Modeling and Monitoring of Dynamic Processes

Yingwei Zhang, Tianyou Chai, Fellow, IEEE, Zhiming Li, and Chunyu Yang

Abstract—In this paper, a new online monitoring approach is proposed for handling the dynamic problem in industrial batch processes. Compared to conventional methods, its contributions are as follows: 1) multimodes are separated correctly since the cross-mode correlations are considered and the common information is extracted; 2) the expensive computing load is avoided since only the specific information is calculated when a mode is monitored online; and 3) after that, two different subspaces are separated, and the common and specific subspace models are built and analyzed, respectively. The monitoring is carried out in the subspace. The corresponding confidence regions are constructed according to their respective models.

Index Terms—Common and specific correlations, industrial processes, multimode process monitoring, subspace separation.

I. INTRODUCTION

IN ORDER to ensure the safety and quality of products, the monitoring of the process performance is a key issue. Multivariate statistical process monitoring has been intensively researched in the last few decades, which have been applied widely in industrial processes. In particular, principal component analysis (PCA) and partial least squares (PLS) have achieved great success in process monitoring [1]–[7]. Kernel PCA [8] and kernel PLS [9], [10] have been used to deal with nonlinear fault detection problem. Multiway PCA has been applied to batch processes [11]–[14]. However, in an industrial process, the system has different operation conditions, control objectives, or raw materials. Therefore, there are often different production modes, which makes multimode batch processes quite complicated [15]. Multiscale principal component analysis is used for fault detection and diagnosis in some multimode batch processes. Using wavelets, the data model is decomposed at different scales. Contributions from each scale are collected in separate matrices, and a new model is then constructed at each scale [16]. Under different assumptions, various researchers have proposed a variety of methods to solve the dynamic problems [17]–[24]. Flury [20] assumed that different operating modes retained the same principal components, and proposed the common subspace model. In addition, Lane [21] proposed a multigroup model to monitor batch processes with multiple modes. Hwang [22] assumed that different operating modes have the same number of retained principal components, and proposed a super PCA model to monitor multimode batch processes. However, in an actual production process, it is very hard to have the same principal components or the same number under different operating modes. Recently, recursive and adaptive PCA and PLS methods have been introduced [23], [24]. Although these methods can be applied to adapt the online process changes, they still lack the ability to cope with processes with multiple operating modes. Recently, novel models have been built based on modified PCA methods [1], [4]. In [1], the fault probabilities could be determined. In [4], the unbalanced feature extraction could not be avoided. Zhao et al. [25], [26] proposed a multiple PCA model that adopted principal angles to measure the similarities of any two models. Yoo et al. [27] proposed local monitoring, where multiple models divide the entire historical dataset into separate regions, which are then modeled separately. Liu and Chen [28] proposed a Bayes classifier to identify abnormal events data. Measuring the differences between normal and faulty states can identify the faulty variables since the faults are formed in a new operating region. Yu and Qin [29] used the Figueiredo–Jain algorithm to determine the cluster parameters of Bayesian classification. In their approach, a fault detection index was derived based on Mahalanobis distance and the posterior probability of each cluster. However, faulty variables were not isolated when the index exceeded its control limits. In [30], a switching control strategy was proposed based on the indirect observation of the process state where fuzzy classification was used for pattern recognition. Although the switching speed was improved, the safety and reliability could not be analyzed [30]. Qiu et al. proposed that a number of multivariate cumulative sum can detect a shift in the mean vector of the measurements when the multivariate distribution of the measurement is non-Gaussian [31]–[33] and exponentially weighted moving average (EWMA) can provide protection against the various shifts [34], [35]. These methods have shown good performance in faults detection. However, the switching problems of multimodes could not be investigated.

In this paper, new statistical analysis and online monitoring approaches are proposed for handling the problem of multimode in batch processes. The basic idea is to first analyze the similarity and dissimilarity of different modes: that is, to analyze different types of correlations from the cross-mode viewpoint. The common part is the similar variable correlations over modes, and the specific part is the correlations that are not shared by all modes. The dissimilarity is the characteristic of the specific subspace. The common and specific explanatory variations are separated from each other. In the traditional method, the global mode is often divided into several different modes and it is separately monitored in each mode. However, in the proposed statistical analysis, the global mode is divided into several different modes. Then the common information from each mode is extracted.

Manuscript received January 7, 2011; revised November 23, 2011; accepted December 3, 2011. Date of publication December 29, 2011; date of current version February 8, 2012. This work was supported in part by China’s National 973 Program under Project 2009CB320600 and by the National Science Foundation of China under Grant 60974057 and Grant 61020106003. The authors are with the State Key Laboratory of Synthesis Automation of Process Industry, Northeastern University, Shenyang 110819, China (e-mail: zhangyingwei@mail.neu.edu.cn; tychai@mail.neu.edu.cn; lizhiming406@yahoo.cn; chunyuyang@yahoo.cn).

Digital Object Identifier 10.1109/TNNLS.2011.2179669
Then, EWMA is added to the method to help detect faults. By analyzing both the common part and the specific part, this method can identify the different operating modes and can effectively diagnose faults of multimode processes. The contributions of the proposed method are as follows.

1) Multiple modeling may give each mode a higher resolution, but it neglects the cross-mode correlations and may result in some false alarms. In the proposed approach, the multimodes are separated correctly since the cross-mode correlations are considered.

2) When the multimodes are separated, the common information is extracted.

3) Compared to the conventional method, the expensive computing load is avoided since only the specific information is calculated when a mode is monitored online.

The remainder of this paper is organized as follows. A model of multimode processes is proposed in Section II. A monitoring method of multimode processes is proposed in Section III. The proposed monitoring approach is applied to a fused process in Section IV. Conclusions are summarized in Section V.

II. MODELING OF MULTIMODE PROCESSES

Multiple modeling may give each mode a higher resolution, but it neglects the cross-mode correlations and may result in some false alarms. In the proposed approach, the multimodes are separated correctly since cross-mode correlations are considered. The differences are shown in Figs. 1 and 2.

Suppose that there are \( M \) industrial production modes in the same production line. There are multiple datasets \( \mathbf{X}_1(K_{1,p} \times J) \), \( \mathbf{X}_2(K_{2,p} \times J) \), \ldots, \( \mathbf{X}_m(K_{m,p} \times J) \), \ldots, \( \mathbf{X}_M(K_{M,p} \times J) \) (where \( K \) denotes the number of samples, subscript \( p \) denotes the batch, \( J \) denotes the number of variables, and subscript \( m = 1, 2, \ldots, M \) denote different modes). Different modes have the same number of variables and the same number of samples within each batch. For analysis, the datasets are stacked in a variable-unfolding way. That is, \( \mathbf{X}_1(K_{1,p} \times J) \), \( \mathbf{X}_2(K_{2,p} \times J) \), \ldots, \( \mathbf{X}_M(K_{M,p} \times J) \) are put together one after another, forming \( \mathbf{X}_m(\sum_{i=1}^{J} K_{m,i} \times J) \) (where \( i = 1, 2, \ldots, I \) denote the number of the batch, \( m = 1, 2, \ldots, M \) denote the different modes). Then, they are mean-centered and scaled to unit variance, designated as \( \mathbf{X}_1(N_1 \times J) \), \( \mathbf{X}_2(N_2 \times J) \), \ldots, \( \mathbf{X}_M(N_M \times J) \) (where \( N_1 \) is equal to \( \sum_{i=1}^{J} K_{1,i} \), and \( N_2 \) is equal to \( \sum_{i=1}^{J} K_{2,i} \), \ldots, \( N_M \) is equal to \( \sum_{i=1}^{J} K_{M,i} \)).

The number of the models in the datasets is determined by using clustering method [5]. The basic idea of multimode PCA is to reveal the common information among multiple datasets. In each data space, it is always possible to find out a set of vectors that are representative enough to the other samples and can substitute all samples by their linear combinations. They are called sub-basis vectors. Therefore, the sub-basis vectors can be employed as the evaluation index of similarity and dissimilarity over multiple datasets.

So, \( \mathbf{p}_{m,j}(j = 1, 2, \ldots, J; m = 1, 2, \ldots, M) \) are defined as the sub-basis vectors in each dataset, and there exist linear combination coefficients \( \mathbf{a}_m^j = [\mathbf{a}_{1,j}^m, \mathbf{a}_{2,j}^m, \ldots, \mathbf{a}_{n,j}^m] \) (where \( n \) denotes the lines of \( \mathbf{a}_m^j \)), such that

\[
\mathbf{p}_{m,j} = \sum_{n=1}^{N_m} \mathbf{a}_{n,j}^m \mathbf{x}_n = \mathbf{X}_m^j \mathbf{a}_m^j
\]

where \( \mathbf{x}_n^m \) is a sample of \( \mathbf{X}_m(N_m \times J) \). Therefore, the sub-basis vector \( \mathbf{p}_{m,j} \) is actually a linear function of the original observations in each dataset.

The degree of similarity of sub-basis vectors should be measured in terms of “how close with each other over modes.” However, it would be complicated if all mode-to-mode interactions are simultaneously and directly evaluated. Here, the similarity of variable correlations over sets can be analyzed through the introduction of a global and common basis vector \( \mathbf{p}_g \). Next, \( \mathbf{p}_g \) is defined and extracted.

First, \( \mathbf{p}_g \) is defined to make the correlation between \( \mathbf{p}_g \) itself and the sub-basis vectors \( \mathbf{p}_m \) \((m = 1, 2, \ldots, M) \) of each mode as close as possible, that is, to find the maximum of the polynomial \( (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_1^1)^2 + (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_2^2)^2 + \cdots + (\mathbf{p}_g^T \mathbf{X}_M^j \mathbf{a}_M^2)^2 \). At the same time, the values of \( (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_1^1)^2, (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_2^2)^2, \ldots, (\mathbf{p}_g^T \mathbf{X}_M^j \mathbf{a}_M^2)^2 \) should be approximately the same. The inequality \( \varepsilon \leq (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_1^1)^2/(\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_2^2)^2 = ((\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_2^2)^2/(\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_3^3)^2) \cdots = (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_M^2)^2/(\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_{M-1}^M)^2 \leq (1/\varepsilon) \) is introduced to make the difference of \( (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_1^1)^2, (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_2^2)^2, \ldots, (\mathbf{p}_g^T \mathbf{X}_1^j \mathbf{a}_M^2)^2 \) very small. \( \varepsilon \) is a constant scalar, which is defined to meet the inequality. Then the global basis \( (\mathbf{p}_g) \) together with the sub-basis vectors \( (\mathbf{p}_m) \) and the object function is as follows:

\[
\max R^2 = \max \left( \sum_{m=1}^{M} \left( \mathbf{p}_g^T \mathbf{p}_m \right)^2 \right).
\]
Equation (1) is substituted into (2) to get
\[
\max R^2 = \max \left( \sum_{m=1}^{M} \left( p_g^T X_m^T \alpha_m \right)^2 \right) .
\] (3)

To get the target results, certain constraints are introduced
\[
\begin{align*}
\varepsilon & \left( p_g^T X_{m-1}^T \alpha_{m-1} \right)^2 - \left( p_g^T X_m^T \alpha_m \right)^2 \leq 0 \\
\varepsilon & \left( p_g^T X_m^T \alpha_m \right)^2 - \alpha_m^2 \leq 0 \\
p_g^T p_g &= 1
\end{align*}
\] (4)

where \( m = 1, 2, \ldots, M \), and the combination coefficient vector \( \alpha_m \) is set to unit length. And \( (p_g^T X_m^T \alpha_m) \) actually model the covariance information between the sub-basis vector \( X_m^T \alpha_m \) and global basis vector \( p_g \). So the objective function involves the covariance information, which is much better than the pure correlation analysis.

Without loss of generality, the case of two modes, which are called mode A and mode B, is discussed here. To get the target results, the initial objective function is defined as follows, which can be expressed as an extreme-value problem
\[
F(p_g, \alpha, \lambda) = \left( p_g^T X_A^T \alpha_A \right)^2 + \left( p_g^T X_B^T \alpha_B \right)^2 - \lambda_g \left( p_g^T p_g - 1 \right) - \lambda_A \left( \alpha_A^T \alpha_A - 1 \right) - \lambda_B \left( \alpha_B^T \alpha_B - 1 \right)
\] (5)

where \( \lambda_g, \lambda_A, \lambda_B, \lambda_1, \) and \( \lambda_2 \) are constant scalars.

The optimization \( p_g \) should meet the following conditions:
\[
\begin{align*}
\frac{\partial F(p_g, \alpha, \lambda)}{\partial p_g} &= 0 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
Correspondingly, \((T^C_A, T^C_B)\) and \((T^S_A, T^S_B)\) are their associated variations.

For different modes, a different monitoring system can be designed to check online whether the current operation pattern is normal and which mode it belongs to. The \(T^2\)-statistic and the Q-statistic are calculated as follows for each subspace:

\[
T^m_{C} = (t^m_{C})^T (A^m_{C})^{-1} t^m_{C} \tag{17}
\]

\[
T^m_{S} = (t^m_{S})^T (A^m_{S})^{-1} t^m_{S} \tag{18}
\]

\[
\text{SPE}^m = (s^m)^T s^m \tag{19}
\]

where the superscript \(m\) denotes different mode. \(A^m_{C}\) and \(A^m_{S}\) are the variance and covariance matrices of components, respectively.

EWMA and PCA method are combined to help detect faults in the proposed method. The EWMA model is first built based on the originally measured variables of the process, and then PCA is used to extract the correlation between the EWMA variables.

In the EWMA control chart, suppose that \(x_1, x_2, \ldots, x_k\) are some observations, the EWMA is expressed as

\[
Z_k = \zeta x_k + (1 - \zeta) Z_{k-1}
\]

where \(k = 1, 2, \ldots, z_0 = 00 < \zeta \leq 1\). A multivariable version of EWMA (MEWMA) is a statistical method that gives less weight to older data. Substituting recusively for \(Z_k\), the following equation is obtained:

\[
Z_k = \zeta \cdot \sum_{j=1}^{k} (1 - \zeta)^{k-j} x_j
\]

where \(k = 1, 2, \ldots, Z_0 = 0, 0 < \zeta_j \leq 1, j = 1, \ldots, p\). The choice of \(\zeta_j\) values is important because it reflects the ability to track the behavior of the system and to monitor sensitive degrees. Generally, based on experience, \(\zeta_j = 0.2 \pm 0.1\) [34].

Then the MEWMA is added into the proposed method. The new common score, specific score, and \(\text{SPE}^m_k\) can be represented as follows:

\[
t^m_{C, \text{ewma}, k} = Z^T_{C,k} \cdot (P^T g)
\]

\[
t^m_{S, \text{ewma}, k} = Z^T_{S,k} \cdot (P^T S)
\]

\[
\text{SPE}^m_{\text{ewma}, k} = \zeta \cdot \text{SPE}^m_k + (1 - \zeta) \text{SPE}^m_{\text{ewma}, k-1}.
\]

The common and specific Hotelling-\(T^2\) and \(\text{SPE}^m\) can be calculated as follows:

\[
(T^m_{C,k})^2 = (t^m_{C, \text{ewma}, k})^T (A^m_{C})^{-1} t^m_{C, \text{ewma}, k} \tag{25}
\]

\[
(T^m_{S,k})^2 = (t^m_{S, \text{ewma}, k})^T (A^m_{S})^{-1} t^m_{S, \text{ewma}, k} \tag{26}
\]

\[
\text{SPE}^m_{\text{ewma}, k} = Z^T_{S,k} \cdot [I - (P^T S)^T P^T S] \cdot Z^T_{S,k}
\]

\[
= \zeta \cdot \sum_{j=1}^{k} (1 - \zeta)^{k-j} (x^m_{S,j})^T \cdot [I - (P^T S)^T P^T S]
\]

\[
(27)
\]

In each mode, the original measurement samples are deemed to follow a multivariate Gaussian distribution. So the control limits in the systematic subspace for each mode are defined by the \(F\)-distribution with \(\alpha\) as the significance factor [36], [37]

\[
T^m \sim p(N^2_{d} - 1) \quad N_{m}(N_{m} p) F_{p,Nm-p,\alpha}.
\]

Similarly, in the residual subspace, the spectral estimation (SPE) confidence limit for each mode can be approximated by a weighted Chi-squared distribution

\[
\text{SPE}^m_{\text{ewma}} \sim g_m \chi^2_{h_m} \tag{29}
\]

where \(g_m = v_m/2l_m\) and \(h_m = 2(l_m)^2/v_m\), in which \(l_m\) is the average of all the SPE values for the \(m\)th mode, and \(v_m\) is the corresponding variance.

For the new observation vector \(x_{\text{new}}(J \times 1)\), it is first normalized by the grand mean and variance of the current phase. The corresponding common systematic variation scores are calculated by projecting it onto the unified common and specific subspace, respectively

\[
t^C_{\text{new}, \text{ewma}, k} = P^T g \cdot x_{\text{new}}\tag{30}
\]

\[
x^C_{\text{new}, \text{ewma}, k} = P^T g \cdot x_{\text{new}}\tag{31}
\]

\[
x^{S}_{\text{new}, \text{ewma}} = P^S x^{S}_{\text{ewma}}\tag{32}
\]

\[
t^{S}_{\text{new}, \text{ewma}} = (P^S m)^T x^{S}_{\text{ewma}}\tag{33}
\]

\[
x^S_{\text{ewma}, \text{new}} = P^S x^{S}_{\text{ewma}}\tag{34}
\]

\[
SPE^{S}_{\text{ewma, new}} = (x^S_{\text{ewma, new}})^T x^S_{\text{ewma, new}}\tag{35}
\]

Then, the MEWMA method is used to calculate the weighted \(t^{C, \text{ewma}, k}\) and \(t^{S, \text{ewma}, k}\), and \(\text{SPE}^{C, \text{ewma}, k}\) and \(\text{SPE}^{S, \text{ewma}, k}\)

\[
t^{C, \text{ewma, k}} = \zeta t^{C, \text{ewma, k}} + (1 - \zeta) t^{C, \text{ewma, k-1}}\tag{37}
\]

\[
t^{S, \text{ewma, k}} = \zeta t^{S, \text{ewma, k}} + (1 - \zeta) t^{S, \text{ewma, k-1}}\tag{38}
\]

\[
\text{SPE}^{C, \text{ewma, k}} = \zeta \text{SPE}^C_{\text{ewma, k}} + (1 - \zeta) \text{SPE}^C_{\text{ewma, k-1}}\tag{39}
\]

\[
\text{SPE}^{S, \text{ewma, k}} = \zeta \text{SPE}^S_{\text{ewma, k}} + (1 - \zeta) \text{SPE}^S_{\text{ewma, k-1}}\tag{40}
\]

where \(0 < \zeta \leq 1, \quad t^{C, \text{ewma, 0}} = 0, \quad k = 1, 2, \ldots, R, \quad \text{SPE}^{C, \text{ewma, 0}} = 0\).

Subsequently, the monitoring statistics, namely, the common \(T^2\)-statistics and specific \(T^2\)-statistics, can be calculated as

\[
T^C_{m, \text{ewma}} = (t^{C, \text{ewma, k}})^T (A^C_m)^{-1} t^{C, \text{ewma, k}}\tag{41}
\]

\[
T^S_{m, \text{ewma}} = (t^{S, \text{ewma, k}})^T (A^S_m)^{-1} t^{S, \text{ewma, k}}\tag{42}
\]

By calculating the Hotelling-\(T^2\) of the common features, the faults in common information can be detected. If there are no faults in the common features, the calculation of the specific Hotelling-\(T^2\) can reveal the faults in the specific features [4]. And the calculation of SPE can help detect the process faults.
IV. ILLUSTRATIVE EXAMPLE

The electro-fused magnesia furnace (EFMF) is one of the main pieces of equipment used to produce electro-fused magnesia and is a kind of mine hot electric arc furnace. With the development of the technology of melting, EFMF has already gained extensive application in the industry. EFMF refining technology can enhance the quality and increase the production variety. The whole equipment of the EFMF has a transformer, a short net, an electrode holder, an electrode, a furnace, etc. An operating board beside the furnace controls the electrode up and down. The furnace shell is round and slightly tapered, and facilitates melting during processing. There are rings on the furnace wall and a trolley under the furnace. When melting process has completed, the trolley is removed to cool down. The EFMF smelting process is shown in Fig. 3.

In the melting process, voltage flicker, which is a kind of power quality problem, occurs in the power system when the gas aggregates quickly in the magnesia. The EFMF in this paper takes light-burned magnesia as the raw material. It makes use of the heat generated both by the burden resistance when the current flows through the burden and the arc between the electrodes and the burden to melt the burden and then obtain fused magnesia crystals with higher purity. The materials are powdered magnesium in mode A. The materials are massive magnesium lumps in mode B.

In total, 40 normal batch runs are generated in mode A and 40 normal batches runs in mode B. After the data processing in the above section, the datasets \(X_A\) and \(X_B\) are obtained. There are three variables in both datasets \(X_A\) and \(X_B\). First, a batch run is used to test the feasibility of the method. As is shown in Fig. 4(a) and (b), from the contrast of the modes A and B, the consecutive \(T^2\)-statistics values can only be enclosed by the confidence region in mode A, but go beyond the confidence regions of mode B. However, noticing that some part of \(T^2\) are in mode B, the common systematic information can also be enclosed by the confidence region. Then the group affiliation can be further checked by monitoring specific systematic information as shown in Fig. 4(c). Obviously, due to use of different specific parts, the out-of-control indications in mode B are more obvious compared with those shown in Fig. 4(a). For example, the \(T^2\) values occur in mode B. Combining their monitoring results in both subspaces, the affiliation of the two modes is definitely fixed and the operation status is also checked for systematic information in both subspaces. Finally, the residual information is supervised, as shown in Fig. 4(c), which also indicates that the current batch is operating normally.

At the same time, from the above analysis, it can be judged as to which mode the EFMF is working. From Fig. 4, both \(T^2\) of the common and the specific part are enclosed by the confidence region in mode A, and the SPE of mode A also shows there are no faults. While \(T^2\) of the specific part goes beyond the confidence region in mode B, \(T^2\) of the common part of mode B and SPE show that the test data is normal. From the test results of mode A, the test data is deemed to be...
normal data. From the test results of mode B, the common part of mode B is under control, but its specific part is abnormal. That means the test data is normal data and belongs to mode A.

Then an abnormal batch run, which belongs to mode A, is applied in the proposed method. Process faults are introduced from the 78th sample. As is shown in Fig. 5(a), in the common part there are some samples that are not enclosed by the confidence region and some part of the faults is not very obvious. On further checking by monitoring specific systematic information, it is obvious that there are faults that start from 78 approximately in Fig. 5(b). And the residual information is supervised, which reveals the faults in the batch in Fig. 5(c). Generally, clear and stable alarms are revealed especially by the $T_2$ monitoring system in a specific systematic subspace, which means the abnormal behavior mainly disturbs the specific part of information.

In the proposed method, the common information is used for the overall analysis and the specific information is used for the local analysis. The conventional method is used to deal with the problems. In this section, the conventional multimodel method is used for the detection of multimode faults for comparison purposes. An abnormal batch run is applied that belongs to mode B. Applications of the proposed method and the conventional method to detect the abnormal batch are shown in Figs. 6 and 7, respectively. In Fig. 6, the results reveal good fault detection performance by the proposed method. Generally, clear and stable alarms are revealed especially by the $T_2$ monitoring system in the common part, which means that the abnormal behavior mainly disturbs the common part of information. However, in Fig. 7, there are some false alarms in the $T_2$ monitoring chart, and the SPE chart is not as good. By comparing the results, it can be concluded that the faults can be detected more effectively using the proposed method.

V. CONCLUSION

In this paper, a new method was proposed for the analysis of multimode batches. Since the cross-mode correlations were considered, the multimodes were separated more correctly in the proposed method. The similarity and dissimilarity of different modes were first analyzed. By dataset decomposition and subspace separation, the common information could be obtained from the specific information in each mode. The
common information was the similar variable correlations over modes, and the specific information was the difference in each mode. By dataset decomposition and subspace separation, the underlying variations of different modes could be analyzed more comprehensively. The strength of the proposed strategy lies in not only the effective monitoring but also in the appealing analysis results and comprehension for multimode problems. Fault detection can be accurate and obvious. The case of the EFMM illustrated its effectiveness. The monitoring for between-mode transition and the irregular transition dynamics over batches can be topics for future work. It can first reveal the problem of irregular transition dynamics over batches. Instead of identifying the definite boundary between mode and transition, all process patterns are regarded as possible transitions to accommodate their irregular dynamics. By between-mode modeling, the underlying mode information was decomposed more meaningfully for between-mode transition analysis. In addition, considering the nonlinearity of data, the kernel trick may be introduced into the proposed method, which can deal with the nonlinear problem.

APPENDIX

From (6)–(8), the following equations are obtained:

\[
(1 - \lambda_1 e + \lambda_2) (p_g^T X_A^T \alpha A X_A^T \alpha A ) + (1 - \lambda_2 e + \lambda_1) (p_g^T X_B^T \alpha B X_B^T \alpha B ) = \lambda_g p_g
\]

\[
\alpha_A = \sqrt{\frac{(1 - \lambda_1 e + \lambda_2)}{\lambda_A}} X_A p_g = \frac{1}{p_g^T (X_A X_A) p_g} X_A p_g
\]

\[
\alpha_B = \sqrt{\frac{(1 - \lambda_2 e + \lambda_1)}{\lambda_B}} X_B p_g = \frac{1}{p_g^T (X_B X_B) p_g} X_B p_g
\]

where the suboptimal objective parameters \(\lambda_A, \lambda_B\), can be calculated from (6) and (7)

\[
\lambda_A = (1 - \lambda_1 e + \lambda_2) p_g^T (X_A^T X_A) p_g
\]

\[
\lambda_B = (1 - \lambda_2 e + \lambda_1) p_g^T (X_B^T X_B) p_g
\]

From (41)–(43) and (46) is obtained

\[
(1 - \lambda_1 e + \lambda_2) X_A^T X_A p_g + (1 - \lambda_2 e + \lambda_1) X_B^T X_B p_g = \lambda_g p_g
\]

To get the solutions, the parameters \(\lambda_1\) and \(\lambda_2\) should be discussed.

1) If \(\lambda_1 \neq 0 \) and \(\lambda_2 \neq 0\), (49) and (50) are contradictory. Therefore, equation has no solution in this case.

2) If \(\lambda_1 = 0 \) and \(\lambda_2 = 0\), then (49) and (50) will lose effectiveness. And (46) can be rewritten as follows:

\[
X_A^T X_A p_g + X_B^T X_B p_g = \lambda_g p_g
\]

The solution leads to the difference between \(p_g^T (X_A X_A) p_g\) and \(p_g^T (X_B X_B) p_g\) being very large. That means \(p_g\) may be very close to one of the datasets but have little relationship with the other datasets. Take, for instance, the global vector \(p_g\) has close relationship with dataset \(X_A\) but may have no relationship with dataset \(X_B\). Hence, the solution in this case is not the optimal solution.

3) We discuss the cases of \(\lambda_1 = 0, \lambda_2 \neq 0\), or \(\lambda_1 \neq 0, \lambda_2 = 0\). From (48) and (49), it can be known that, if the optimization problem meets the situation of \(\lambda_1 = 0 \) and \(\lambda_2 \neq 0\), it would not meet the situation of \(\lambda_1 \neq 0 \) and \(\lambda_2 = 0\); the reverse is also true. So the two situations can be discussed together. Suppose the optimization problem follows the situation \(\lambda_1 = 0, \lambda_2 \neq 0, \lambda_1 = 0\) and \(\lambda_2 \neq 0\) are substituted into (10) and (46), and (52), (53) can be obtained

\[
(1 + \lambda_2) X_A^T X_A p_g + (1 - \lambda_2 e) X_B^T X_B p_g = \lambda_g p_g
\]

From (52) and (53), we know that there exist \(J\) eigenvectors. \(J\) eigenvectors \(p_g\) are obtained according to the descending eigenvalues (where \(R\) vectors are selected corresponding to the principal eigenvalues). Thus, the \(R\) vectors can construct a global basis matrix \(P_g (J \times R)\).

Besides, global vector \(p_g\) should approximate \(p_m\) as close as possible. That is, \(p_m\) should be able to be comprehensively described and substituted by the global basis vector \(p_g\). So \(p_g\) can meet \(p_g = X_A^T \alpha_1\) and \(p_g = X_B^T \alpha_2\), in which \(\alpha_1\) and \(\alpha_2\) are the linear combination coefficients and they are set to unit length. The above two equations can be rewritten as \(\alpha_1 = X_A p_g\) and \(\alpha_2 = X_B p_g\). We can determine the number of global components by using the accumulating contribution rate.

REFERENCES


学霸图书馆
www.xuebalib.com

本文献由“学霸图书馆-文献云下载”收集自网络，仅供学习交流使用。

学霸图书馆（www.xuebalib.com）是一个“整合众多图书馆数据库资源，提供一站式文献检索和下载服务”的24小时在线不限IP图书馆。

图书馆致力于便利、促进学习与科研，提供最强文献下载服务。

图书馆导航：
- 图书馆首页
- 文献云下载
- 图书馆入口
- 外文数据库大全
- 疑难文献辅助工具