Exponential Curvature-Compensated BiCMOS Bandgap References

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Abstract—An exponential curvature compensation technique for bandgap references (BGR’s) which exploits the temperature characteristics of the current gain β of a bipolar transistor is described. This technique requires no additional circuits for the curvature compensation; only a size adjustment of a bias transistor in a conventional first-order compensated BGR is required. Positive and negative versions of the exponential curvature-compensated BGR have been fabricated using 1.5-μm BiCMOS process. Average temperature coefficients (TC’s) of the negative BGR are measured as 2.4 and 6.7 ppm/°C, and those of the positive BGR are measured as 3.5 and 8.9 ppm/°C over the commercial (0 ~70 °C) and military (−55 ~125 °C) temperature ranges, respectively. These circuits dissipate 0.37 mW with a 5-V single supply, and occupy 270 × 150 μm² and 290 × 150 μm², respectively.

I. INTRODUCTION

Since introduced by Widlar [1], [2], the bandgap reference (BGR) has been used extensively to make on-chip voltage references in A/D and D/A converters, voltage regulators, and measurement systems. Several BGR’s using Widlar BGR or its variants with bipolar [3], [4] and CMOS [5], [6] processes have been reported. As the resolution of data converter systems increases, requirements for the temperature stability of BGR’s have also increased. However, due to the curvature in the reference output voltage [7], there exists a limit for improving the temperature stability of the first-order compensated BGR. To overcome this drawback, several compensation techniques which compensate the curvature in the reference output voltage of the first-order compensated BGR have been developed [8]–[11]. With these techniques, the temperature stability of a BGR is increased significantly. However, these techniques need complex circuits, have large power consumption, and occupy significant chip area for curvature compensation. Therefore, applying these techniques to the on-chip voltage references in analog/digital mixed systems is inappropriate.

This paper describes a simple curvature compensation technique for a BGR and realization of BiCMOS BGR family using this technique. In Section II, an exponential curvature compensation technique is introduced and compared with the conventional curvature compensation technique. In Section III, BiCMOS BGR’s employing this technique are presented. In Section IV, experimental results are summarized.

II. EXPONENTIAL CURVATURE COMPENSATION OF A BGR

The main idea of the first-order compensated BGR is to cancel the negative temperature dependency of the base-emitter voltage of a bipolar transistor $V_{BE}$ by adding a voltage source proportional to the thermal voltage $V_T$ [1], [2]. However, compared with $V_T$, which is a fully linear function of $T$, $V_{BE}$ is a complex function of $T$ containing many higher order terms. Consequently, even in the optimally compensated case, the reference voltage $V_{REF}$ has some temperature drift terms. Since this drawback is inherent, it is impossible to improve the temperature stability of the first-order compensated BGR above a certain limit.

The temperature characteristics of $V_{BE}$ are studied extensively by Tsividis [7]. This work suggests that the analytical form of $V_{BE}$ is:

$$V_{BE}(T) = V_G(T) + \left( \frac{T}{T_r} \right) [V_{BE}(T_r) - V_G(T_r)]$$

$$- \left( \frac{kT}{q} \right) \ln \left( \frac{T}{T_r} \right) + \left( \frac{kT}{q} \right) \ln \left( \frac{I_C(T)}{I_C(T_r)} \right)$$

where $V_G$ is the bandgap voltage of silicon, $\eta$ is the temperature dependency parameter of silicon mobility, and $T_r$ is a reference temperature. Since the collector current $I_C$ is PTAT (proportional to absolute temperature) in most BGR’s [1], [5], [6], [7], [10], the reference voltage $V_{REF}$ of the first-order compensated BGR is expressed as:

$$V_{REF}(T) = V_{BE}(T) + kT$$

$$= V_G(T) + \left( \frac{T}{T_r} \right) [V_{BE}(T_r) - V_G(T_r)]$$

$$- \left( \frac{\eta}{\eta - 1} \right) \left( \frac{kT}{q} \right) \ln \left( \frac{T}{T_r} \right) + kT$$

where $K$ is the optimization parameter which is adjusted to minimize the temperature drift of $V_{REF}$. In order to understand the temperature behavior of $V_{REF}$, an analysis of the temperature characteristics of $V_G$ should be preceded. There are many empirical models of $V_G$, and the model of Bludau et al. [12] is the most accurate available. This model matches the bandgap voltage $V_G$ of silicon to the following empirical equation within 0.2 mV:

$$V_G(T) = a - bT - cT^2$$
The temperature drift of $V_{REF}$ of the optimally compensated first-order BGR is determined by $\eta$ only, since the influences of other process variations are all canceled out by adjusting $K$. Fig. 1 shows the temperature drifts of $V_{REF}$ with several $\eta$s. They show the quadratic temperature behavior, and the magnitude of the quadratic term increases as $\eta$ increases. The main contributions of the quadratic term are the nonlinearity of the bandgap voltage $V_{BE}$ and the logarithmic characteristic of $V_{BE}$. As we see from (2) and (3), the temperature drift of $V_{REF}$ of the optimally compensated first-order BGR is determined by $\eta$ only, since the influences of other process variations are all canceled out by adjusting $K$. Fig. 1 shows the temperature drifts of $V_{REF}$ with several $\eta$s. They show the quadratic temperature behavior, and the magnitude of the quadratic term increases as $\eta$ increases. The main contributions of the quadratic term are the nonlinearity of the bandgap voltage $V_{BE}$ and the logarithmic characteristic of $V_{BE}$ to $I_C$.

Varying with the process variation, $\eta$ has the value of 2.3 generally [7]. Using (2) with $\eta$ of 2.3, the temperature coefficient (TC) at the optimal condition is calculated as 15 ppm/°C over the military temperature range. This is the theoretical limit of the achievable TC of the first-order compensated BGR. The practical TC of it, however, increases significantly with nonideal factors such as process variation, incomplete compensation. The reported TC's of the first-order compensated BGR lie in the range from 20 to 60 ppm/°C [1], [4], [13], [14].

In order to increase the temperature stability beyond the limit of the first-order compensated BGR, several methods are proposed to compensate the curvature of $V_{BE}$. One is to linearize $V_{BE}$ directly [9], another to add higher order terms of $T$ [8], [10]. Among these methods, the second-order curvature compensation technique which is to add the $PTAT^2$ (proportional to absolute temperature squared) term to cancel the curvature of $V_{BE}$ is widely used [10]. The reference voltage $V_{REF}$ of the second-order compensated BGR is expressed as:

$$V_{REF}(T) = V_{BE}(T) + K_1 T + K_2 T^2. \quad (4)$$

In (4), $K_1$ and $K_2$ are both optimization parameters, $K_1$ compensates the linear term of $V_{BE}$, $K_2$ the curvature term. After optimization, only higher order terms of order greater than or equal to three remain in $V_{REF}$. As the shape of the curvature is similar to a quadratic function of temperature, this technique offers the possibility of increasing the temperature stability significantly.

Although this second-order compensation technique is conceptually quite simple, its practical realization introduces some problems to the design. Making $PTAT^2$ voltage requires complex circuits which consume significant chip area and power [10]. For this reason, the second-order compensation technique is appropriate for the construction of a single-chip BGR or as a building block of the hybrid IC, and is not appropriate for the on-chip voltage reference in a data conversion application.

In order to make a simple and efficient BGR suitable for data conversion applications, which require a precision voltage reference with small size and low power consumption, we have developed a curvature compensation technique which uses the temperature characteristics of the current gain $\beta$ of a bipolar transistor. This technique was named exponential curvature compensation technique, as the temperature behavior of $\beta$ shows an exponential function of temperature. Fig. 2 describes the concept of the exponential compensation technique.

In Fig. 2, the two current sources $I_1$ and $I_2$ are PTAT sources, so the reference voltage $V_{REF}$ is:

$$V_{REF}(T) \approx V_{BE}(T) + c_1 RT + \frac{c_2 RT}{\beta(T)} \quad (5)$$

where the term $c_2 RT/\beta$ is the voltage drop at $R$ due to the base current of $Q$, and it makes a nonlinear curvature compensation voltage. In this equation, $c_1$ and $c_2$ are design parameters that circuit designers can adjust the value, $c_1$ for a linear term and $c_2$ for a curvature term. By adjusting $c_1$ and $c_2$, the temperature drift of $V_{REF}$ can be minimized.

The advantage of the exponential compensation technique over the conventional second-order compensation technique is that this technique needs no extra circuitry required in the second-order compensated BGR to generate bias currents for curvature compensation [10]. This technique, which uses the base current of $Q$ as the curvature compensation source,
has the same topology that the first-order compensated BGR, since the PTAT biasing technique of $Q$ which generates $V_{BE}$ is common in the first-order compensated BGR. The only additional work is sizing the current source $I_2$ to minimize the nonlinear temperature drift of $V_{REF}$.

Another advantage of this technique is that the temperature stability of the proposed exponential compensation technique is superior to the second-order compensation technique. In the following sections, the temperature stability of the exponential and the second-order compensation techniques is compared extensively.

A. Temperature Characteristics of $\beta$

The temperature dependency of $\beta$ is an exponential function of temperature, and an inverse exponential function of the emitter doping level [15]-[17]. It can be expressed as [17]:

$$\beta(T) = \beta_\infty \exp \left(-\frac{\Delta E_G}{kT}\right)$$  \hspace{1cm} (6)

This temperature dependency is the consequence of the emitter bandgap decrease, caused by the large number of dislocations and lattice deformations at high doping levels [15]. $\Delta E_G$ is the bandgap narrowing factor of emitter proportional to the emitter doping level. Combining (5) and (6), the reference voltage $V_{REF}$ of the exponential-compensated BGR is derived as:

$$V_{REF}(T) = V_{BE}(T) + c_1 RT + \frac{c_2 R}{\beta_\infty} T \exp \left(\frac{\Delta E_G}{kT}\right)$$

$$= V_{BE}(T) + K_1 T + K_2 T \exp \left(\frac{\Delta E_G}{kT}\right)$$  \hspace{1cm} (7)

where $K_1$ and $K_2$ are the same optimization parameters as $K_1$ and $K_2$ in (4), $K_1$ for a linear term, $K_2$ for a curvature term.

As the device size scales down and the doping level increases, $\Delta E_G$ tends to increase. To calculate $\Delta E_G$ of the 1.5-μm BiCMOS process which was used to fabricate the prototype BGR's, the temperature dependency of $\beta$ has been measured and plotted in Fig. 3. This graph clearly shows the inverse exponential characteristics of $\beta$. From the measurement result of 10 samples, the mean and the standard deviation of $\Delta E_G$ is calculated as 63.9 meV and 2.86 meV, and those of $\beta_\infty$ is calculated as 117 and 92.3, respectively. The measurement results show that there is a strong correlation between the distribution of $\Delta E_G$ and $\beta_\infty$. The correlation coefficient $r$ is calculated as 0.97.

B. Comparison Between the Second-Order and the Exponential-Compensated BGR

The TC is widely used as an indicator of the temperature stability of BGR [1]. To minimize TC, the temperature drift $\Delta V_{REF}$ of the reference voltage should be minimized in a given temperature range. So, optimization procedures of the curvature-compensated BGR’s are to find the optimal values of $K_1$ and $K_2$ in (4) and (7) which minimize the temperature drift $\Delta V_{REF}$. However, the approximation method, which finds $K_1$ and $K_2$ that make the first and the second derivatives of $V_{REF}(T)$ to give zero at the reference temperature $T_r$ [1], [10], has been used instead in order to get an analytic solution of $V_{REF}$, but is not efficient for the curvature-compensated BGR. Temperature behaviors of curvature-compensated BGR’s show cubic or more complex forms rather than the quadratic one. So, if this approximation method is used to optimize the curvature-compensated BGR, the optimized BGR may show a slope in its temperature behavior, which increases TC. To avoid this problem arising in the optimization of the curvature-compensated BGR, a numerical method which finds the global minimum of TC should be used. In this paper, we developed a simple algorithm for finding optimal values of $K_1$ and $K_2$. This algorithm, first, finds the optimal value of $K_2$ which minimizes the curvature term of $V_{REF}$ using least-square method, and then moves the slope of $V_{REF}$ to zero by adjusting $K_1$. The algorithm is described in detail in the Appendix.

As we see from (a6) in the Appendix, the optimized temperature characteristics of the second-order and the exponential-compensated BGR’s don’t depend on any other process parameters except for the mobility temperature dependency $\eta$ and the bandgap narrowing factor $\Delta E_G$. Therefore, if $\eta$ and $\Delta E_G$ are given, the temperature characteristics of the two BGR’s are fully determined. We compared the performances of the two BGR’s using the algorithm described in the Appendix. The value of $\Delta E_G$ was set to 63.9 meV as calculated in Section II-A; the value of $\eta$ was set to 2.3, which is the typical value of $\eta$ [7]. Since the temperature stability of both the second-order and the exponential-compensated BGR’s degrades as $\eta$ increases, the value of $\eta$ does not affect the performance comparison of the two methods. After optimization, $K_2$ is calculated as $2.1 \times 10^{-6}$ V/K for the exponential compensation technique, $3.0 \times 10^{-6}$ V/K$^2$ for the second-order compensation technique. These values of $K_2$ are determined by given $\eta$ and $\Delta E_G$ entirely; no other process parameters affect them. On the
other hand, the values of $K_1$ vary with process variation, since $V_{BE}(T_0)$ in (a5) has different values with different processes. The nominal reference voltage $V_{REF}$ of the second-order compensated BGR is 1209.1 mV, and 1257.2 mV for the exponential-compensated BGR.

Fig. 4 shows the simulated temperature drift $\Delta V_{REF}$ of the two methods over the military temperature range. In case of the second-order compensated BGR, $\Delta V_{REF}$ behaves like a cubic function of temperature since the linear and the quadratic terms are canceled out, and in case of the exponential-compensated BGR, $\Delta V_{REF}$ behaves as a somewhat complicated function of temperature since the added exponential term to cancel the curvature of $V_{BE}$ has many higher order terms in itself. The exponential-compensated BGR shows more stable temperature behavior than the second-order compensated BGR. The TC of the exponential-compensated BGR is 1.45 ppm/°C and that of the second-order compensated BGR is 3.07 ppm/°C. These are theoretical limits of achievable TC's using these methods with given $\eta$ and $\Delta E_G$. The exponential compensation method delivers a temperature characteristic, which is twice as good as the second-order compensation method.

However, there is a problem in comparing the two methods. The second-order compensated BGR is not affected by $\Delta E_G$, so the temperature characteristic of the optimized second-order compensated BGR is constant over the variation of $\Delta E_G$. However, in case of the exponential-compensated BGR, the temperature characteristic of it is a function of $\Delta E_G$. Fig. 5 is the relationship of the TC of the optimized exponential-compensated BGR and $\Delta E_G$. From Fig. 5, though the temperature stability of the exponential-compensated BGR decreases as $\Delta E_G$ increases, the TC value is still smaller than the second-order-optimized compensated BGR whose TC is drawn as a straight line of 3.07 ppm/°C during the variation of $\Delta E_G$ from 10 to 100 meV, which is 10% of silicon bandgap.

From the relationship of TC and $\Delta E_G$, the smaller the doping level of emitter, the smaller TC of the exponential-compensated BGR. However, for a $\Delta E_G$ of less than 40 meV, it is impractical to compensate the curvature using the exponential compensation method, as the bias current required to generate the curvature compensation voltage is excessively large.

### III. IMPLEMENTATION OF BiCMOS BGR'S

A 1.5-µm BiCMOS process is used to implement the prototype circuit. The advantage of a BiCMOS process over a CMOS process is that it gives more degrees of freedom than the conventional CMOS process does. Designing a BGR with a BiCMOS process, a circuit designer does not need to bias the collector of a p-n-p (or n-p-n) transistor to the lowest (or highest) potential of the chip as required in CMOS process.

The circuits of Fig. 6 illustrate BiCMOS BGR's which are designed employing the exponential curvature compensation technique. These circuits are direct implementations of the conceptual circuits of Fig. 2. $Q_1$, $Q_2$, $R_1$, $MP1$ and $MP2$ compose a self-biased PTAT current generation circuit using Widlar current source; the emitter size ratio of $Q_1$ and $Q_2$ is 1:4. $R_{comp}$ is added to compensate the mismatch of base resistance between $Q_1$ and $Q_2$ [10]. Cascode transistors $MN1$, $MN2$, $MP3$ and $MP4$ are added to improve the PSRR (power supply rejection ratio) of this self-biased circuit. $MN4$ is a start-up transistor which is turned off during normal operation.

In the positive BGR of (a), $R_2$, $Q_3$, and $MP5$ are equivalents of $R$, $I_1$, and $I_2$ in Fig. 2, respectively. $R_2$ is trimmed externally to minimize the temperature drift of $V_{REF}$. The $W$ ratio of $MP5$ and $MP2$ are calculated from optimal $K_2$, $\Delta E_G$ and $\beta_{\infty}$ in (7). However, because of base currents of $Q_1$ and $Q_2$, the collector current of $Q_3$ and the drain current of $MP5$ are slightly different from the PTAT currents as we expected in the conceptual circuit of Fig. 2. Since the exponential compensation technique exploits the base current of the bipolar transistor, these small deviations should not be neglected. Putting the base currents into consideration, the collector current of $Q_3$ and the drain current of $MP5$ are expressed as $\beta/(\beta+1)c1T$ and $\beta/(\beta+1)c2T$, respectively.
So the voltage drop $V_{R2}$ at $R2$ is expressed as:

$$V_{R2} = c_1 RT + RT \left( \frac{c_2}{(\beta + 1)^2} - \frac{c_1}{\beta + 1} \right)$$

$$\approx c_1 RT + \frac{c_2 RT}{\beta} \left[ 1 - \left( \frac{c_2}{c_1} \right) \right]$$

(8)

This expression is slightly different from (7). From (7) and (8), the $W$ ratio of $MP5$ and $MP2$ is calculated, which is:

$$\frac{c_2}{c_1} = 1 + \left[ \left( \frac{K_2}{\beta} \right) \left( \frac{B_e}{R_1} \right) \ln \left( \frac{A_2}{A_1} \right) \right].$$

(9)

The emitter size ratio of $Q5$ and $Q3$ in the negative BGR of (b) is calculated using the same method as the positive BGR of (a). Letting $K_2 = 2.10 \times 10^{-6} \text{ V/K}$, $\beta_\infty = 1172$, $R2/R1 = 13$, and $A_2/A_1 = 4$, which is the emitter size ratio of $Q2$ and $Q1$, the ratio $c_2/c_1$ is calculated as 2.58. We used 3 instead of 2.58 as the optimum ratio since it is common in IC layout to set the emitter size ratio as the integer multiple of unit transistor.

To analyze the influence of $\beta_\infty$ and $\Delta E_G$ variation, a statistical simulation has been performed. Using Monte Carlo simulation method with jointly Gaussian random variables of $\beta_\infty$ and $\Delta E_G$ given in Section II-A, and with the emitter size ratio $c_2/c_1$ of 3, the simulated TC shows a distribution with a mean of 4.07 ppm/°C and standard deviation of 1.89 ppm/°C.

IV. EXPERIMENTAL RESULTS

Experimental prototypes of the exponential curvature-compensated BGR were fabricated using a 1.5-µm BiCMOS process with minimum emitter size of 3.5 × 3.5 µm$^2$. A 7.0 × 7.0 µm$^2$ n-p-n transistor, which is 4 times as large as the minimum transistor, was used as a unit transistor to increase matching properties. To adjust the linear compensation parameter $K_1$, $R2$ was trimmed externally to give a minimum temperature drift. To adjust the curvature compensation parameter $K_2$, an array of 5 negative BGR's with emitter size of $Q5$ from $x_1$ to $x_5$ of the emitter size of $Q3$ and 4 positive BGR's with gate width of $MP5$ from $x_2$ to $x_5$ of the gate width of $MP2$ was made since the trimming of an active device is impractical. Fig. 7 is the microphotograph of this test array.

From the measurement result, BGR's with $x_4$ sized transistors show the best performances both in the positive and in the negative cases. These results are not the same as the theoretical prediction in Section III that the BGR's with $x_3$ sized transistors would show the best performance. It is assumed to be that the nonideal factors not considered in the modeling procedure such as the temperature dependency of resistors [10] and the drift of $\beta$ from nominal values, cause this discrepancy.

Fig. 8 shows the microphotograph of the prototype BGR with $x_4$ sized curvature compensation bias transistor. Areas of the positive and the negative BGR's are 68 mil$^2$ and 63 mil$^2$, respectively. These are less than one tenth of typical BGR's [9], [10], [13], [14], [18]. The power consumption of this prototype BGR is about 370 µW with a 5-V single supply. This small area and low power consumption match the requirement of on-chip voltage reference well.

Average PSRR's are measured as 73.0 dB for positive BGR's, 66.2 dB for negative BGR's with a 5-V ±10% single supply. The PSRR is mainly limited by the Early effect of $Q3$. In case of the positive BGR, the collector voltage of $Q3$ is relatively stable to the $V_{CC}$ variation, since it is the base-emitter voltage of $Q4$. In case of the negative BGR, however, the supply variation affects the collector voltage of $Q3$ through the cascode transistor $MN5$. This is why the positive BGR shows superior PSRR characteristics to the negative BGR.

Output impedances are measured as 6 Ω for the positive BGR, and 295 Ω for the negative BGR. This high output impedance of the negative BGR is due to the fact that the
output current cause the voltage drop at R2 through Q4. Some bipolar BGR’s with relatively high output impedances have been reported [19], [20]. A simple buffer amplifier may be used to reduce the output impedance and to increase the driving capability.

The nominal voltages of positive and negative BGR’s are 1264.0 mV and 1275.6 mV respectively. Average TC’s of the negative BGR measured on 10 samples are 2.41 and 6.65 ppm/°C over the commercial (0 ~ 70°C) and military (−55 ~ 125°C) temperature ranges, respectively, and those of the positive BGR are 3.50 and 8.94 ppm/°C, respectively. The distribution of TC matches the prediction in section III with minute deviation.

The overall performance of the exponential curvature-compensated BiCMOS BGR is summarized in Table II.

V. CONCLUSION

A simple and efficient curvature compensation technique for a BGR and the prototype BiCMOS BGR family which employs this technique have been reported. The proposed exponential curvature compensation technique has advantages of superior temperature stability and no need for extra circuits to generate curvature compensation voltage source over the conventional one.

Positive and negative versions of the exponential curvature-compensated BGR have been designed and fabricated using 1.5-µm BiCMOS process. With a temperature range from −55 to 125°C and with a 5-V ± 10% single supply, these positive and negative BGR’s achieve average TC’s of 8.9 and 6.7 ppm/°C and average PSRR’s of 73 and 66 dB, respectively. These experimental prototypes dissipate only 0.37 mW, and occupy less than 70 mil².

With this excellent temperature stability, besides small size and low power consumption, the proposed BGR is suitable
for on-chip voltage references in high precision analog/digital mixed systems.

APPENDIX: ALGORITHM FOR OPTIMIZING $K_1$ AND $K_2$ OF THE CURVATURE-COMPENSATED BGR

From (4) and (7), the reference voltage $V_{\text{REF}}$ of the curvature-compensated BGR is expressed as:

$$V_{\text{REF}}(T) = V_G(T) + \left( \frac{T}{T_r} \right) [V_{\text{BE}}(T_r) - V_G(T_r)] - (\eta - 1) \left( \frac{kT}{q} \right) \ln \left( \frac{T}{T_r} \right) + K_1 T + K_2 H(T)$$  

(1)

where $H$ is a nonlinear term added to compensate the curvature of $V_{\text{BE}}$. In case of the second-order compensation and the exponential compensation, $H(T)$ is expressed as $T^2$ and $T \cdot \exp(\Delta E_G/kT)$, respectively.

Inserting (3) into (1) and rearranging it, $V_{\text{REF}}$ is expressed as the sum of linear and nonlinear terms of temperature, which is:

$$V_{\text{REF}}(T) = a + \left[ K_1 + \left( \frac{1}{T_r} \right) [V_{\text{BE}}(T_r) - V_G(T_r)] \right] - (\eta - 1) \left( \frac{kT}{q} \right) \ln(T_r) + b T^2$$

$$+ \left[ K_2 H(T) - (\eta - 1) \left( \frac{kT}{q} \right) \ln(T) - c T^2 \right]$$

(2)

where $b$ is the coefficient of the linear term. Now, we can approximate the third term of (2) with a straight line using least-square method, the result of which is:

$$K_2 H(T) - (\eta - 1) \left( \frac{kT}{q} \right) \ln(T) - c T^2 \approx m + n T$$  

(3)

where $m$ and $n$ are the fitting parameters determined by least-square method.

From (2) and (3), $V_{\text{REF}}^*$, which is the straight line approximation of $V_{\text{REF}}$, is:

$$V_{\text{REF}}^*(T) = (a + m) + (B + n) T$$  

(4)

If $B + n = 0$ in (4), the linear term disappears and the nominal reference voltage $V_{\text{REFN}}$ of the BGR becomes $a + m$. In this case, the optimal value of $K_1$ is calculated as:

$$K_1 = b + \left( \frac{1}{T_r} \right) [V_{\text{BE}}(T_r) - V_{\text{BE}}(T)] - (\eta - 1) \left( \frac{kT}{q} \right) \ln(T_r)$$  

(5)

and, the temperature drift voltage $\Delta V_{\text{REF}}$ is:

$$\Delta V_{\text{REF}}(T) = V_{\text{REF}}(T) - V_{\text{REF}}^*(T)$$

$$= K_2 H(T) - (\eta - 1) \left( \frac{kT}{q} \right) \ln(T) - c T^2 - m - n T.$$  

(6)

As we can see from (4) and (6), $V_{\text{REFN}}$ and $\Delta V_{\text{REF}}$ are calculated using $K_2$ only. So, if $K_2$ is given, the TC of the optimally curvature-compensated BGR is determined. Therefore, the optimization procedure is composed of these steps:

1) finding optimal $K_2$ which minimizes TC of the curvature-compensated BGR using exhaustive search.
2) calculate $K_1$ using (5).

Strictly speaking, the least-square method is not a method of finding a straight line minimizing the maximum difference between a curve and the line. Thus, the values of $K_1$ and $K_2$ calculated with this method may be slightly different from the optimum values which minimize the TC of the curvature-compensated BGR. In practical application, however, the difference is negligible.

ACKNOWLEDGMENT

The authors would like to thank Dr. Ook Kim of ETRI for his valuable suggestions and discussions, Dr. Joongsik Kih of Hyundai Electronics Co. for his support during the chip testing and measurement, and the staff members of Samsung Electronics Co. for fabricating the prototype circuits.

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