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An accurate measure of callable bond price sensitivity to interest rates and passage of time

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Abstract
This paper explores a general mathematical expression for approximating the callable bond return in terms of passage of time and yield change. The callable bond can be decomposed into the contributions from the effect of call-adjusted and the effect of passage of time. The study examines each component of the approximation of the callable bond return that is sensitive to changes in the passage of time and changes in market yield. Finally, based on a numerical example, comparisons are made with other approximation approaches based

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on effective duration and convexity. The results indicate that the proposed approximation measurement is closer to the theoretical measurement than the effective approximation when the market yield is greater than the coupon rate.

Keywords and phrases: Callable bond, duration, convexity, Taylor series expansion.

1. Introduction

To analyze the price and yield behavior of option-free bonds (or “plain vanilla” bonds), the conventional approach is to examine the convexity of a graph of price against yield over the duration of the bond, assuming that the expected cash flow does not change with interest rates. However, for bonds with embedded options, the conventional approach can be inappropriate as a measure of a bond’s price sensitivity to interest rates changes. This is because the expected cash flows change as interest rates change. In practice, effective duration is used to measure the responsiveness of a bond’s price, taking into account that the expected cash flow will change as a result of the embedded option.

For a callable bond, the holder has given the issuer the right to redeem the bond before its maturity date. In these circumstances, the cash flows of the bond are uncertain. To estimate the value of such a callable bond, the intuitive method is to treat it as a pure bond with an embedded short-call option, owned by the issuer, to repurchase the callable bond at a predetermined price and date. Using the effective duration and convexity to estimate the interest rate sensitivity of the callable bonds, it should be possible to value the embedded call option or estimate the callable bond directly (Sarkar (2001)). However, the estimation and valuation of the instrument are quite complex procedures.

The pioneering work of Dunetz and Mahoney (1988) provided an option premium approach to analyze the price-yield behavior of a callable bond by considering the callable bond as though it consisted of a non-callable bond and a call option. The measures of callable bond returns then determine the call-adjusted duration and convexity. Mehran and Homaifar (1993) extended the call-adjusted duration and convexity analysis to convertible bonds. They concluded that the traditional duration and convexity of option-free bonds provide an inadequate approximation to the duration and convexity of convertible bonds.
To employ bond portfolio strategies effectively, it is necessary to predict price volatility at the end of a pre-specified date. The time dimension is thus a significant element in determining the bond return. With a view to managing the risk of bond portfolios through ‘immunization’ strategies, Christensen and Sorensen (1994) introduced the concept of ‘time change’, which was defined as the bond price sensitivity with respect to the passage of time. Chance and Jordan (1996) showed that the effect of traditional convexity is far less important than that of the passage of time, even for one-day increments. Their findings revealed that accounting for the effect of passage of time usually produces a fairly accurate estimation of bond price change.

The conventional duration and convexity are proportional to the first and second derivatives of bond prices with respect to only bond yield changes. Both Dunetz and Mahoney (1988), and Mehran and Homaifar (1993) neglected to incorporate the effect of the passage of time into the price change for a callable and convertible bond. In contrast, the present study derives a model of callable bond return using a Taylor series expansion with respect to yield and passage of time. The model presented here stops with the second derivative. The first-order derivative terms include: (i) the call-adjusted duration; (ii) the passage of time; and (iii) the effects of their interaction. The second-order derivative terms include: (i) the call-adjusted convexity; and (ii) the second-order effect of passage of time. The present study verifies the proposed approximation measurement of the callable bond return and examines the relative influence of each term in the context of a wide range of market yields and passages of time.

The remainder of this paper is arranged as follows. The next section explores the proposed expression for the callable bond return. Numerical examples are then used to show that each element of the callable bond return generates a different impact on the effect of passage of time at different market yields. The study then compares the proposed model of callable bond return with other methods at different market yields. The paper concludes with a summary of the main findings.

2. Proposed model of a callable bond return

According to Chance and Jordan (1996), any analysis of bond returns can be improved by including the effect of passage of time. The callable bond equals the price of the non-callable bond minus the price of
the embedded option. To examine the behavior of callable bond returns during a period, the non-callable bond price and the bond’s return is defined as follows:

\[ P_t = \sum_{i=1}^{N} \frac{CF_i}{Y_t^{(i-t)}} \]  

(1)

in which:

\( P_t \) = the non-callable bond price at date \( t \);
\( N \) = number of years;
\( CF_i \) = cash flow of the non-callable bond at \( i \)th payment; and
\( Y_t \) = one plus the yield to maturity at date \( t \).

To measure the sensitivity of bond prices to the changes in yield and the passage of time, the return of the non-callable bond is defined as follows:

\[ R_t = \frac{P_{t+1} - P_t}{P_t} = \frac{\Delta P_t}{P_t} \]  

(2)

in which \( \Delta P_t \) is the change of bond prices through time \( t \).

Because the relationship between bond price and its yield is not linear, it is not possible to use only the duration to measure the percentage change of the yield change of the bond. Consequently, a second-order Taylor series expansion is described to approximate the change in bond prices. This is a function of the change in yield and passage of time, described as follows:

\[
\frac{\Delta P_t}{P_t} \approx \frac{\partial P_t}{\partial Y_t} \frac{1}{P_t} (\Delta Y_t) + \frac{\partial P_t}{\partial t} \frac{1}{P_t} (\Delta t) + \frac{\partial^2 P_t}{\partial Y_t \partial t} \frac{1}{P_t} (\Delta Y_t)(\Delta t) \\
+ \frac{1}{2} \frac{\partial^2 P_t}{\partial Y_t^2} \frac{1}{P_t} (\Delta Y_t)^2 + \frac{1}{2} \frac{\partial^2 P_t}{\partial t^2} \frac{1}{P_t} (\Delta t)^2 
\]  

(3)

in which \( \Delta t \) = the passage of time.

Thus, the second-order Taylor approximation of \( \Delta P_t \) is as follows:

\[
\Delta P_t \approx \frac{\partial P_t}{\partial Y_t} (\Delta Y_t) + \frac{\partial P_t}{\partial t} (\Delta t) + \frac{\partial^2 P_t}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) \\
+ \frac{1}{2} \frac{\partial^2 P_t}{\partial Y_t^2} (\Delta Y_t)^2 + \frac{1}{2} \frac{\partial^2 P_t}{\partial t^2} (\Delta t)^2. 
\]  

(4)

On the basis of Equation (4), \( \Delta P_{NCB} \) can be derived as follows:

\[
\Delta P_{NCB} \approx \frac{\partial P_{NCB}}{\partial Y_t} (\Delta Y_t) + \frac{\partial P_{NCB}}{\partial t} (\Delta t) + \frac{\partial^2 P_{NCB}}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) 
\]
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\[ + \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial Y_t^2} (\Delta Y_t)^2 + \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial t^2} (\Delta t)^2 \]  

(5)
in which \( P_{NCB} \) = the price of the non-callable bond; and

\[
\Delta P_O \cong \frac{\partial P_O}{\partial Y_t} (\Delta Y_t) + \frac{\partial P_O}{\partial t} (\Delta t) + \frac{1}{2} \frac{\partial^2 P_O}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t)
\]

\[ + \frac{1}{2} \frac{\partial^2 P_O}{\partial Y_t^2} (\Delta Y_t)^2 + \frac{1}{2} \frac{\partial^2 P_O}{\partial t^2} (\Delta t)^2 \]  

(6)
in which \( P_O \) = the price of the embedded option of the callable bond.

Accordingly, the callable bond equals the price of the non-callable bond minus the price of the embedded option that allows the issuer to call the bond when interest rates fall below the coupon rate of the bond. Furthermore, because the call option belongs to the issuers, it has a negative sign for the investor in the callable bond. That is:

\[ P_{CB} = P_{NCB} - P_O \]  

(7)
in which \( P_{CB} \) = the price of the callable bond.

Hence, the price percentage change of the callable bond can be expressed as follows:

\[ \frac{\Delta P_{CB}}{P_{CB}} = \frac{\Delta P_{NCB}}{P_{CB}} - \frac{\Delta P_O}{P_{CB}}. \]  

(8)

Therefore, by substituting Equations (5) and (6) into Equation (8), the Taylor approximation of \( \frac{\Delta P_{CB}}{P_{CB}} \) can be expressed as follows:

\[
\frac{\Delta P_{CB}}{P_{CB}} \cong \frac{1}{P_{CB}} \left( \frac{\partial P_{NCB}}{\partial Y_t} (\Delta Y_t) - \frac{\partial P_O}{\partial Y_t} (\Delta Y_t) \right)
\]

\[ + \frac{1}{P_{CB}} \left( \frac{\partial P_{NCB}}{\partial t} (\Delta t) - \frac{\partial P_O}{\partial t} (\Delta Y_t) \right) \]

\[ + \frac{1}{P_{CB}} \left( \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) - \frac{1}{2} \frac{\partial^2 P_O}{\partial Y_t^2} (\Delta Y_t)^2 \right) \]

\[ + \frac{1}{P_{CB}} \left( \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial t^2} (\Delta t)^2 - \frac{1}{2} \frac{\partial^2 P_O}{\partial t^2} (\Delta t)^2 \right). \]  

(9)

The following section illustrates the derivation of each effect from Equation (9) separately.
2.1 Call-adjusted duration effect

The first partial derivative of the non-callable bond price with respect to yield provides modified duration. It is the bond price elasticity, which is an approximation of the percentage change in bond price for a percentage change in yield:

\[
D_{NCB} = -\frac{\partial P_{NCB}}{\partial Y_t} = -\frac{1}{P_{NCB}} \left[ \sum_{i=1}^{N} \frac{(i-t)CF_i}{Y_{i-t}} \right]. \tag{10}
\]

From Equations (8) and (10), the callable bond price-yield relationship can be expressed as:

\[
\frac{\partial P_{CB}}{\partial Y_t} = \frac{\partial P_{NCB}}{\partial Y_t} - \frac{\partial P_{O}}{\partial Y_t}
\]

\[
= \frac{\partial P_{NCB}}{\partial Y_t} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) \tag{11}
\]

\[
D_{CB} \left( \Delta Y_t \right) = -\frac{\partial P_{CB}}{\partial Y_t} \left( \Delta Y_t \right)
\]

\[
= P_{NCB} \left[ \frac{\partial P_{NCB}}{P_{CB}} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) \right] \left( \Delta Y_t \right). \tag{12}
\]

\(D_{CB}\) is an approximation of the percentage change in the callable bond price response to a percentage change in yield. From Equation (12), the negative relationship between the yield change and the callable bond return can be established.

2.2 Effect of passage of time

An approximation of the percentage change in bond price in response to the passage of time is defined as (Chance and Jordan (1996), Eqn. (6)):

\[
\theta_{NCB} = \frac{\partial P_{NCB}}{P_{NCB}} = \frac{1}{P_{NCB}} \left[ \sum_{i=1}^{N} \frac{CF_i}{Y_{i-t}} \ln(Y_i) \right]
\]

\[
= \frac{1}{P_{NCB}} P_{NCB} \ln(Y_i) = \ln(Y_i). \tag{13}
\]
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From Equations (8) and (13), the callable bond price-time relationship can be written as:

\[
\frac{\partial P_{CB}}{\partial t} = \frac{\partial P_{NCB}}{\partial t} - \frac{\partial P_{O}}{\partial t} \\
= \frac{\partial P_{NCB}}{\partial t} - \frac{\partial P_{O}}{\partial t} \frac{\partial P_{NCB}}{\partial t} \\
= \frac{\partial P_{NCB}}{\partial t} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) \\
= -P_{NCB} \theta_{NCB} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right). \tag{14}
\]

It is then logical to define the effect of passage of time on a callable bond as follows:

\[
\theta_{CB}(\Delta t) = \left( \frac{\partial P_{CB}}{\partial t} \right) (\Delta t) \\
= \frac{P_{NCB}}{P_{CB}} \theta_{NCB} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) (\Delta t). \tag{15}
\]

\(\theta_{CB}\) is thus an approximation of the percentage change in callable bond price in response to the passage of time. There is therefore a positive relationship between the passage of time and the callable bond return.

2.3 Interaction effect

From the third term of Equation (9), the second derivative of the callable bond can be rearranged by yield and time separately as follows:

\[
\frac{\partial^2 P_{CB}}{\partial Yt \partial t} = \frac{\partial}{\partial t} \left( \frac{\partial P_{NCB}}{\partial Y} - \frac{\partial P_{O}}{\partial Y} \right) \\
= \frac{\partial}{\partial t} \left( \frac{\partial P_{NCB}}{\partial Y} - \frac{\partial P_{O}}{\partial P_{NCB}} \frac{\partial P_{NCB}}{\partial Y} \right) \\
= \frac{\partial^2 P_{NCB}}{\partial Y^2} - \frac{\partial^2 P_{O}}{\partial P_{NCB}^2} \frac{\partial P_{NCB}}{\partial Y} \frac{\partial P_{NCB}}{\partial t} - \frac{\partial^2 P_{NCB}}{\partial Y \partial t} \frac{\partial P_{O}}{\partial P_{NCB}} \frac{\partial P_{NCB}}{\partial t}. \tag{16}
\]

For simplicity, Equations (10) and (13) can be substituted into Equation (16) as follows:

\[
\frac{\partial^2 P_{CB}}{\partial Yt \partial t} = \frac{\partial^2 P_{NCB}}{\partial Y^2} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) + \frac{\partial^2 P_{O}}{\partial P_{NCB}^2} P_{NCB}^2 \theta_{NCB} D_{NCB}. \tag{17}
\]

Then, by substituting Equation (14) into the third term of Equation (9), the interaction derivative term of a Taylor expansion can be represented by only \(D_{NCB}\) and \(\theta_{NCB}\):

\[
\frac{\partial^2 P_{CB}}{\partial Yt \partial t} = \frac{\partial^2 P_{NCB}}{\partial Y^2} \left( 1 - \frac{\partial P_{O}}{\partial P_{NCB}} \right) + \frac{\partial^2 P_{O}}{\partial P_{NCB}^2} \theta_{NCB} D_{NCB}.
\]
\[
\frac{\partial^2 P_{CB}}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) = \frac{\partial^2 P_{NCB}}{\partial Y_t \partial t} \left( 1 - \frac{\partial P_O}{\partial P_{NCB}} \right) (\Delta Y_t)(\Delta t) \\
+ \frac{\partial^2 P_O}{\partial P_{NCB}^2} P_{NCB}^2 \theta_{NCB} D_{NCB} (\Delta Y_t)(\Delta t). \quad (18)
\]

The second partial derivative of non-callable bond price with respect to the yield and passage of time can be defined as:

\[
\frac{\partial^2 P_{NCB}}{\partial Y_t \partial t} = \frac{\partial}{\partial t} \left\{ - \frac{1}{Y_t} \sum_{i=1}^{N} \frac{[(i-t)C_i]}{(Y_t)^{i-t}} \right\} \\
= - \frac{1}{Y_t} \left\{ - \sum_{i=1}^{N} CF_i (Y_t)^{-(i-t)} + \sum_{i=1}^{N} (i-t)CF_i (Y_t)^{-(i-t)} \ln(Y_i) \right\} \\
= - \frac{1}{Y_t} \left\{ - P_{NCB} + \sum_{i=1}^{N} (i-t)CF_i (Y_t)^{-(i-t)} \frac{P_{NCB}}{P_{NCB}} \ln(Y_i) \right\} \\
= \frac{1}{Y_t} P_{NCB} - \frac{1}{Y_t} \sum_{i=1}^{N} (i-t)CF_i (Y_t)^{-(i-t)} \frac{P_{NCB}}{P_{NCB}} \ln(Y_i) \\
= \frac{1}{Y_t} P_{NCB} - D_{NCB} P_{NCB} \ln(Y_i) \\
= - P_{NCB} \left[ \theta_{NCB} D_{NCB} - \left( \frac{1}{Y_t} \right) \right]. \quad (19)
\]

Thus, Equation (18) can be rewritten as:

\[
\frac{\partial^2 P_{CB}}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) \\
= - P_{NCB} \left[ \theta_{NCB} D_{NCB} - \left( \frac{1}{Y_t} \right) \right] \left( 1 - \frac{\partial P_O}{\partial P_{NCB}} \right) (\Delta Y_t)(\Delta t) \\
+ \frac{\partial^2 P_O}{\partial P_{NCB}^2} P_{NCB}^2 \theta_{NCB} D_{NCB} (\Delta Y_t)(\Delta t). \quad (20)
\]

By dividing Equation (20) by \( P_{CB} \), the interaction effect on the callable bond return is defined as follows:

\[
\frac{\partial^2 P_{CB}}{\partial Y_t \partial t} (\Delta Y_t)(\Delta t) \\
= - P_{NCB} \left[ \frac{\theta_{NCB} D_{NCB} - \left( \frac{1}{Y_t} \right)}{P_{CB}} \right] \left( 1 - \frac{\partial P_O}{\partial P_{NCB}} \right) (\Delta Y_t)(\Delta t) \\
+ \frac{\partial^2 P_O}{\partial P_{NCB}^2} \frac{P_{NCB}^2}{P_{CB}} \theta_{NCB} D_{NCB} (\Delta Y_t)(\Delta t). \quad (21)
\]
2.4 Call-adjusted convexity effect

From the fourth term of Equation (9), the second-order Taylor approximations of \( \Delta P_{NCB} \) and \( \Delta P_O \) can be combined with the yield change. That is:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial Y_i^2} = \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial Y_i^2} (\Delta Y_i)^2 - \frac{1}{2} \frac{\partial^2 P_O}{\partial Y_i^2} (\Delta Y_i)^2. \tag{22}
\]

The derivation of call-adjusted convexity is then defined as follows:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial Y_i^2} = \frac{1}{2} \left\{ \frac{\partial^2 P_{NCB}}{\partial Y_i^2} \right\} - \left\{ \left( \frac{\partial^2 P_O}{\partial Y_i^2} \right)^2 + \left( \frac{\partial P_O}{\partial Y_i^2} \right)^2 \right\}, \tag{23}
\]

The definition of convexity of the non-callable bond is:

\[
C_{NCB} = \frac{\partial^2 P_{NCB}}{\partial Y_i^2} = \frac{1}{P_{NCB}} \frac{1}{Y_i^2} \sum_{i=1}^{\infty} \left[ \left( i - t \right)^2 + \left( i - t \right) CF_i \right]. \tag{24}
\]

From Equations (10) and (24), Equation (23) can be rearranged as:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial Y_i^2} = \frac{1}{2} \left\{ C_{NCB} P_{NCB} \right\} - \left\{ \left( \frac{\partial^2 P_O}{\partial Y_i^2} \right)^2 + \left( \frac{\partial P_O}{\partial Y_i^2} \right)^2 \right\} \tag{25}
\]

By dividing Equation (25) by \( P_{CB} \), the call-adjusted convexity is defined as follows:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial Y_i^2} (\Delta Y_i)^2 = \frac{1}{2} \left\{ \frac{P_{NCB}}{P_{CB}} C_{NCB} \left( 1 - \frac{\partial P_O}{\partial P_{NCB}} \right) - \left( \frac{\partial^2 P_O}{\partial P_{NCB}^2} \right) P_{NCB} D_{NCB}^2 \right\} (\Delta Y_i)^2. \tag{26}
\]

2.5 Second-order effect of passage of time

Finally, from the last term of Equation (9), the second-order Taylor approximations of \( \Delta P_{NCB} \) and \( \Delta P_O \) are combined with the passage of time again.

That is:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial t^2} = \left( \frac{1}{2} \frac{\partial^2 P_{NCB}}{\partial t^2} (\Delta t)^2 - \frac{1}{2} \frac{\partial^2 P_O}{\partial t^2} (\Delta t)^2 \right). \tag{27}
\]
So, the derivation of the second order of the effect of passage of time can be stated as:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial t^2} = \frac{1}{2} \left\{ \frac{\partial^2 P_{NCB}}{\partial t^2} - \left[ \left( \frac{\partial^2 P_O}{\partial P_{NCB}^2} \right) \left( \frac{\partial P_{NCB}}{\partial t} \right)^2 + \frac{\partial P_O}{\partial P_{NCB}} \frac{\partial^2 P_{NCB}}{\partial t^2} \right] \right\}. \tag{28}
\]

The second order of the effect of passage of time on the pure bond return can be reasonably defined as follows:

\[
\Phi_{NCB} = \frac{\partial^2 P_{NCB}}{\partial t^2} = \frac{1}{P_{NCB}} \frac{\partial}{\partial t} \left\{ \sum_{i=1}^{N} \frac{C_i}{Y_i} \ln(Y_i) \right\} \tag{29}
\]

From Equation (29), Equation (28) can be rearranged as follows:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial t^2} = \frac{1}{2} \left\{ \Phi_{NCB} P_{NCB} - \left[ \left( \frac{\partial^2 P_O}{\partial P_{NCB}^2} \right) P_{NCB}^2 \theta_{NCB}^2 + \frac{\partial P_O}{\partial P_{NCB}} \Phi_{NCB} P_{NCB} \right] \right\}. \tag{30}
\]

By dividing Equation (28) by \( P_{CB} \), the second-order effect of passage of time on the callable bond return is defined as follows:

\[
\frac{1}{2} \frac{\partial^2 P_{CB}}{\partial t^2} (\Delta t)^2 = \frac{1}{2} \left\{ P_{NCB} \left[ \Phi_{NCB} \left( 1 - \frac{\partial P_O}{\partial P_{NCB}} \right) - \left( \frac{\partial^2 P_O}{\partial P_{NCB}^2} \right) P_{NCB} \theta_{NCB}^2 \right] \right\} (\Delta t)^2. \tag{31}
\]

### 2.6 Generality of the proposed approximation

For the sake of simplicity, the model of Black (1976) is used to assume two important elements of an embedded bond option, which are defined as:

\[
\frac{\partial P_O}{\partial P_{NCB}} = \text{delta} = N(d_1) \tag{32}
\]

and

\[
\frac{\partial^2 P_O}{\partial P_{NCB}^2} = \text{Gamma} = \frac{e^{-d^2}}{P_{NCB} \sigma \sqrt{2\pi t}}, \tag{33}
\]

in which:
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\[ d_1 = \frac{\ln \left( \frac{P_{NCB}}{P_{CB}} \right) + \frac{1}{2} \sigma^2 t}{\sigma \sqrt{t}}, \text{ and } \sigma \text{ represents the volatility of } P_{NCB}. \]

Finally, by substituting Equations (32) and (33) into each effect described above, a measure of callable bond price sensitivity against interest rates and passage of time is obtained as follows:

\[
\frac{\Delta P_{CB}}{P_{CB}} \approx \frac{P_{NCB}}{P_{CB}} \left( -D_{NCB}(\Delta Y_t) + \theta_{NCB}(\Delta t) - \left( \theta_{NCB}D_{NCB} - \left( \frac{1}{Y_t} \right) \right) \right)
\times (1 - \text{Delta})(\Delta Y_t)(\Delta t) + \text{Gamma} \frac{P^2_{NCB}}{P_{CB}} \theta_{NCB} D_{NCB} \frac{(\Delta Y_t)}{(\Delta t)}
\frac{1}{2} \left( \frac{P_{NCB}}{P_{CB}} \left[ C_{NCB}(1 - \text{Delta}) - (\text{Gamma})P_{NCB}D^2_{NCB} \right] \right) (\Delta Y_t)^2
\frac{1}{2} \left( \frac{P_{NCB}}{P_{CB}} \left[ \Phi_{NCB}(1 - \text{Delta}) - (\text{Gamma})P_{NCB}\theta^2_{NCB} \right] \right) (\Delta t)^2. \quad (34)

From Equation (34), the return of the callable bond can be decomposed into five components (i) the call-adjusted duration effect; (ii) the effect of passage of time; (iii) the interaction effect; (iv) the call-adjusted convexity; and (v) the second-order effect of passage of time. Therefore, the callable bond price sensitivity to interest rates and passage of time can be estimated by the following variables: (i) duration; (ii) convexity; (iii) theta of pure bonds; and (iv) the delta and gamma of the options.

However, if the call-adjusted effect is ignored, then both the delta and gamma are equal to zero, and \( P_{CB} = P_{NCB} \). Equation (34) can be simplified to the following expression:

\[
\frac{\Delta P_{CB}}{P_{CB}} \approx -D_{NCB}(\Delta Y_t) + \theta_{NCB}(\Delta t) + P_{NCB}\theta_t
\frac{1}{2} C_{NCB}(\Delta Y_t)^2 + \frac{1}{2} \Phi_{NCB}(\Delta t)^2. \quad (35)
\]

In addition, if the effect of passage of time is ignored, then Equation (34) can be simplified to the following expression:

\[
\frac{\Delta P_{CB}}{P_{CB}} \approx - \frac{P_{NCB}}{P_{CB}} D_{NCB}(1 - \text{Delta})(\Delta Y_t) + \frac{1}{2} \left( \frac{C_{NCB}}{P_{CB}} \right) \left( \frac{P_{NCB}}{P_{CB}} \right) \left( \frac{D_{NCB}}{C_{NCB}} \right) (\Delta Y_t)^2. \quad (36)
\]
This is exactly the same as the result in Dunetz and Mahoney (1988), which is only a special case of Equation (34) with $\theta_{NCB} = 0$ and $\Delta t = 0$.

3. Numerical examples

3.1 Decomposition of the callable bond returns among different holding periods

Assume that a bond portfolio manager holds a callable (European-style) bond, with a 10-year, 5% coupon. This can be redeemed by the issuer in the third year at the $105 call price. To estimate the embedded option value using Black (1976), the risk-free rate of the non-callable bond price is assumed to float with the market yield (minus 100 basis points), and the volatility of the non-callable bond price is set at 5%. For hedging purposes, the bond portfolio manager might wish to estimate the callable bond return thirty days later. The model proposed in the present study can provide the bond manager with an accurate measure of callable bond price sensitivity to interest rates and passage of time.

Using the data described above, the prices for both the callable and non-callable bonds can be obtained. The price-yield relationship is illustrated in Figure 1, which shows that the price-yield relationship for the callable bond and the non-callable bond are quite different. This is because the issuer will benefit from calling the bond back as yields in the market decline, especially when the yield is less than the 5% coupon rate. In addition, the callable bond price in Figure 1 will approach $105$ (that is, the call price) when market yield converges to zero. This phenomenon is expressed in terms of the negative convexity. However, the callable bond is unlikely to be called back if the market yield is greater than the coupon rate. Therefore, the callable bond will have the same price and yield relationship as that of a pure bond. As noted above, the negative convexity of the callable bond in Figure 1 makes the estimated option price reasonable.

A numerical example can demonstrate the two main virtues of the approximation measurement proposed in the present study:

- identification of the characteristics of the decomposition of callable bond returns among different holding periods; and
- a better estimation of the callable bond returns than is achievable by the traditional approach.
Callable Bond Price Sensitivity

Figure 1
Price-yield relationship for a callable and a non-callable bond
(This figure assumes a 10-year, 5% coupon callable bond with a $105 call price and a 10-year, 5% coupon non-callable bond. The callable and non-callable bond prices are plotted against market yield. The embedded option is estimated by Black (1976). The callable bond price will approach $105 (that is, the call price) when market yield converges to zero. This phenomenon is expressed in terms of the negative convexity. The negative convexity of callable bond makes the estimated option price reasonable)

Table 1 shows five elements (obtained from Equations 12, 15, 21, 26 and 31) of the proposed approximation measurement among different levels of market yields when the holding period is set at 30 days. The second and third columns of Table 1 illustrate that the absolute values of call-adjusted duration and passage of time generally tend to increase (from near zero to 0.061 and 0.004 respectively as the market yield rises). It is apparent that the effect of passage of time has less influence than the call-adjusted duration on the callable bond return. The fourth column of Table 1 is the interaction item for the effects of call-adjusted duration and passage of time, which tends to have maximum volatility as the market yield stays around the 5% coupon rate. The fifth and sixth columns of Table 1 illustrate that the callable bond is irredeemable in the environment of higher market yields. This leads to a positive effect by the call-adjusted
convexity and the second-order passage of time on the callable bond return. Both of their influences are volatile when the market yield stays around the 5% coupon rate.

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Call-adjusted</th>
<th>Time passage</th>
<th>Interaction</th>
<th>Call-adjusted convexity</th>
<th>Second-order effect of passage of time</th>
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</table>

The 10-year, 5% coupon callable bond with a $105 call price is considered. This table shows five elements (obtained from Equations 12, 15, 21, 26, and 31) of the proposed approximation measurement among different levels of market yields when the holding period is set at 30 days.

To analyze the impact of the passage of time on each component of Equations (12), (15), (21), (26) and (31), it is necessary to examine the variations of each callable bond return component for a given holding period (30 days, 60 days, 180 days and 360 days). From Equation (12), the call-adjusted duration effect is estimated using the duration of the non-callable bond and then multiplying by one minus delta. Figure 2 shows that the impact of the call-adjusted duration effect is negative with respect to the callable bond return. In addition, it indicates a positive relationship between the call-adjusted duration effect and the market yield over different periods. When the market yield approximates to the 5%
coupon rate, the curvature of each curve in Figure 2 reaches maximum for different passages of time. In addition, it approaches the asymptotic value when the market yield moves to a higher level, and zero when the market yield moves to a lower level. The reason is that the delta of the embedded option approaches one or zero.

**Figure 2**

*Comparison of the call-adjusted duration effect with market yield at different holding period* (The 10-year, 5% coupon callable bond with a $105 call price is considered. To analyze the impact of the passage of time on each component of Equations (12), (15), (21), (26), and (31), This figure examine the variations of each callable bond return component for a given holding period (30 days, 60 days, 180 days, and 360 days). The solid line presents period 30 days; the (+) line presents period 60 days; the (◦) line presents period 180 days; the (square) line presents period 360 days)

However, the curves in Figure 2 show that the increment of holding periods dilutes the impact of the call-adjusted duration effect on the callable bond return. Furthermore, the dilution effect from the holding periods is stronger on the callable bond return when the market yield increases.

The second component of the callable return from the proposed approximation measurement is the effect of passage of time, which is the first derivative of the callable bond price with respect to time, multiplied by
the time change. The variations of the effect of passage of time can be examined for a 100-basis-point yield change at holding periods of 30 days, 60 days, 180 days, and 360 days. Figure 3 plots the positive relationship between the effect of passage of time and the market yield over these different holding periods.

As shown in Figure 3, when the market yield moves over the 5% coupon rate, the slope of each curve begins to increase for different holding periods. However, the effect of passage of time over different holding periods approaches zero because the part value of a callable bond (the value of the non-callable bond) vanishes, and the delta equals zero.
when the market yield moves beyond the coupon rate. Therefore, when
the market yield increases above the coupon rate, the curve in Figure 3
with the longer period will approach a larger asymptotic value because the
value of the embedded option vanishes (delta = 0) and the time change
(\Delta t) increases from Equation (15). The results in Figures 2 and 3 show
that the call-adjusted duration effect and the effect of passage of time have
opposite influences on the callable bond return.

![Figure 4](image-url)

**Figure 4**
Comparison of the interaction effect with market yield at different
holding period (The 10-year, 5% coupon callable bond with a $105 call price
is considered. This figure shows that the Interaction Effect (as shown in Equation
(21)) between call-adjusted convexity and market yield has a negative hump curve
over different holding periods. The solid line presents period 30 days; the (+) line
presents period 60 days; the (◦) line presents period 180 days; the (square) line
presents period 360 days)

The interaction effect (the call-adjusted effect and the time-passage
effect) on the callable bond return is the first derivative of the callable bond
with respect to both market yield and time change from Equation (21). The
formula for the interaction effect includes the delta, duration, theta, and
gamma. To understand the relationship between the time change and the
interaction effect, the variations of the interaction effect are examined at a
100-basis-point yield change for a given set of time changes (30 days, 60
days, 180 days and 360 days). Figure 4 plots the curve of the interaction
effect, which shows more fluctuation as holding periods increase. As shown in Figure 4, a hump shape occurs in each curve when market yields range from 5% to 9%. The explanation for this is the pattern of gamma of the embedded option (as will be shown in Figure 9). The results show that the interaction effect depends upon the gamma of the embedded option. A larger time change will make the gamma more sensitive, which makes the positive impact of the interaction effect on the callable return more important.

The second partial derivative of the callable bond return in a Taylor series expansion combines two terms—the call-adjusted convexity and the second-order time effect—as shown in Equations (26) and (31). Figure 5 shows that the relationship between call-adjusted convexity and market yield has a negative hump curve over different holding periods. The slope of each curve in Figure 5 increases as the holding period increases. When the market yield moves to a lower level, the call-adjusted convexity effect over different holding periods approaches zero since the value of the embedded option (delta = 1, gamma = 0) increases. When the market yield becomes greater than the coupon rate of 5%, the call-adjusted convexity effect on callable bond return begins to decrease over different periods. Furthermore, when the market yield goes far beyond the 5% coupon rate, the call-adjusted convexity effect tends towards an asymptotic zero value. The results in Figure 5 show that the effect of the call-adjusted convexity has an influence on the callable bond return for different holding periods. However, longer holding periods have little influence on the call-adjusted convexity effect in estimating the callable bond return.

The next term of the second partial derivative of the callable bond return in a Taylor series expansion is the second-order effect of passage of time. As shown in Figure 6, the relationship of this effect with market yield still has a negative hump curve over different holding periods. The slope of each curve in Figure 6 increases as the holding period increases. When the market yield moves above the 5% coupon rate, the slope of the curve becomes more volatile. A comparison of Figures 5 and 6 reveals that the callable bond return due to the second-order effect of passage of time is smaller than that of the call-adjusted convexity effect; however, both have the same influence on the callable bond return. Furthermore, the callable bond return is more sensitive to the second-order effect of passage of time than the call-adjusted convexity with different holding periods.
Comparison of the call-adjust convexity with market yield at different holding period (The 10-year, 5% coupon callable bond with a $105 call price is considered. This figure shows that the relationship between call-adjusted convexity and market yield has a negative hump curve over for a given holding period (30 days, 60 days, 180 days, and 360 days). The solid line presents period 30 days; the (+) line presents period 60 days; the (◦) line presents period 180 days; the (square) line presents period 360 days)

The results in Figure 6 illustrate that the second-order effect of passage of time cannot be ignored in estimating the callable bond return for different holding periods. In addition, longer holding periods have a stronger influence on the second-order effect of passage of time when estimating the callable bond return, which is different from the call-adjusted convexity effect.

3.2 Superiority of the proposed approximation measurement

The proposed approximation measurement captures the effects of the first and second partial derivatives of the callable bond return over different holding periods. To establish the superiority of the proposed approximation measurement, it is necessary to calculate the effective approximation with the duration and convexity and the theoretical measurement with the estimated callable bond price percentage change.
Comparison of the second-order time effect with market yield at different holding period (The 10-year, 5% coupon callable bond with a $105 call price is considered. The rtest term of the second partial derivative of the callable bond return in a Taylor series expansion is the second-order effect of passage of time. This figure shows that the relationship of this effect with market yield still has a negative hump curve over for a given holding period (30 days, 60 days, 180 days, and 360 days). The solid line presents period 30 days; the (+) line presents period 60 days; the (◦) line presents period 180 days; the (square) line presents period 360 days)

Table 2 displays three kinds of measurements for the callable bond return based on a 100 basis points ($\Delta y = 0.01$) increment of the market yield and zero time change ($\Delta t = 0$). The results in Table 2 show that the callable bond return measured by all methods declines as the market yield increases. The return of the proposed approximation is close to that of the effective approximation and the theoretical measurement only when the market yield rises above the coupon rate. However, when the market yield falls below the coupon rate (5%), smaller callable bond returns are obtained by all three methods, especially in the proposed approximation measurement. This result in Table 2 illustrates that a lower market yield causes the issuer to call back the callable bond. Therefore, the appreciation of the callable bond price is less than that of the non-callable bond.
Table 2

Three measurement instantaneous return versus yield with a 100-bp yield increment

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Proposed approximation measurement</th>
<th>Effective approximation</th>
<th>Theoretical measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-0.0001</td>
<td>-0.0289</td>
<td>-0.0303</td>
</tr>
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<td>0.02</td>
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<td>0.06</td>
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<td>-0.0390</td>
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<tr>
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<td>-0.0651</td>
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</tbody>
</table>

The 10-year, 5% coupon callable bond with a $105 call price is considered. The proposed approximation measurement captures the effects of the first and second partial derivatives of the callable bond return over different holding periods. To establish the superiority of the proposed approximation measurement, it is necessary to calculate the effective approximation with the duration and convexity and the theoretical measurement with the estimated callable bond price percentage change. This table displays three kinds of measurements for the callable bond return based on a 100 basis points (\(\Delta y = 0.01\)) increment of the market yield and zero time change (\(\Delta t = 0\)). The second column gives the estimated callable bond return by the new approximation in Equation (34); the third column gives the estimated callable bond return as measured by the effective duration plus convexity method; From a textbook of bond markets, the effective duration and convexity are defined as \(\frac{(P_{CB}−P_{CB})−(P_{CB}+P_{CB})}{2P_{CB}\Delta y}\) and \(\frac{(P_{CB}−P_{CB})−(2P_{CB})}{P_{CB}\Delta y^2}\). The fourth column gives the callable bond return from Equations (7) and (8). The embedded option is estimated by Black (1976)

Figure 7 compares the callable bond instantaneous return based on three measurements for a given set of market yields. When the market yield falls below the coupon rate, the callable bond return from the proposed approximation measurement is much smaller than that from the effective approximation and the theoretical measurement. In accordance with intuition, when the market yield falls well below the coupon rate, the value of the callable bond leaves only the embedded option. The callable bond return is thus relatively insensitive to the change in the market yield.

However, when the market yield rises above the coupon rate, the return from the proposed measurement approximates that of the theoretical
measurement. However, the effective approximation measurement gives smaller callable bond returns than the other measurements for higher market yields. Therefore, the proposed approximation measurement, with only the call-adjustment effect considered, could measure the callable bond return quite well (as shown in Figure 7).

![Figure 7](image)

**Figure 7**
Comparison of instantaneous return for three measurements (The 10-year, 5% coupon callable bond with a $105 call price is considered. This figure compares the callable bond instantaneous return based on three measurements for a given set of market yields. The solid line presents Proposed Approximation Measurement; the (o) line presents Effective Approximation the (square) line presents Theoretical Measurement)

The results in Table 3 and Figure 8 provide three measurements over thirty days ($\Delta t = 30$). The return from the proposed approximation measurement is very close to that from the theoretical measurement as market yield rises above the coupon rate of 5%. The effective approximation measurement gives smaller callable bond returns than the other methods for higher market yields. These results verify that the estimated callable bond return from the proposed approximation measurement, which includes both the call-adjusted effect and the effect of passage of time, is likely to be more consistent with that from the theoretical measurement as market yields rise above the coupon rate.
### Table 3
Three measurement return versus yield with a 100-bp yield increment over 30 days

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Proposed approximation measurement</th>
<th>Effective approximation</th>
<th>Theoretical measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
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The 10-year, 5% coupon callable bond with a $105 call price is considered. This table provides three measurements with a 100-bp yield increment over thirty days ($\Delta t = 30$).

However, as shown in Figures 7 and 8, when market yields move below the coupon rate, the callable bond return for the proposed approximation measurement is close to zero and different from those of other measurements.

The question then arises as to why the callable bond return from the proposed approximation measurement is smaller than that of other measurements when the market yield falls below the coupon rate. The explanation is that it includes the delta of the embedded option, which is defined as the change of the embedded option with respect to the price of the non-callable bond. The results in Table 4 decompose the embedded option of the callable bond to the delta, gamma, and option price with different market yields. When market yields in Figure 9 fall below the coupon rate, the delta is very close to one. Therefore, the price of a callable bond approximates that of a non-callable bond, which makes the return from the proposed approximation measurement near zero. In contrast, when the market yield in Figure 9 rises above the coupon rate, the delta approaches zero, which creates a large change in the callable bond price.
Comparison of return for three measurements over 30 days (The 10-year, 5% coupon callable bond with a $105 call price is considered. This figure plots each shape of three measurements with a 100-bp yield increment over thirty days ($\Delta t = 30$). The solid line presents Proposed Approximation Measurement; the dashed line presents Effective Approximation; the dotted line presents Theoretical Measurement.)

Therefore, the delta makes return from the proposed approximation measurement different from those of other measurements with lower market yield, but close to that of the theoretical measurement with higher market yield.

In addition, both Table 4 (column 3) and Figure 9 show that gamma tends to rise when the market yield is around the coupon rate of 5%. In this example, the large gamma implies that the delta is very sensitive to the price of the non-callable bond when the market yield stays around the coupon rate.

4. Summary and conclusions

The contribution of this paper is to propose a new approximation for the callable bond return that relates the call-adjusted effect and the effect of the passage of time on the first and second partial derivatives of the callable bond return with respect to yield and time. The study has
### Table 4

Components of the embedded option by different market yields

<table>
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<th>Yield to maturity</th>
<th>Delta</th>
<th>Gamma</th>
<th>Option price</th>
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<tr>
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</tr>
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In an explanation of Figure 7 and 8, two components of Equation (34) in this table, delta and gamma, are considered based on the same 10-year, 5% coupon callable bond with a $105 call price. The delta, gamma, and option prices are estimated by Black (1976) for fifteen different market yields demonstrated that the methods in Chance and Jordan (1996), and Dunetz and Mahoney (1988) are only special cases of the new proposed general approximation measurement.

Numerical results show that the callable bond return from using the proposed approximation measurement with respect to yield and time change is less than that from using the effective approximation and theoretical measurement when the market yield falls near zero. It would seem that this difference among the measurements of the callable bond return is due to the call-adjusted duration and convexity in the proposed measurement, which include the delta and gamma items. This excludes other factors that might affect the price of the embedded option, and considers only the market the holding period. Numerical results also show yield and that the callable bond return with the call-adjusted effect and the effect of passage of time is close to the callable bond return from the theoretical measurement because the value of the embedded option wanes as the market yield moves to a significantly higher level than the coupon rate.
Delta and gamma of the embedded option with different market yields

(This figure plots the delta and gamma of the embedded option estimated by Black (1976) from Table 4 for fifteen different market yields. The table shows that gamma tends to rise when the market yield is around the coupon rate of 5%, and a large gamma implies that the delta is highly sensitive to the price of the non-callable bond when the market yield stays around the coupon rate. The solid line presents Delta; the (◦) line presents Gamma)

Thus, both the delta and the gamma from the embedded option are not significant influences on the callable bond return as the market yield moves well above the coupon rate. The study has shown that the proposed approximation measurement leads to less estimation error (the difference between the return by the proposed approximation and the theoretical measurement) than the effect approximation when the market yield rises above the coupon rate.

The measurement of the call-adjusted effect and the effect of passage of time for the first derivative of callable bonds can decompose the call-adjusted duration, the time-passage effect, and the interaction effect. Given a passage of time from 30 days to 360 days, the passage-of-time effect and the interaction effect have positive influences on the callable bond return, which differs from the call-adjusted duration effect. Therefore, the analysis of the callable bond return could not depend only
on the modified duration multiplied by the yield change. When the time changes considerably, the positive time-passage and interaction effects are more significant.

Chance and Jordan (1996) noted that the effect of the convexity is not obviously greater than the effect of the interaction effect between the time and yield changes for non-callable bonds. Furthermore, the present study has found that the interaction effect has a larger influence on the callable bond return than the call-adjusted convexity and second time-passage effects in that the market yield approximates to the coupon rate when the holding period changes significantly. Moreover, the impact of the interaction effect has an opposite influence to that of the call-adjusted convexity and second time-passage effects on callable bonds.

References


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