVY model in optimal futures hedging is demonstrated empirically by applying it to S&P 500 spot and futures contracts and comparing its results with those of competing models, namely the HEAVY, MRS-GARCH, and GARCH models. The MRS-HEAVY model is associated with the smallest variance reduction, exhibiting an improvement of up to 1.24% (1.00%) in performance for the in-sample (out-of-sample) period compared with the competing models. Regarding economic variability, the results indicate that the improvement in variance reduction can be translated into substantial utility gains, particularly for hedgers with higher risk aversion. The more risk averse hedgers are, the higher their willingness to pay high fees so that they can switch from conventional hedging models to the proposed MRS-HEAVY model. The EV gains in a MRS-HEAVY hedging over competing models range from 9.25 to 102.11 per annum for the in-sample period and from 28.67 to 87.68 per annum for the out-of-sample period. Our results also support the superiority of the MRS-HEAVY model for effective hedging, which prevails even when transaction costs are considered.

REFERENCES


