Behavior and design of slender circular tubed-reinforced-concrete columns subjected to eccentric compression

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ABSTRACT

The tubed-reinforced-concrete (TRC) column is a relatively new kind of confined reinforced-concrete (RC) columns, where the outer enclosing thin-walled steel tube is discontinued at the beam–column joint and thus the axial load is transferred to the RC core only. In this paper, the behavior of slender circular TRC columns under eccentric compression loads was studied. A total of sixteen specimens considering the following primary system parameters were tested: two slenderness ratios (24, 40), two load eccentricities (25 mm, 50 mm), two diameter-to-thickness ratios of the steel tube (133, 160), and two continuity conditions for the steel tube (continuous, discontinuous at mid-height). The test results indicate that the slender circular TRC specimens exhibit fairly ductile behavior and the discontinuity of the steel tube at mid-height has a small effect on the bearing capacity. A finite element (FE) model was developed to simulate the behavior of circular TRC columns under eccentric compression loads. The predicted load versus mid-span lateral displacement curves are generally in good agreements with the measured ones. To identify the influence of the key parameters on the second order effects of circular TRC columns, an extensive parametric study was carried out using the FE model. Lastly, a regression formula is suggested to estimate the moment magnification factor and simplified design equations for slender circular TRC columns under eccentric compression loads are proposed based on the section capacity analysis.

ARTICLE INFO

Article history:
Received 26 November 2015
Revised 19 May 2016
Accepted 23 May 2016
Available online 20 June 2016

Keywords:
Circular tubed-reinforced-concrete (TRC)
Slender column
Eccentric compression
Finite element
Moment magnification factor

1. Introduction

The tubed-reinforced-concrete (TRC) column is a special reinforced-concrete (RC) column where the closely arranged stirrups are replaced by an outer enclosing thin-walled steel tube with only a few stirrups, as shown in Fig. 1. Note that the steel tube is made slightly shorter than the column and thus not passing through the beam–column joints to ensure that the axial load is transferred to the RC core only. Contrasting to the traditional RC columns (Fig. 2a and b), the steel tube in TRC columns serves two purposes: (1) Protect the concrete cover from spalling off during an earthquake event; and (2) enhance the load-carrying capacity, deformability, and seismic performance of RC columns [1,2]. In addition, the TRC columns possess higher construction efficiency than RC columns as less transverse reinforcement is needed and the steel tube can serve as the permanent formwork for concrete pouring. Compared to the traditional hollow composite steel members such as concrete filled steel tube (CFT) columns (Fig. 2b and c), the steel tube in TRC columns does not carry a direct axial load. As such, the confinement effect is maximized and the potential of local buckling in the steel tube is minimized [3–7]. Moreover, the TRC columns exhibit a better fire-resistance performance than CFT columns since the majority of reinforcing steel is embedded in the concrete [8–11].

Tomii et al. [12,13] first investigated the TRC columns in buildings to improve the shear strength and ductility of short RC columns. Priestley et al. [14,15] conducted theoretical and experimental investigations to study the effectiveness of steel jackets for retrofitting and shear strength enhancement of RC bridge columns. Sun et al. [16–18] examined the earthquake-resisting performance of square TRC columns. In their studies, the effects of the wall thickness of steel tubes and the shear span ratio of column on the seismic behavior of square TRC columns were discussed. Aboutaha et al. [19,20] experimentally investigated the cyclic response of tubed high-strength RC columns and concluded that TRC columns were more ductile and had better seismic performance than ordinary RC columns. Han et al. [21] conducted a test on thin-walled steel tube confined concrete column to RC beam joints subjected to cyclic loading and showed the good seismic...
performance. Zhang and Liu et al. [22–24] carried out a series of experimental and theoretical studies to investigate the axial strength and seismic behavior of circular/square TRC columns and proposed the corresponding design methods. Zhou et al. [25] tested six short TRC columns and two ordinary short RC columns subjected to a constant axial compression load combined with lateral cyclic loading. They concluded that the confinement from the steel tube could effectively improve the brittle shear failure mode of short RC columns. Liu et al. [26,27] developed a nonlinear three-dimensional finite element model to simulate the behavior of TRC columns and proposed the corresponding design methods. The TRC column and beam–column connection. Fig. 1. The TRC column and beam–column connection.

2. The experimental study

2.1. Specimens

A total of 16 slender circular TRC columns were prepared and tested in this study. The investigated parameters of the specimens

\[ r \] radius of the specimen cross section

\[ r_b \] radius of reinforcing bar circle

\[ \tau \] wall thickness of steel tube

\[ u \] lateral displacement along the column height

\[ z \] strength reduction factor in stress block method

\[ a_b \] longitudinal reinforcement to concrete area ratio

\[ a_t \] steel tube to concrete area ratio

\[ \delta \] mid-span lateral displacement of the specimen

\[ \delta_m \] mid-span lateral displacement corresponding to the peak axial load

\[ \varepsilon \] tensile strain of concrete corresponding to \( f_{ct} \)

\[ \varepsilon_{co} \] strain of concrete corresponding to \( f_{co} \)

\[ \varepsilon_{cc} \] strain of confined concrete corresponding to \( f_{cc} \)

\[ \varepsilon_{ut} \] ultimate tensile strain of concrete

\[ \phi \] angle between the horizontal line and the line connecting the centroid of the section and the centroid of the \( i \)th longitudinal reinforcement bar

\[ \lambda \] slenderness ratio of the specimen

\[ \theta \] one-half of the angle subtended at the center of the cross section by concrete compression stress block

\[ \sigma_{bi} \] stress of the \( i \)th longitudinal reinforcement bar in the critical state

\[ \sigma_{h} \] circumferential stress of steel tube

\[ \sigma_{y} \] longitudinal stress of steel tube

\[ \sigma_2 \] equivalent stress of steel tube

\[ \xi \] the confinement factor = \( \frac{A_{fc}}{A_{st}} \)

2.1. Specimens

A total of 16 slender circular TRC columns were prepared and tested in this study. The investigated parameters of the specimens

\[ A_b \] cross-sectional area of longitudinal reinforcing bars

\[ A_{bi} \] cross-sectional area of the \( i \)th longitudinal reinforcing bar

\[ A_c \] cross-sectional area of concrete

\[ A_t \] cross-sectional area of steel tube

\[ D \] diameter of the cross-section

\[ e \] load eccentricity

\[ E_c \] elastic modulus of concrete

\[ f_{cu,100} \] 100 mm cube compressive strength of concrete

\[ f_{cc} \] compressive strength of the confined concrete

\[ f_{ty} \] yield strength of steel tube

\[ f_{by} \] yield strength of longitudinal reinforcing bar

\[ f_y \] yield strength of stirrup

\[ f_i \] effective lateral confining stress

\[ f_{bol/fo} \] the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress defined in ABAQUUS

\[ K \] initial stiffness

\[ K_f \] the ratio of the second stress invariant on the tensile meridian defined in ABAQUUS

\[ L \] length of the specimen

\[ n_b \] number of longitudinal reinforcements

\[ N_t \] nominal axial load-carry capacity

\[ M_b \] nominal bending moment

\[ P_a \] axial peak load

\[ \delta \] mid-span lateral displacement of the specimen

\[ \delta_m \] mid-span lateral displacement corresponding to the peak axial load

\[ \varepsilon \] tensile strain of concrete corresponding to \( f_{ct} \)

\[ \varepsilon_{co} \] strain of concrete corresponding to \( f_{co} \)

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\[ \sigma_2 \] equivalent stress of steel tube

\[ \xi \] the confinement factor = \( \frac{A_{fc}}{A_{st}} \)
include two slenderness ratios (24, 40), two load eccentricities (25 mm, 50 mm), two diameter-to-thickness ratios of steel tubes (133, 160), and two continuity conditions for the steel tube (continuous, discontinuous at mid-height). In order to study the effect of continuity of the steel tube at critical bearing capacity sections, the specimens were separated into two groups: Type A and Type B. In Type A specimens, the steel tube is disconnected at the specimen’s both ends, while in Type B specimens, the steel tube is disconnected at the mid-height additionally, as shown in Fig. 3. Each test specimen consists of three basic parts: outer steel tube, reinforcement, and concrete. The steel tube was cold-formed by rolling the steel plate and the line of butt weld joint was reinforced by a 30 mm width steel plate to prevent a premature weld failure. Six longitudinal reinforcing bars with a diameter of 20 mm were symmetrically arranged in the steel tube with a separation of 15 mm between the bar’s outer perimeter and the tube’s inner face. Stirrups with a diameter of 8 mm were placed at 200 mm intervals to erect the longitudinal bars. All specimens were cast from the same batch of concrete and the layered concrete pouring process was adopted to ensure more uniform and compact concrete. Besides, the 10 mm thick enlarged end plate was welded at each end of a test specimen. For Type A specimens, two 10 mm wide girth strips were cut off from the steel tube at 30 mm away from the end plates. For Type B specimens, the additional strip was cut off at the mid-height. Details of the specimens are shown in Table 1, where $e$ is the load eccentricity, $L$ is the length of the specimen, $D$ is the diameter of the cross-section, $\zeta$ is the confinement factor $= \frac{A_{ct}}{A_{c}}$ (implying the degree of confinement), $\lambda$ is the slenderness ratio $= \frac{4L}{D}$ for circular specimens, $t$ is the wall-thickness of steel tubes, $\alpha_s$ is the steel tube to concrete area ratio ($A_{st}/A_{c}$), $\alpha_{r}$ is the longitudinal reinforcements to concrete area ratio ($A_{r}/A_{c}$), $f_{ty}$, $f_{by}$, and $f_{sy}$ are respectively the yield strength of steel tube, longitudinal reinforcement, and stirrup, $f_{c100}$, $f_{co}$, and $E_c$ are respectively the average 100 mm cube strength, prism strength, and elastic modulus of the concrete (obtained by testing the concrete cubes and prisms prepared and cured under the same condition as the specimens). Using the designation “C200-6-25-a” as an example, “C200” denotes a circular TRC column with a diameter of 200 mm, “6” means that the length-to-diameter ratio is 6, “25” implies that the load eccentricity is 25 mm, and the last letter “a” indicates a Type A specimen.

2.2. Test setup and instrumentations

The columns were tested under monotonically increasing axial compression loads using a hydraulic testing machine at the Structural and Seismic Test Research Centre, Harbin Institute of
An adjustable knife-edge articulation system was used to provide the required end eccentricities and pin supports (Fig. 4). Five linear variable displacement transducers (LVDTs) were applied to measure the lateral displacements of the specimens and four additional LVDTs to monitor the overall axial displacement (Fig. 4). To record the strains in the steel tube, four pairs of orthogonally arranged longitudinal and transverse strain gauges were symmetrically installed on the outer surface of steel tube at mid-height (Fig. 4).

### 2.3. Failure mode

Fig. 5 shows the typical failure modes of the specimens. For Type A specimens, the deformation curvature increases slowly and linearly with the loading at the initial stage. The first local buckling due to the compressive loading is observed near the tube’s mid-height when the load approaches to 75–80% of the peak value. After the peak load, more local buckling appears near the mid-height section and the distance between two adjacent buckling locations is approximately equal to be the diameter of the cross-section. The concrete is crashed in the region where severe tube buckling occurs and smeared flexural cracks are observed on the opposite side. For Type B specimens, no obvious local buckling in the steel tube is seen prior to the peak load. The damage of concrete occurs mainly at the mid-height where the gap/disconnection is located. The concrete is crashed in the compression region and a significant crack accompanied with several small cracks appear in the tension region. The steel tube in the tension region is slightly separated from the concrete at mid-height. Similar failure modes were also observed for specimens with different length-to-diameter and diameter-to-thickness ratios of the steel tube. The typical lateral deformed shapes of the specimens are in good agreement with the half sine waves, as shown in Fig. 6. The curvature at mid-height of Type B specimens is slightly greater than that of Type A specimens.

### 2.4. Load versus mid-span lateral displacement curves

The relationship between axial load and mid-span lateral displacement for the test specimens is shown in Fig. 7. It can be seen...
that all test specimens exhibit good ductile behavior during the loading. The stiffness and axial bearing capacity of the specimens decrease with the increasing eccentricity and slenderness ratio. When the load reaches about 80% of the peak value, the steel tube in the compression region (Test Point 1, Fig. 4) begins to yield (denoted as the tube yield point in Fig. 7), except Specimen C200-10-25-b in which the steel tube yields beyond the peak load. Primary mechanical properties calculated based on the relationship...
curves (Fig. 7) are listed in Table 2, in which $K$ is the initial stiffness and $P_u$ and $d_m$ are the peak load and the corresponding mid-span displacement, respectively. Most of Type A specimens exhibit slightly higher ($\approx 5\%$ on average) axial load-carrying capacity and greater initial stiffness ($\approx 16\%$ on average) than Type B specimens.

2.5. Analysis of stresses in the steel tube

During the loading, the longitudinal and transverse strains in the steel tube were recorded at the mid-height. The stress state of each test point was calculated based on the elastic–plastic analysis method [33] and the measured strains. The steel tube was assumed to be an elastic–perfectly plastic material in the analysis. Fig. 8 shows the typical analysis results for the specimens and each single graph includes a Type A specimen and a corresponding Type B specimen (shown in the graph title), where $\sigma_{av}$, $\sigma_{ah}$, and $\sigma_{az}$ are respectively the longitudinal stress, transverse stress, and equivalent stress of the steel tube for Type A specimens and $\sigma_{bh}$ is the transverse stress for Type B specimens. The equivalent stress can be determined by

$$\sigma_{eq} = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{av} - \sigma_{ah})^2 + \sigma_{av}^2 + \sigma_{ah}^2}$$  \hspace{1cm} (1)

![Fig. 7. Axial load versus mid-span lateral displacement curves.](image)

Table 2

<table>
<thead>
<tr>
<th>Specimens</th>
<th>$K$ (kN/mm)</th>
<th>$P_u$ (kN)</th>
<th>$d_m$ (mm)</th>
<th>Test Point 1</th>
<th>Average of Test Point 2 and 4</th>
<th>Test Point 3</th>
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<tr>
<td></td>
<td>$\sigma_{av}$</td>
<td>$\sigma_{ah}$</td>
<td>$\sigma_{az}$</td>
<td>$\sigma_{av}$</td>
<td>$\sigma_{ah}$</td>
<td>$\sigma_{az}$</td>
</tr>
<tr>
<td>C200-6-25-a</td>
<td>416</td>
<td>1926</td>
<td>13.2</td>
<td>-0.75</td>
<td>0.38</td>
<td>1.00</td>
</tr>
<tr>
<td>C200-6-25-b</td>
<td>246</td>
<td>1758</td>
<td>16.1</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C200-6-50-b</td>
<td>140</td>
<td>1238</td>
<td>17.1</td>
<td>-0.18</td>
<td>0.90</td>
<td>1.00</td>
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<tr>
<td>C200-10-25-a</td>
<td>152</td>
<td>1602</td>
<td>21.7</td>
<td>-0.57</td>
<td>0.48</td>
<td>1.00</td>
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<tr>
<td>C200-10-50-a</td>
<td>119</td>
<td>1520</td>
<td>23.0</td>
<td>0.00</td>
<td>0.80</td>
<td>0.80</td>
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<tr>
<td>C200-10-50-b</td>
<td>68</td>
<td>1121</td>
<td>28.9</td>
<td>-0.60</td>
<td>0.55</td>
<td>1.00</td>
</tr>
<tr>
<td>C200-10-50-b</td>
<td>68</td>
<td>1136</td>
<td>26.5</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C240-6-25-a</td>
<td>421</td>
<td>2518</td>
<td>13.1</td>
<td>-0.54</td>
<td>0.61</td>
<td>1.00</td>
</tr>
<tr>
<td>C240-6-25-b</td>
<td>418</td>
<td>2464</td>
<td>12.8</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C240-6-50-a</td>
<td>332</td>
<td>1957</td>
<td>15.1</td>
<td>-0.57</td>
<td>0.59</td>
<td>1.00</td>
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<tr>
<td>C240-6-50-b</td>
<td>225</td>
<td>1872</td>
<td>18.6</td>
<td>0.00</td>
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<td>1.00</td>
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<tr>
<td>C240-10-25-a</td>
<td>207</td>
<td>2219</td>
<td>25.4</td>
<td>-0.83</td>
<td>0.28</td>
<td>1.00</td>
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<tr>
<td>C240-10-25-b</td>
<td>235</td>
<td>2212</td>
<td>24.0</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C240-10-50-a</td>
<td>156</td>
<td>1909</td>
<td>25.9</td>
<td>-0.54</td>
<td>0.14</td>
<td>0.60</td>
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<tr>
<td>C240-10-50-b</td>
<td>118</td>
<td>1687</td>
<td>29.5</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: The forward slash \ means invalid data.
For Type B specimens, the longitudinal strains are quite small compared to the transverse strains as the steel tube at mid-height is disconnected. Thus, the longitudinal stress may be assumed to be zero and only the transverse stress ($\sigma_{bh}$) is considered for Type B specimens. As such, the equivalent stress $\sigma_{bz}$ is equal to the transverse stress $\sigma_{bh}$ according to Eq. (1). The yield point in Fig. 8 indicates that Test Point 1 of the steel tube (Fig. 4) begins to yield, i.e., yielding when the equivalent stress equals to the yield strength of the tube.

Although no load is applied on the steel tube directly, the longitudinal stress at the mid-height section of Type A specimens increases rapidly during the initial loading stage due to the bond and frictional forces between the steel tube and the concrete. As the columns are loaded eccentrically, the longitudinal stress forms an obvious stress distribution around the tube section: compressive stress on the compression side and tensile stress on the tension side. The longitudinal stress of the steel tube in Type B specimens is zero at the mid-height, thus the bending resistance of Type B specimens is slightly smaller than that of Type A specimens. The transverse stresses of Type A specimens maintain at a relatively low level, but increase rapidly when the concrete begins to dilate plastically near or at the peak load. Differently from Type A specimens, the transverse tensile stress of Type B specimen starts increasing at the first stage of loading as the steel tube at the mid-height is fully utilized to confine the RC core.

The stresses at the peak load point in each specimen are summarized in Table 2 with positive values denoting tensile stresses and negative values denoting compressive stresses. The transverse stress (indicating the magnitude of confinement) of each test point is not uniform and the stress on the compression side is larger than that on the tension side. Furthermore, the transverse stress of Type A specimens on the compression side tends to decrease with the increasing length-to-diameter ratio: the average value of Test Point 1 is 0.62$\sigma_y$ when the length-to-diameter ratio is 6 and it is 0.43$\sigma_y$ when the length-to-diameter ratio is 10. For Type B specimens, the transverse stress is close to the yield strength.

3. Finite element (FE) analysis

A three-dimensional (3D) nonlinear finite element model was developed using the program ABAQUS [34] to investigate the behavior of slender circular TRC columns under eccentric compression loads.

3.1. Materials

Both the steel tube and the reinforcing bars are assumed to be the elastic perfectly-plastic material in the FE model and the stress–strain relationship is given as

$$
\sigma = \begin{cases} 
E_\varepsilon & \varepsilon < \varepsilon_y \\
\sigma_y & \varepsilon \geq \varepsilon_y 
\end{cases}
$$

Fig. 8. Typical load versus steel tube stresses curves.

Fig. 9. FE model of circular TRC column (1/2 model).
Fig. 10. Comparisons of load versus mid-span lateral displacement curves among the predicted, published, and test results.
where $E_c$ and $f_y$ are respectively the elastic modulus and yield strength of the steel tube (or reinforcing bars) and $\epsilon_y$ is the yield strain $= f_y/E_c$.

The Concrete Damaged Plasticity Model (CDPM) [35] provided in ABAQUS was adopted in the analysis. This model implicitly includes the effects of hydrostatic pressure and the third deviatoric stress invariant on the strength of concrete. While the hardening-/softening rule, the flow rule, and the damage variable in this model are not confinement-dependent, thus the model does not perform satisfactorily in simulating the elastic stiffness reduction and strain softening behavior of the confined concrete [36–38]. A simple and practical method to consider the plastic behavior of confined concrete is to modify the equivalent stress–strain relationship in the CDPM [39–42]. In this study, the uniaxial constitutive relationship proposed by Popovics [43] was employed as the equivalent stress–strain relationship of concrete confined by the steel tube, and the plastic properties in CDPM were taken as $0.3$ and $0.4$ N/mm$^2$, respectively. The deformation of longitudinal reinforcing bars, and the concrete, respectively. The interaction behavior between the steel tube and the concrete was simulated by the contact interaction algorithm called “Surface-to-Surface Contact”, in which the normal behavior is defined as “Hard Contact” allowing separation after the contact and the tangential behavior is defined as “Penalty Model” with the friction coefficient and bond strength taken as 0.3 and 0.4 N/mm$^2$, respectively. The deformation of longitudinal reinforcing bars is constrained by the concrete using the “Embedded Element Technique”. In order to reduce the computational work, a 1/2 column model using symmetry boundary condition was adopted, as shown in Fig. 9. The end plates were simulated by two rigid body constraints which are respectively tied to the top surface and the bottom surface of the concrete core. The distance between the reference point of the rigid body and the center of the section is equal to the load eccentricity. The pinned boundary conditions are assigned at the reference points and the compression load was applied to the reference point of the top end plate in form of displacements.

3.3. Validation of the FE model

The FE model was validated against the test results reported in this paper and the published results [28], as shown in Fig. 10. The load versus mid-span lateral displacement curves predicted by the FE model generally agree well with the experimental and published results. The following two notes should be made.

1. The tested peak load of specimen C240–10–50-a (Fig. 10d) is found to be significantly higher (23%) than that predicted by the FE model due to the abnormal test behavior found in this particular specimen. This single irregularity may be caused by some uncertain reasons such as the increase in concrete strength with age, experimental errors, etc.

2. Except for specimen C240–10–50-a, the mean value and standard deviation of the peak loads predicted by the FE model to the test result ratio are 0.94 and 0.06, respectively. Overall, the predicted results are smaller than the test results. As discussed in Section 4 below, the smaller axial peak load generally leads to a larger moment magnification factor, resulting in a conservative practical design.

4. Suggestions for practical design

It is generally regarded that the nonlinear second-order analysis like the FE method described above is an exact design approach for slender columns. To simplify the computation, most of the current design codes for RC columns [45–47] approximate the second-order effects by amplifying the first-order moments in critical sections, however. Similar to the practical approach, the design of a slender column with axial load $P_c$ and first-order moment $M_{pe}$ can be treated as the design of a section with the same axial load and equivalent moment $\eta P_c e$, where $\eta$ is a moment magnification factor.

4.1. Sectional strength of slender circular TRC columns

The behavior of short circular TRC columns under eccentric compression loads has been investigated and reported recently by the authors [28], where design equations to predict the sectional capacity were proposed as

$$\begin{align*}
N_i &= a f_{c2} A_d 2 \theta - \sin 2 \theta
\end{align*}$$

$$\begin{align*}
M_i &= a f_{c2} A_d D \sin^3 \theta / 3 \pi
\end{align*}$$

in which the contribution of longitudinal stress in the steel tube is ignored and the transverse stress is assumed to be the yield strength of the tube so that the confining stress can be obtained by

<table>
<thead>
<tr>
<th>Table 3</th>
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<tr>
<td>Details of studied parameters.</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$f_{c2}$ (MPa)</td>
</tr>
<tr>
<td>$f_{c2}$ (MPa)</td>
</tr>
<tr>
<td>$D/t$</td>
</tr>
<tr>
<td>$n_h$ ($x_h$)</td>
</tr>
<tr>
<td>e/r</td>
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<tr>
<td>$\lambda$</td>
</tr>
</tbody>
</table>
In Eqs. (9) and (10), $f_{cc}$ is the confined concrete strength proposed by Mander et al. [48,49], given as

$$f_{cc} = \frac{f_{co}}{C_0^{1/2} + 2^{1/4}f_l f_{co}/C_16/C_{17}} f_{co} > 50 \text{ MPa}$$

$$f_{cc} = \sqrt{1 + 7.94\frac{f_l f_{co}}{C_0}} - 2\frac{f_l}{f_{co}} f_{co} \leq 50 \text{ MPa}$$

In Eqs. (9) and (10), $f_{cc}$ is the confined concrete strength proposed by Mander et al. [48,49], given as

$$f_{cc} = \frac{2f_{ty}}{D - 2t} \approx \frac{2f_{ty}}{D}$$

Since the above equations were derived based on the test results of short columns, their applicability for slender TRC columns needs a further examination. By contrast with short TRC column, the confinement reduction and the contribution of steel tube's longitudinal stress at the critical section should be considered in the section capacity analysis for slender TRC column. The plastic hinges caused by seismic forces usually appear near the column ends. However, in TRC columns the steel tube is intentionally shortened at both ends. As such, the steel tube is fully utilized to confine the concrete core in the vicinity of column ends and the induced longitudinal stress is very small. Thus, they are deemed similar to short TRC columns. Additionally, the continuity of the steel tube affects little the load-carrying capacity of a TRC column as evidenced from the test results between Type A and Type B specimens. As a result, Eqs. (9) and (10) may also be adopted to develop the sectional capacity interaction curves of slender circular TRC columns.
The moment magnification factor \( \eta \) proposed in this paper is intended for the particular case of an initially straight TRC column bent in single curvature due to an eccentric load \( P \) applied at each end with equal eccentricity \( e \). For a given failure axial load \( P_u \), the corresponding sectional bending moment \( M_u \) can be obtained according to the sectional strength curve. Then the moment magnification factor \( \eta \) can be theoretically calculated by \( M_u / P_u \). To examine the \( \eta \) values of slender circular TRC columns, an extensive parametric study was carried out using the above FE model of Type A specimens. The sample model has a diameter of 200 mm and is longitudinally reinforced by evenly distributed steel bars with diameter = 12 mm and yield strength = 400 MPa. The diameter of reinforcement circle to the diameter of the cross-section is 0.85. The details of considered parameters are given in Table 3.

The parametric investigations are presented in Fig. 11. The moment magnification factor approximately linearly increases with the slenderness ratio, and the increasing rate is greatly affected by the eccentricity ratio. Compared to the eccentricity ratio \( e/r \) and the slenderness ratio \( \lambda \), the influence of other parameters \( (f_{co}, f_{py}, D/t, \eta_s) \) on the moment magnification factor is relatively small. Moreover, a total of 1008 models were analyzed considering 7 slenderness ratios \((\lambda = 24, 32, 40, 48, 56, 64, 72)\), 6 eccentricity to cross-sectional radius ratios \((e/r = 0.125, 0.25, 0.375, 0.5, 0.75, 1)\), 2 concrete compressive strengths \((f_{co} = 50 \text{ MPa}, 70 \text{ MPa})\), 2 steel tube yield strengths \((f_{py} = 400 \text{ MPa}, 600 \text{ MPa})\), 2 numbers of longitudinal reinforcing bars \((n_b = 8, 10)\), and 3 diameter to thickness ratios of the steel tube \((D/t = 85, 115, 145)\). Based on this results from the extensive parametric study and a regression analysis, the following formula is proposed to estimate the moment magnification factor \( \eta \) for slender circular TRC columns

\[
\eta = \begin{cases} 
0.0078 \frac{e}{r} + 0.8 & f_{co} \leq 50 \text{ MPa} \\
0.0078 \frac{e}{r} + 0.7 & f_{co} > 50 \text{ MPa} 
\end{cases} \quad \text{and} \quad \eta \geq 1 \tag{13}
\]

The moment magnification factors predicted by Eq. (13) were compared with the results obtained from the FEAs and the sectional strength analyses, as shown in Fig. 12, which indicates an acceptable agreement. Note that the mean value and standard deviation of the predicted to the analysis value ratio are 1.02 and 0.06, respectively.

4.3. Comparison between the suggested design method and the test results

Based on the moment magnification method and the sectional strength, the design equations for slender circular TRC columns under eccentric compression loads can be expressed as follows

\[
P_a \leq \frac{f_{co} A_c}{2 \pi} \left( \frac{h}{2} - \frac{\sin \theta}{\theta} \right) + \sum_{i=1}^{n_b} \sigma_{bi} A_{bi} \tag{14}
\]

\[
\eta P_u e \leq \frac{f_{co} A_c D}{3 \pi} \left( \frac{\sin^2 \theta}{\theta} \right) + \sum_{i=1}^{n_b} \sigma_{bi} A_{bi} r_s \cos \varphi_i \tag{15}
\]

The developed section capacity interaction curves and the test results \( (P_a = \text{tested axial peak load}, \text{and } M_u = \text{magnified nominal moment} = \eta P_u e) \) are compared to each other, as shown in Fig. 13 in which an acceptable agreement is observed.

5. Conclusions

Sixteen slender circular TRC columns subjected to eccentric compression loads were tested, which were divided into two groups: Type A specimens (8 specimens) where the steel tube is continuous at the mid-height; and Type B specimens (8 specimens) where the steel tube is disconnected with a girth gap at the mid-height. The experimental results including failure modes, specimens’ ultimate capacity and deformation, and steel tube stresses are described in detail and discussed. The results predicted by the nonlinear finite element analysis are in good agreement with the experimental results. An extensive parametric study was car-

![Fig. 12. Comparison of \( \eta \) factors between the FE regression analysis and the equation-predicted results.](image)

![Fig. 13. Axial force versus moment capacity interaction diagrams.](image)
ried out and simplified design expressions based on the moment magnification method are proposed for slender circular TRC columns under eccentric compression loads. Based on this study, the following primary observations and findings are offered.

1. The tested slender circular TRC columns exhibit good ductile behavior during the eccentric loading. The initial stiffness and axial compression capacity decrease with the increasing eccentricity and slenderness ratios.

2. The failure mode of the examined slender circular TRC columns is characterized by global bending failure with a critical section at mid-height. The discontinuity of the steel tube at mid-height has a small effect (≈5% on average) on the bearing capacity of the columns.

3. The derived regression formula for estimating the moment magnification factor (η) of slender circular TRC columns under eccentric compression loads, i.e., Eq. (13), is practically acceptable.

4. Based on the moment magnification method and section capacity analysis, simplified design equations, i.e., Eqs. (14) and (15), are proposed to account for the second order effects on slender circular TRC columns under eccentric compression loads.

Acknowledgments

The research work reported herein is made possible through the financial support from the National Natural Science Foundation of China (\#51178210, \#51438001) and the Fundamental Research Funds for the Central Universities of China (106112014CDJZK200001), to which the authors are grateful.

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