A systematic design approach for two planetary gear split hybrid vehicles

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Multiple power sources in a hybrid vehicle allow for flexible vehicle power-train operations, but also impose kinematic constraints due to component characteristics. This paper presents a design process that enables systematic search and screening through all three major dimensions of hybrid vehicle designs – system configuration, component sizing and control, to achieve optimal performance while satisfying the imposed constraints. An automated dynamic modelling method is first developed which enables the construction of hybrid vehicle model efficiently. A screening process then narrows down to configurations that satisfy drivability and operation constraints. Finally, a design and control optimisation strategy is carried out to obtain the best execution of each configuration. A case study for the design of a power-split hybrid vehicle with optimal fuel economy is used to demonstrate this overall hybrid vehicle design process.

Keywords: hybrid vehicle system; modelling; configuration design; systematic design; optimisation

1. Introduction

Improving fuel economy of ground vehicles is an important topic in recent years because of tightening global crude oil supplies and growing environmental concerns. With the announcement of the new Corporate Average Fuel Economy (CAFE) rule in the USA and the carbon dioxide-emission standard in the European Union countries, the automotive industry faces substantial new challenges. One of the most promising technologies to satisfy these fuel economy or emission regulations is the hybrid vehicle (HV) technologies.

A hybrid vehicle adds an additional power source (e.g. battery, ultra capacitor, etc.) and one or multiple actuators (often electric machines) to the conventional power-train. The additional power devices helps to improve system efficiency and fuel economy by engine right-sizing, load levelling, regenerative braking and pure electric mode. A right-sized engine has better fuel efficiency and lower heat loss. The reduced engine power is augmented by the electrical machine. Compared with internal combustion engines, electric machines provide higher torque more quickly, especially at low vehicle speed. Therefore, launching performance can be improved even with the reduced overall rated power. Load levelling can be
achieved by adding the electrical path, which enables the engine to operate more efficiently, and independent from the road load. Regenerative braking allows the electric machine to capture part of the vehicle kinetic energy and recharge the battery, when the vehicle is decelerating. The pure electric mode allows the engine to shut off entirely which eliminates the unnecessary losses during the idle mode.

These promising advantages can only be realised through proper design of the hybrid vehicle, including the configuration, component sizing and control. Hybrid vehicles can be crudely divided into three types: parallel, series and split. In a parallel configuration (Figure 1(a)), the electric path works on the side of the internal combustion engine (ICE), and can be designed to be strong enough to drive the vehicle individually or mild to have limited capabilities. The additional power path can also be added as a series configuration (Figure 1(b)). In a series configuration, the vehicle is always driven by the electric motor (or motors). The motor power is supplied either by a battery, or a generator transforming the engine power, or both. In general, the parallel configuration requires the conventional engine-transmission path and lack of freedom of controlling the engine operation, the series configuration improves the engine operation but the additional losses of the electric machine(s) will reduce the overall power-train efficiency, especially at high-speed cruising [1]. A power-split configuration combines these two configurations with a power-split device, as shown in Figure 1(c). Through the power-split device, the electric motor can deliver the power from the battery or engine to the vehicle as a series hybrid, while the engine can also directly drive the vehicle as a parallel. It can be designed to take advantage of both the parallel and series types and avoid their disadvantages. Here the configurations are categorised in three different types, but in each type, there can be many different configurations depending on different gear linkage and component connections.

The sizes and efficiency of the components are also important to HV system designs. Typical engines and electric machines operate at their peak efficiency only in a small speed and torque range (sweet spot). Proper sizing and control enables these components to operate close to the sweet spot. Frequently, this simple idea cannot be realised because of the constraints imposed by the mechanical/electrical linkage and performance requirements. The design of a

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![Figure 1. Hybrid vehicle configurations: (a) parallel; (b) series; (c) power-split.](image-url)
hybrid vehicle is still an art of balancing the configuration, component sizing and component efficiency.

Over the last several years, hybrid vehicles were commercialised by several major manufacturers. Toyota hybrid system (THS), the core of the first commercial power-split hybrid vehicle, the Toyota Prius, was described in [2–5]. It was tested by the Argonne National Lab and compared with the Honda Insight [6], a mild parallel hybrid. Both systems are modified and implemented to several different models (upgraded Prius in 2004 [7–9], Honda Civic and Accord [10], Toyota Highlander and Lexus RH400 [11], etc.). The General Motors (GM) hybrid vehicle design uses a different power-split design and has two electronically variable transmission (EVT) modes. It reduces the requirement on electric machines in terms of both rotational speed and power. GM has developed several different dual-mode designs [12–15].

A review of the operation characteristics of dual-mode hybrids can be found in [16]. The many different possible configurations and additional propulsion components brings new challenge and research opportunities to vehicle designers.

To design a hybrid vehicle, the engineer typically first selects one configuration to focus on. The design parameters (e.g. motor size, battery size, planetary gear sizes, etc.) and control strategy then need to be determined. Obviously, to achieve the near-optimal overall performance, an iterative process needs to be executed. However, the problem for this approach is that even with the optimal performance, whether the selected configuration offers the best solution among all possible configurations is not known. To achieve this goal, the exact same process from selecting another configuration and iteratively approaching the optimal performance has to be repeated. Moreover, only when the optimal performance is gained for each configuration, then the comparison between them is a sensible exercise. With numerous options for the configuration design variations, such an iterative process can only be achieved with a systematic method with many underlying techniques, including the automated model generation and simulation with optimal design and optimal control techniques.

The math-based method for gear design is not a new concept [17,18]. The studies on power-flow analysis of planetary gear trains were mostly performed as a part of efficiency formulations. Pennestri and Freudenstein [19] used the same fundamental circuits proposed earlier for a complete static force analysis [20]. Hsieh and Tsai [21] applied a similar formulation in conjunction with their earlier kinematics study to determine the most efficient kinematic configurations. The work by Castillo [22] further generalised the efficiency formulations of gear trains formed by single- or double-planet arrangements. However, these studies did not consider hybrid power electric components and their efficiencies. The major contribution of this paper is the development of a systematic exhaustive search method for all possible hybrid configurations for optimal fuel economy, under component and vehicle performance constraints.

The study of possible HV configurations (i.e. how the engine, electric machines and the vehicle are connected with the planetary gears) associated with component efficiency and sizing is of interest both from an industrial viewpoint and from the academic viewpoint. From the industrial viewpoint, it is interesting to explore alternative configurations to circumvent patents from Toyota and GM. Ford and Nissan, for example, are licensing the THS technology from Toyota while Chrysler and BMW are licensing the GM dual-mode technology. From the academic viewpoint, it is useful to develop design methodologies to search all possible split hybrid configurations exhaustively. Many patents were issued based on different hybrid configurations, but not component sizes. In addition, the selected configuration will set the stage for the subsequent component selection and control, and thus is a crucial first step in the hybrid vehicle design.
After a systematic method to generate all possible configurations, all candidate configurations are analysed, through three steps. First, the feasibility of a configuration is examined by ensuring the power sources and the vehicle are connected sensibly. Second, the parameters that can be varied by the designer need to be varied within the admissible range and component-imposed constraints (e.g. maximum generator and motor speed) and vehicle performance constraints (e.g. wide-open-throttle launching performance) needs to be checked. Finally, the surviving configurations will need to have a reasonably high-speed (power-split) mode so that the vehicle operates both at low speed and at highway speed. After these three steps, the surviving configurations will be used as the target, for which optimal control and optimal component sizes will be calculated. The last step is crucial because the full potential of a hybrid configuration can only be assessed based on its performance under the optimal control strategy. In this paper, we use the dynamic programming (DP) [23] technique to calculate the optimal control signal for each design candidate. DP solutions are computationally expensive, but guarantee to reflect the best execution of the hybrid vehicle design being evaluated. Therefore, they are selected to generate the optimal control solutions.

2. Mathematical model of hybrid vehicle configurations

2.1. Manual model derivation

Since the model to be developed is going to be used for searching optimal fuel economy of a large set of configurations/designs, it is appropriate to develop simple mathematical models that capture only the fundamental dynamics of the vehicle. In this section, we will first derive simple models for the THS (one planetary gear) and then the GM dual-mode system (two planetary gears). The core of these two hybrid power-trains is the planetary gear, which consists of the sun gear, the carrier gear (which is attached to the pinion gears) and the ring gear (Figure 2).

As a result of the mechanical connection, the rotational speeds of the ring gear $\omega_r$, sun gear $\omega_s$ and the carrier gear $\omega_c$ satisfy the following relationship:

$$\omega_s S + \omega_r R = \omega_c (R + S)$$

(1)

where $R$, and $S$ are the radii of the ring gear and the sun gear. Equation (1) is a kinematic constraint that limits the three planetary nodes to have only two degrees of freedom (DOF). Equation (1) can be nicely presented by the lever diagram [24] shown on the right-hand side of Figure 2. The length of the vectors shows the rotational speed of the three nodes. When the

Figure 2. Planetary gear set and the lever diagram.
power sources are connected to the planetary gear, as shown in Figure 3, the hybrid power-train dynamics can be derived as

\[
\begin{bmatrix}
  I_e + I_c & 0 & 0 & R + S \\
  0 & \frac{R_{\text{tyre}}^2}{K^2}m + I_{\text{MG2}} + I_t & 0 & -R \\
  0 & 0 & I_{\text{MG1}} + I_s & -S \\
  R + S & 0 & -R & 0
\end{bmatrix}
\begin{bmatrix}
  \dot{\omega}_e \\
  \dot{\omega}_{\text{out}} \\
  \dot{\omega}_{\text{MG1}} \\
  F
\end{bmatrix}
= \begin{bmatrix}
  T_{\text{MG2}} - \frac{1}{K} [T_{fb} + mgf_t R_{\text{tyre}} + 0.5 \rho AC_d \left( \frac{\omega_c^2}{K} \right)^2 R_{\text{tyre}}^3] \\
  T_{\text{MG1}} \\
  0
\end{bmatrix},
\]

where the configuration shown in Figure 3 corresponds to the configuration of THS, with the engine, generator (MG1) and motor (MG2) connected to the carrier, sun and ring gear, respectively. The ring gear output is also connected to a final drive and subsequently to the vehicle. The gain \( K \) represents the ratio gain due to the final drive ratio and tyre radius. The internal force \( F \) between the ring gear and planet gears and between planet gears and the sun gear are assumed to be the same due to the small gear inertia assumption.

Equation (2), together with a simple battery state of charge (SOC) dynamics,

\[
S \dot{OC} = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4(T_{MG1}\omega_{MG1}\eta_{MG1}^k \eta_{\text{c1}}^k + T_{MG2}\omega_{MG2}\eta_{MG2}^k \eta_{\text{c2}}^k) R_{batt}}}{2R_{batt} Q_{\text{max}}}
\]

forms a simple model for THS, which simulates the mechanical motions as well as the battery SOC. Details of this model derivation can be found in [25].

Derivation of the model for the GM dual-mode hybrid power-train from [26] is similar but more tedious. The corresponding free body diagram of an example of GM dual-mode system is shown in Figure 4, which shows two planetary gears with two clutches. The only difference between THS shown in Figure 3 and the dual-mode power-train shown in Figure 4 is the fact the ‘transmission’ consists of two planetary gears with two clutches. The vehicle will launch from mode 1 (only CL1 closed) and switch to mode 2 (only CL2 closed) when the vehicle speed becomes high.
Based on the free body diagram, the state space equations for the mechanical part of vehicle dynamics for the two modes are

\[
\begin{bmatrix}
\dot{\omega}_e \\
\dot{\omega}_{\text{out}} \\
\dot{\omega}_{\text{MG1}} \\
\dot{\omega}_{\text{MG2}} \\
F_1 \\
F_2
\end{bmatrix}
= \begin{bmatrix}
I_e + I_{c1} & 0 & 0 & 0 & R_1 + S_1 & 0 \\
0 & \frac{R_{\text{tyre}}^2 m + I_{c2}}{K^2} & 0 & 0 & 0 & R_2 + S_2 \\
0 & 0 & I_{\text{MG1}} + I_{s1} & 0 & -I_{\text{MG2}} + I_{c1} + I_{c2} & 0 \\
0 & 0 & 0 & I_{\text{MG2}} + I_{c1} + I_{c2} & 0 & -R_1 \\
R_1 + S_1 & 0 & -S_1 & -R_1 & 0 & 0 \\
0 & R_2 + S_2 & 0 & -S_2 & 0 & 0
\end{bmatrix}^{-1}
\times
\begin{bmatrix}
T_e \\
-\frac{1}{K} \left[ T_{\text{fb}} + m g f_r R_{\text{tyre}} + 0.5 \rho A C_d \left( \frac{\omega_c}{K} \right)^2 R_{\text{tyre}}^3 \right] \\
T_{\text{MG1}} \\
T_{\text{MG2}} \\
0 \\
0
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\dot{\omega}_e \\
\dot{\omega}_{\text{out}} \\
\dot{\omega}_{\text{MG1}} \\
\dot{\omega}_{\text{MG2}} \\
F_1 \\
F_2
\end{bmatrix}
= \begin{bmatrix}
I_e + I_{c1} & 0 & 0 & 0 & R_1 + S_1 & 0 \\
0 & \frac{R_{\text{tyre}}^2 m + I_{c2}}{K^2} & 0 & 0 & 0 & R_2 + S_2 \\
0 & 0 & I_{\text{MG1}} + I_{s1} + I_{s2} & 0 & -I_{\text{MG2}} + I_{c1} + I_{c2} & 0 \\
0 & 0 & 0 & I_{\text{MG2}} + I_{c1} + I_{c2} & 0 & -R_1 \\
R_1 + S_1 & 0 & -S_1 & -R_1 & 0 & 0 \\
0 & R_2 + S_2 & 0 & -R_2 & 0 & 0
\end{bmatrix}^{-1}
\times
\begin{bmatrix}
T_e \\
-\frac{1}{K} \left[ T_{\text{fb}} + m g f_r R_{\text{tyre}} + 0.5 \rho A C_d \left( \frac{\omega_c}{K} \right)^2 R_{\text{tyre}}^3 \right] \\
T_{\text{MG1}} \\
T_{\text{MG2}} \\
0 \\
0
\end{bmatrix}
\]

Note that Figure 3 represents only one THS design, and Figure 4 represents only one of many dual-mode designs. If a designer needs to explore all possible power-split configurations, with many possible combinations of power source hook-ups, an automated modelling procedure needs to be developed.
2.2. Automated modelling approach

In order to develop an automated modelling process, we first observe existing power-split configurations, and derived all of them by hand. Soon we were able to obtain rules based on these examples. As an example, the square matrix in Equations (2), (4) and (5) are all symmetric and can be decomposed into four parts \[ \begin{bmatrix} J & D \\ D^T & 0 \end{bmatrix} \], where \( J \) is the inertia matrix and \( D \) shows the gear train connections of the power-train, which reflects the kinetic constraint of the planetary gears for the torque and speed terms of the gear nodes. The dynamic equations that govern the hybrid power-trains can then be rewritten as

\[
\begin{bmatrix} J & D \\ D^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\Omega} \\ F \end{bmatrix} = \begin{bmatrix} T \\ 0 \end{bmatrix}
\]  

(6)

where \( \Omega \) and \( T \) are the speed and torque vectors of the four nodes that are connected to the engine, vehicle, MG1 and MG2, respectively. By introducing a matrix

\[
E \equiv J^{-1/2} D
\]  

(7)

The dynamic equation then, after some manipulations, becomes

\[
\dot{\Omega} = J^{-1/2}(I - E(E^T E)^{-1} E^T)J^{-1/2}T.
\]  

(8)

Equation (8) indicates that the key in obtaining the dynamic equation for a split power-train is the step to define the inertia matrix \( J \) and the constraint matrix \( D \). After these two matrices are specified, the vehicle dynamics are nothing but the combination of Equations (8) and (3). This enables an automated procedure to translate hybrid power-train designs quickly to a dynamic model. In the following, we assume that two electric machines are used, and the planetary gear sets have small inertia and no transmission loss. The matrices \( D \) and \( J \) can then be derived from the following two steps.

2.2.1. Step 1: determine the constraint matrix \( D \)

The constraint matrix \( D \) can be derived from the following rules.

**Rule 1.** The number of columns of \( D \) is equal to the number of planetary gears.

**Rule 2.** The number of rows of \( D \) is equal to the number of columns of \( D \) plus two, each corresponding to a node on the combined lever diagram.

**Rule 3.** For the power source component(s) at each row, a ‘node coefficient’ should be entered. The ‘node coefficient’ is equal to: \(-S_i\) if connected to the sun gear; \(-R_i\) if connected to the ring gear; and \(R_i + S_i\) if connected to the carrier gear. Here the subscript \( i \) represents the \( i \)th planetary gear set.

**Rule 4.** Fill all other entries in matrix \( D \) with zeros.

**Rule 5.** For systems with three or more planetary gears, after the original matrix \( D \) is obtained. A valid design can be simplified to a \( 4 \times 2 \) matrix. This is done by using the kinematic relations derived from the free-rolling node(s) that is not connected to any power source or vehicle.

Although two planetary gear configurations are focused in this paper, rule 5 indicates that the proposed model can be extended to more complex applications. For example, given a triple planetary gear system as shown in Figure 5, first follow rules 1 and 2: matrix \( D \) is a \( 5 \times 3 \) matrix; then, follow rules 3 and 4: the engine is connected to the ring gear of the PG1, therefore, a node coefficient \(-R_1\) is entered into element (1,1). The vehicle final drive is connected to
the carrier gear of the PG3, therefore, \( R_3 + S_3 \) is entered into element (2,3). MG1 is connected to both ring gear of the PG2 and sun gear of the PG1, therefore, \(-S_1\) is entered into element (3,1) and \(-R_2\) is entered into element (3,2). MG2 is connected to both sun gears of the PG2 and PG3, therefore, \(-S_2\) is entered into element (4,2) and \(-S_3\) is entered into element (4,3).

Now the fifth row of matrix \( D \) corresponds to the node/shaft of both carrier gears of the PG1 and PG2 where there is no power source connected. The corresponding node coefficients, in this case, \( R_1 + S_1 \) is entered into element (5,1) and \( R_2 + S_2 \) is entered into element (5,2). After filling the rest of the entries with zeros, the matrix \( D \) is generated as

\[
\begin{bmatrix}
-R_1 & 0 & 0 & 0 \\
0 & 0 & R_3 + S_3 & 0 \\
-S_1 & -R_2 & 0 & -S_3 \\
0 & -S_2 & -S_3 & 0 \\
R_1 + S_1 & R_2 + S_2 & 0 & 0
\end{bmatrix}
\]  \tag{9}

Follow rule 5, this originally derived matrix \( D \) can be further simplified to a \( 4 \times 2 \) matrix \( \tilde{D} \) to construct the dynamic model. In Equation (9), the fifth row corresponds to the free-rolling node that is not connected to any power sources. Because the gear inertia on this node is ignored, the dynamics are

\[(R_1 + S_1)F_1 + (R_2 + S_2)F_2 = 0.\]  \tag{10}

From Equation (10),

\[F_2 = -\frac{(R_1 + S_1)}{(R_2 + S_2)}F_1.\]  \tag{11}

Because in matrix \( D \), the first and second columns consist of the node coefficients that multiply with \( F_1 \) and \( F_2 \), respectively, the relationship between these two forces in Equation (11) can then be substituted to simplify the matrix \( D' \) as in Equation (12). With this simplification, the three planetary gear configuration can be treated as a two planetary configuration and applied to the same design process proposed in this paper. Here we assume only two electric machines and one engine are applied as the power sources.

\[
\begin{bmatrix}
-R_1 & 0 \\
0 & R_3 + S_3 \\
-S_1 + \frac{R_1 + S_1}{R_2 + S_2}R_2 & 0 \\
\frac{R_1 + S_1}{R_2 + S_2} & -S_3
\end{bmatrix}
\]  \tag{12}
2.2.2. Step 2: determine the inertia matrix $J$

Matrix $J$ is a diagonal $4 \times 4$ matrix. The entry of each diagonal term is equal to the inertia of each node plus the inertia connected to it. Since the planetary gear is a compact devise, the node inertia is assumed to be equal to the inertia of the power components (i.e. gear inertia are ignored). The inertias are denoted as $I_e$ for the engine, $I_{MG}$ for electric machines, and $mR_{tyre}^2/K^2$ for the vehicle (where $K$ is the final drive gear ratio). A convention is followed that the first row of both matrix $J$ and matrix $D$ represents the engine node, the second row represents the output node connected to the vehicle, the third row represents the MG1 node and the fourth row represents the MG2 node. The matrix $J$ then has the following format for all configurations

$$J = \begin{bmatrix}
I_e & 0 & 0 & 0 \\
0 & mR_{tyre}^2/K^2 & 0 & 0 \\
0 & 0 & I_{MG1} & 0 \\
0 & 0 & 0 & I_{MG2}
\end{bmatrix}. \quad (13)$$

After the matrices $D$ and $J$ are determined, the dynamic model can be constructed from Equations (8) and (3). The input vector $T$ consists of the node torques, engine torque, motor torque, etc. For the output shaft, the torque ‘input’ is the load torque

$$-\frac{1}{K} \left[ T_{fb} + mg \sin \theta f \cdot R_{tyre} + mg \cos \theta + 0.5 \rho AC_d \left( \frac{\omega_{out}}{K} \right)^2 R_{tyre}^3 \right] \quad (14)$$

which assumes a road gradient $\theta$ is present. To simulate the vehicle behaviour fully and fuel consumption, engine and motor efficiency maps are included in the model as look-up tables, details refer to [25].

It is clear from the above process that the determination of the matrix $D$ is the key step in obtaining the dynamic equations of split hybrid vehicles. Matrix $D$ for a few popular power-trains are derived based on the rules presented above, and validated by comparing against hand-derived results. These matrices are summarised in Table 1.

In Table 1, the lever 1 (on the left-hand side) has the sun gear at the bottom, and lever 2 (the one next to lever 1) has the sun gear on the top, while lever 3 (if exists), has the sun gear at the bottom again. The small elements shown next to the levers are the power sources and the vehicle. Many of the configurations have one or more clutches. When the configuration is capable of operating in two modes, both $D$ matrices are shown in the table.

3. Configuration screening

Now that we have identified the relationship between the configurations and the corresponding matrices, it is possible to search through all configurations, by constructing the matrices systematically. In particular, it is obvious if we stick to a defined sequence of the nodes on the lever (engine, vehicle and electric machines), then the inertia matrix is fixed for all configurations, and the construction of matrix $D$ is the only problem to be solved. In the following, we focus on the design of split power-trains with two planetary gears and two electric machines.

The matrix $D$ for 2-PG power-trains is a $4 \times 2$ matrix. In each column, a power-split PG, of which all three nodes are connected with power sources or vehicle, has one zero and three node coefficients while a power-ratio PG, of which one node is locked with a clutch device and two nodes are connected with the power sources or vehicle, has two zeros and any two of
Table 1. Matrix $D$ of a few hybrid power-trains.

<table>
<thead>
<tr>
<th>Design</th>
<th>$D$ matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>THS-II</td>
<td>$\begin{bmatrix} \frac{R_1 + S_1}{R_1} \ -\frac{R_1}{S_1} \ -\frac{R_1}{S_1} \end{bmatrix}$</td>
</tr>
<tr>
<td>THS-II for SUV</td>
<td>$\begin{bmatrix} \frac{R_1 + S_1}{R_1} \ -\frac{R_1 + S_1 + S_2}{-R_2} \ -\frac{R_1}{-S_2} \ -\frac{R_1}{-S_2} \end{bmatrix}$</td>
</tr>
<tr>
<td>THS-II for Lexus GS450</td>
<td>$\begin{bmatrix} \frac{R_1 + S_1}{R_1} \ -\frac{R_1 + S_1 + S_2}{R_2 + S_2} \ -\frac{R_1}{-S_2} \ -\frac{R_1}{-S_2} \end{bmatrix}$</td>
</tr>
<tr>
<td>GM 2PG</td>
<td>$\begin{bmatrix} \frac{R_1 + S_1}{R_1} \ -\frac{R_1 + S_1 + S_2}{R_2 + S_2} \ -\frac{R_1}{-S_2} \ -\frac{R_1}{-S_2} \end{bmatrix}$</td>
</tr>
<tr>
<td>Timken</td>
<td>$\begin{bmatrix} \frac{-R_1}{R_1 + S_1 + S_2} \ -\frac{R_1}{R_2 + S_2} \ -\frac{R_1}{-S_2} \ -\frac{R_1}{-S_2} \end{bmatrix}$</td>
</tr>
<tr>
<td>GM 3PG</td>
<td>$\begin{bmatrix} \frac{-R_1}{R_1 + S_1 + S_2} \ -\frac{R_1}{R_2 + S_2} \ -\frac{R_1}{-S_2} \ -\frac{R_1}{-S_2} \end{bmatrix}$</td>
</tr>
</tbody>
</table>

the three node coefficients. Therefore, for a single column in matrix $D$, there are 24 different combinations for a power-split PG ($P_4^4 = 4! = 24$), and 36 combinations for a power-ratio PG ($C_3^2 \cdot P_4^2 = 3 \times (4 \times 3) = 36$). A valid power-split configuration must consist of at least one power-split PG. And changing the order of the two columns in the matrix $D$ does not change the configuration design. Therefore, the total combination is $24 \times 24/2 + 24 \times 36 = 1152$. It is quite possible not all of the combinations have been explored before. Also, many of the mathematically possible combinations are physically infeasible. Developing steps to screen through these configurations is the next task. Note that in the process above, each node coefficient is entered at most once into each column. This guarantees that any two of the engines, MGs, and vehicle are not connected to the same gear node of any planetary gear. This rule already screens out a large set of obviously infeasible or impractical designs.

Two other types of configurations are deemed infeasible. The first type is when any row of the matrix $D$ has two zeros, because the power source/vehicle presented by that row is then not connected to the planetary gear. Apparently this type of configuration is not realistic. The second type is when the configuration has the engine connected to the vehicle output directly or with a fixed ratio (e.g. Figure 6). As a result, the EVT ratio is lost.

In both infeasible configuration examples described above, the power-train system violates the fact that the configuration needs to maintain two DOF for the combined lever. In other words, given the engine and the vehicle speeds, the speeds of the two MGs need to be uniquely determined. This requirement ensures that the continuously variable gear ratio is achievable between the engine and the vehicle speeds by manipulating the two electric machines. This can
be checked by examining the rank condition of the two parts of the $D$ matrix. By separating the $D$ matrix into two $2 \times 2$ matrices $D = \begin{bmatrix} D_{EV} \\ D_{MG} \end{bmatrix}$, the rank condition requires that $D_{EV}D_{MG}^{-1}$ is non-singular, because

$$\begin{bmatrix} \omega_{MG1} \\ \omega_{MG2} \end{bmatrix} = -D_{MG}^{-T}D_{EV}^{-1} \begin{bmatrix} \omega_{e} \\ \omega_{out} \end{bmatrix}$$ \hspace{1cm} (15)$$

After checking this condition, only 288 out of 1152 configurations were found to be feasible.

In the second step of configuration screening, the vehicle drivability and electric machines sizing constraints are considered. Vehicle drivability is heavily dependent on the power and torque limits of the power sources. For a hybrid vehicle, the drivability objective can be more easily achieved because the electric power sources and power-train configuration provide additional design degree of freedom. In the search process, we assume a constant power launching performance that needs to be satisfied. As an example, to achieve 0–50 mph in 15 s, an HMMWV-like vehicle (which is our design target) needs to generate 100 kW consistently from 0 to 50 mph. The engine and MG1 power under all possible combinations at the specified vehicle speed are then searched. In each search, the given engine and the MG1 power, the MG2 needs to satisfy the power demand gap, while staying within the motor speed and torque limits (note that the torque limits are SOC dependent). If all possible combinations of the engine and MG1 power result in the MG2 operating condition that cannot satisfy the motor speed and torque constraints, then the vehicle performance at this particular vehicle speed/SOC condition is deemed inadequate. A vehicle configuration-electric machine sizing combination needs to yield a feasible solution at each speed/SOC condition to be declared a feasible candidate.

Given the vehicle parameters (Table 2) and if we limit the total electric machines power to a rather low level of 60 kW, only 17 of the 288 kinematically feasible configurations were
found to be ‘drivable’ and will be allowed to pass on to the next step of screening. Since the motor and engine power are specified, their weights related to the sizing are also fixed.

All the 17 surviving configurations were found to be input-split type, which is beneficial for vehicle launching. However, it is known that the input-split transmission only has one mechanical point (MP) and suffers from high efficiency loss during high-speed cruising. In this step, the mode shifting ability of these input-split configurations are checked, and the MPs of the ‘high-speed mode’ are calculated which determine the range of efficient operation. Notice that the high-speed mode is always a compound-split mode which has two MPs. The input-split mode will be used at low speed, and is assumed to shift to the compound-split mode at a synchronous speed. In our study, we assume that the MPs need to fall within the range suggested by [16]: One of the MPs should be close to the low gear ratio in a conventional transmission (in the range between 1.5 and 4) to be beneficial for the low-speed launch, and the other MP should be close to the high-speed overdrive ratio, ranging from 0.5 to 1 for highway cruising.

The task of calculating MPs is equivalent to solving the input/output speed ratio when one of the MGs has zero speed. This can be easily done with the previously defined matrices $D_{EV}$ and $D_{MG}$. In Equation (11), set either $\omega_{MG1}$ or $\omega_{MG2}$ equal to zero, the input/output speed ratio, which is the corresponding MP, can be calculated. Given the constraint matrix $D$, possible shifting mode can also be derived. For example, if an input-split system is designed and the matrix $D$ is

$$D = \begin{bmatrix}
-R_1 & 0 \\
R_1 + S_1 & R_2 + S_2 \\
-S_1 & 0 \\
0 & -R_2
\end{bmatrix}. \quad (16)$$

The only two possibilities for the compound-split mode are

$$D_{mode21} = \begin{bmatrix}
-R_1 & -S_2 \\
R_1 + S_1 & R_2 + S_2 \\
-S_1 & 0 \\
0 & -R_2
\end{bmatrix} \quad \text{or} \quad D_{mode22} = \begin{bmatrix}
-R_1 & 0 \\
R_1 + S_1 & R_2 + S_2 \\
-S_1 & -S_2 \\
0 & -R_2
\end{bmatrix}. \quad (17)$$

In other words, the sun gear of PG2, which is grounded in the low-speed (input-split) mode, is released and locked into either the ring gear of PG1, or sun gear of PG1 to form the high-speed (compound-split) mode. Given Equation (13), the MPs can be calculated, and whether the synchronous shifting between the input-split mode and the compound-split mode can be checked. If both conditions are satisfactory, then the high-speed mode is deemed feasible, else it will not be practical. After this step, only two configurations were found to be feasible. The first one (PT1) is the design in [26] and presented as the example in Section 2.1. The second one (PT2) is the design presented in [27]. As shown in Figure 7, when CL1 is locked and CL2 is released, it functions as an input-split system, MG2 takes the generated power of MG1 from the engine to support the driving demand. When CL1 is released and CL2 is locked, it functions as a compound split mode.

### 4. Optimal component sizing and control results

Assessment of different configurations cannot be done without some kind of optimal sizing and control strategies. Control strategies based on engineering intuition frequently fail to explore the full potential of a design because the multi-power-source nature of the powertrain systems. An optimal control strategy, on the other hand, represents the best execution...
rather than an execution with unknown quality and refinement. The same argument applies to optimal sizing. Therefore, for each of the surviving configuration, it is necessary to solve for optimal component sizes and optimal control solutions. Both tasks are computation intensive. Fortunately, the related technologies are mature and methods and software are readily available. As an example, the sequential quadratic programming (SQP) and DP are mature methods that are suitable for this purpose. However, to use SQP, a wrapper programme is needed to integrate the SIMULINK file with the DP optimisation together with the SQP code. In addition, because of the iterative nature of the SQP search implemented on commercial codes, the computation will be extremely time-consuming.

In this paper, we will solve the optimal design problem by exhaustively searching through a matrix of cases with varying electric machine sizes (total power is limited to 60 kW) and planetary gear dimensions. Both planetary gear sets are assumed to have the ratio between ring gear and sun gear \((K_i \equiv R_i/S_i)\) limited between 1.6 and 2.4, which are practical ranges to which these gears can be constructed. For each of the three design parameter dimensions we are optimising, electric machine size, \(K_1\) and \(K_2\), five values are used. That results in the solution of 125 DP solutions for each of the two surviving configurations. DP is applied to solve the best driving sequence on the Federal Test Procedure (FTP) urban cycle by optimising the objective function

\[
J = \alpha(SOC_N - SOC_f)^2 + \sum_{k=0}^{N-1} fuel_k
\]

where the fuel consumed at each step and a terminal constraint on SOC are considered. The fuel consumption only presents the steady-state engine operation. The effects of thermal transients, affecting both fuel consumption and emissions, are ignored. \(SOC_f\) is the desired SOC at the final time, and \(\alpha\) is a positive weighting factor. Deviation from the desire \(SOC_f\) will be penalised. In addition, to compensate for the change in battery SOC from the start to the end of the driving cycle, three runs are executed based on which the effect of SOC variation can be compensated. In each of the DP problem, the vehicle dynamics are discretised into 1-s

![Figure 8. Best performance of the two candidate configurations over the FTP75 cycle.](image-url)
steps with three states and three inputs (Figure 8). The DP solution involves backward solution of all possible combination of inputs along state grids.

The optimal results (best DP results over all of the 125 design parameter combinations) of the two design candidates are shown in Figure 9. It can be seen that both power-train configurations achieve about 50% fuel economy improvement over the conventional (non-hybrid) power-train. Figure 7 is based on DP results, which are non-causal control signals, rather than implementable control laws. The slightly better results of PT#2 should not be interpreted as solid evidence that PT#2 is superior to all other configurations. The fact this particular configuration stands out in this study is dependent on the vehicle spec (weight, engine), driving cycle, constraints of electric machines, among other factors. We selected the total electric machine power to be limited to 60 kW, which is roughly what the Toyota Prius has. The vehicle weight, however, is more than 10,000 pounds, roughly three times that of Prius. The available electric machine power is thus quite small, which severely constrained the pool of feasible configurations. What is important is the overall model construction, configuration screening and optimisation process, not the final results. In addition, one still needs to design implementable control algorithm, which reflects the actual performance of the selected configuration.

On reviewing the selected configuration designs, it can be found that the power-split ratio defined by the gear design is strictly related to the electric power capability. When varying the electric machine sizing, if any one of the MG is relatively small, the powertrain fails to satisfy the driving demand. This is because of the power circulation in the power-split vehicle. Part of the engine input power is circulated after it is split. The split power in the electrical path goes through both MGs to reach the final wheel. Figure 10 shows the circulated electric power under a launching portion of the driving cycle. This means that in PT2, both of the MGs should be sized above this value to generate or motor the power.

The effect of varying the PG ratios can be explained by using a contour plot. Figure 11 shows the result of PT2. It appears that the fuel efficiency increases as $K_2$ increases for this power-train configuration. To understand the reason, the results from one design ($K_1 = 1.6$ and $K_2 = 2.2$) with higher fuel efficiency (18.43 mpg) and one design ($K_1 = 1.6$ and $K_2 = 1.6$) with lower fuel efficiency (17.57 mpg) are compared. The difference mainly lies in the performance of the electric machines. Figure 12 shows the MG2 operating points of both cases in the power efficiency map. As marked, the lower-efficiency case has more points (triangles) in regions with poor electric efficiency. This can also be observed in Figure 13. When the vehicle is launching and requires a large amount of power (e.g. between 20 and 75 s, and between 170
and 200 s on a EPA Urban cycle), the MG2 is driven at lower a speed (Figure 13(a)) with high torques (Figure 13(b)) for the lower fuel efficiency case, which is not efficient. The lower power efficiency of MG2 results in the lower fuel efficiency since more power is lost in the electrical path.

To explain why $K_2$ has such an effect on the MG2 operation, let us look at the configuration of PT#2. In the launching mode of PT#2 (as shown in Figure 14), because the ring gear of PG2 is grounded, increasing $K_2$ will increase the speed ratio of MG2 over the vehicle output shaft. This means, for the same vehicle speed, a larger $K_2$ results in a higher MG2 speed. When the vehicle speed is low and the MG2 torque is high, the configuration with larger $K_2$ pushes the MG2 operating point to avoid the low power efficiency region and achieves better fuel efficiency.
Figure 12. MG2 efficiencies of two different design cases (high fuel efficiency case: $K_1 = 1.6$ and $K_2 = 2.2$, and low fuel efficiency case: $K_1 = 1.6$ and $K_2 = 1.6$).

Figure 13. MG2 speeds and torques of two different design cases (high fuel efficiency case: $K_1 = 1.6$ and $K_2 = 2.2$, and low fuel efficiency case: $K_1 = 1.6$ and $K_2 = 1.6$).

Figure 14. In the PT2 configuration, increasing $K_2$ results in higher speed of MG2 at the same vehicle speed.

5. Conclusions

A systematic approach to select the configuration, component sizing and optimal control for the hybrid vehicles is presented in this paper. A super-sized HMMWV is the target vehicle with the
purpose of optimising for fuel economy. The approach involves using an automated process for generating dynamic models for all possible two-planetary gears, two electric machines vehicle power-train. Out of the 288 kinematically feasible configurations, 17 were found to be able to launch the vehicle with very small electric machines. And only two of them have a valid compound-split (high-speed) mode and thus can produce practical power-train designs for the given challenging performance and component constraints. The results from this paper should not be interpreted as endorsement for any particular configuration. Rather, the design process which integrates automatic model generation, configuration screening, feasibility checking and optimal sizing and control is the main contribution of this paper. This process is flexible and powerful and can be used for practical hybrid vehicle designs. The optimal objective function (Equation 18) can be modified to fit other design purposes. For instance, changing the SOC target value can set to solve a charge depletion scenario which is common in a plug-in hybrid application.

It is important to emphasise that optimal design and control techniques should be used to obtain ‘best executions’ to ensure fair comparisons of configurations. Preliminary results showed that the fuel economy results are sensitive to driving cycles, component constrains and engine/vehicle parameters. Therefore, best configuration or component sizing for one driving cycle may be bad choices for another driving cycle. An improvement is to take statistical driving information and apply a stochastic DP solution as described in [28]. In that case, the selected configuration and components serve for the best interest of a general driving pattern.

It is also important to emphasise that we have not explored all possible design directions in this study. For example, the battery used is ‘large enough’ and was not a design parameter in this study. In practice, one would probably want to optimise battery size, which is one of the most expensive sub-system of the vehicle. The same thing can be argued for engine size, and several other vehicle parameters. However, we have adequately demonstrated the concept of the overall design process which we did not see in any prior publications. We believe this process is useful for a wide array of hybrid vehicle design problems.

Finally, as implied in this paper, more possible configurations are expected to survive the screening process if some of the design requirements are more relaxed. And in addition, increase the number of design criteria can exponentially increase the search matrix. Considering these facts, the final combinations to be analysed by DP can be in the order of thousands. It will be a huge computational cost even with today’s computer technology. In this case, the automated modelling method can be still used to screen through every possible design, more traditional and faster algorithms can be then added to estimate and evaluate the candidates. And DP is only integrated as an outer optimisation loop for the design variables. In this way, a sub-optimal solution can be reached quicker with less computational cost.

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