Fuzzy controller design for synchronous motion in a dual-cylinder electro-hydraulic system

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Abstract

In this paper, an integrated fuzzy controller is proposed to achieve a synchronous positioning objective for a dual-cylinder electro-hydraulic lifting system with unbalanced loadings, system uncertainties, and disturbances. The control system consists of one-fuzzy coordination controller for both cylinders and an individual cylinder controller, comprised of a feedforward controller, and a fuzzy tracking controller, for each of the hydraulic cylinders. In the integrated fuzzy controller design, the fuzzy coordination controller is responsible for dispatching motion synchronization commands to the individual cylinder controllers. Each cylinder controller then adaptively enforces the position tracking to its controlled actuator. The experimental results show that the proposed integrated fuzzy controller design can effectively achieve the objective of position synchronization in the dual-cylinder electro-hydraulic system, with maximum synchronization error within $7\frac{4}{4}$ mm.

\begin{figure}[h]

\end{figure}

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1. Introduction

Hydraulic systems have been widely used in industrial motion applications, such as active suspension control and construction machines, because of their small size-to-power ratios and their ability to provide large force and torque. Backe (1993) summarized the development of hydraulic applications from 1975 to 1990 and forecasted continuing growth in demand as well as the need for advanced control techniques. Jelali and Kroll (2003) summarized recent developments in nonlinear identification, nonlinear control and the application of both to hydraulic servo-systems, and arrived at the same conclusion. Advanced control methods are necessary for high-performance hydraulic motion control.

Motion synchronization has been an important research area in the manufacturing industry, driven by the need to generate precise geometric curves with multi-axis machines and robotic arms. Chiu and Tomizuka (2001) formulated the synchronization of multi-axis motion for machine tools in a geometrical framework. Xiao, Zhu and Liaw (2005) studied a generalized synchronization controller for multi-axis motion systems by incorporating cross-coupling control. For hydraulic systems, traditional motion synchronization among multiple actuators has been implemented by fluid flow-dividers and/or linkage mechanisms in open-loop operations (Jelali & Kroll, 2003; Merritt, 1967). However, open-loop flow dividers and mechanical linkages are not adequate to provide high-precision motion synchronization in the presence of load variation, nonlinear characteristics in system dynamics, variability in hydraulic components, and the compressibility of the circulating fluid. These issues increase the difficulty of achieving high-performance motion synchronization, and present safety concerns when positioning inaccuracies occur in applications with heavy loads and/or large workspace requirements. With advanced electronic
flow-control elements, such as proportional valves and servo valves, the feedback control approach has become an attractive option for electro-hydraulic systems, as it permits accuracy and bandwidth requirements to be maintained. For example, Kim and Lee (2006) proposed an experimental optimization formulation to determine the effective control gains for an electro-hydraulic position control system, such that the variations of the system dynamic caused by various factors can be compensated for properly. Many researchers (Hogan & Burrows, 1994; Jelali & Kroll 2003; Sun & Chiu, 2001, 2002) have also investigated different approaches to achieve motion synchronization for multiple hydraulic actuators with even and uneven loadings. Sun and Chiu (2002) developed a nonlinear control algorithm to address motion synchronization of a dual-cylinder electro-hydraulic lifting system. Wey and Lemmen (1998) proposed a flatness-based, two-degree-of-freedom tracking controller for synchronizing cylinder drives. The flatness-based controller functions as a feedforward controller (FFC) to generate the reference command according to the system inverse dynamic model so that the dynamical system can be steered into the region around a desired trajectory. Complex differential geometry as well as availability of the system state variables are needed to compute the reference command. Additional closed-loop controls can be employed to compensate for the error associated with the modeling uncertainty.

Nonlinearities and parameter uncertainties are the two main challenges associated with developing control algorithms for electro-hydraulic systems. Various control approaches have been investigated to address uncertain system nonlinearities and uncertain parameters in hydraulic systems. In particular, fuzzy control has been shown to be effective in handing complex, ill-defined, and uncertain systems. One of the key factors in developing a useful fuzzy controller is the ability to create a quantitative mapping among a set of fuzzy variables, in order to achieve a desired set of control objectives without the need for traditional mathematical descriptions of the plant model. Berger (1996) studied the self-tuning of PI controllers using fuzzy logic and demonstrated that fuzzy control was able to shorten start-up time and reduce the initial cost of building an automatic tuning system. Kim and Han (2006) presented a robust PID-like neuro-fuzzy controller with on-line adjusting of controller gains to control the speed of the induction motor. Other researchers (Branco & Dente, 2000; Mudi & Pal, 2000) have investigated the feasibility of using fuzzy control to reduce the influence of unmodeled nonlinearities and parameter uncertainties in hydraulic systems.

In this paper, an integrated fuzzy controller design that consists of a pair of cylinder controllers and a motion synchronization controller (also named a fuzzy coordination controller, FCC) is proposed for synchronous motion and positioning in a hydraulic lifting system. The experimental dual-cylinder lifting system includes two electro-hydraulic actuators driven by independent control circuits. The motion synchronization controller (FCC) is designed to synchronize the position of two hydraulic actuators by using coordinating fuzzy control. Each individual cylinder controller includes a FFC and a fuzzy tracking controller (FTC). The FFC generates the desired cylinder control command, based on the mathematical model of the electro-hydraulic actuator and on the desired trajectory profile. The FTC, working in conjunction with the FFC, is designed to reduce the position-tracking error due to the parameter uncertainties and unmodeled nonlinearities of the hydraulic components. The motion synchronization controller coordinates the two individual cylinder controllers to reduce the synchronization error. Both simulations and experimental results demonstrate that the proposed integrated fuzzy controller can effectively improve the performance of both motion synchronization and position tracking along the desired trajectory.

The remainder of this paper is organized as follows. Section 2 presents the system description and modeling of the dual-cylinder electro-hydraulic system. In Section 3, the proposed controller designs for the synchronous motion and positioning of a dual-cylinder electro-hydraulic lifting system is introduced. Section 4 presents simulation results and demonstrates the performance of the integrated fuzzy control applied to the experimental dual-cylinder electro-hydraulic system. Conclusions and on-going investigations are summarized in Section 5.

2. System description and modeling

An experimental dual-cylinder electro-hydraulic system is shown in Fig. 1. The system includes two double-acting electro-hydraulic actuators, a set of pressure transmission pipelines, and one hydraulic oil supply. A constant-flow type of vane pump supplies the required hydraulic pressure and flow. The desired working pressure is regulated by a gas-loaded accumulator and a proportional pressure-relief valve. Two proportional directional valves are used to control the motion of the respective cylinders. Fig. 2 shows the flow direction of the hydraulic oil circulating in the cylinders and the locations of the equipped proportional directional valves. A potentiometer is mounted on the side of each cylinder to measure the cylinder position. Two pressure transducers are mounted at the inlet and the outlet of the cylinder to measure the cylinder position. Two pressure transducers are mounted at the inlet and the outlet of the cylinder to measure the cylinder position.

2.1. System modeling

Fig. 2 illustrates the structure of two sets of single-rod hydraulic cylinders and valve lift assemblies. Assuming that the two hydraulic lift assemblies use the same model of components, it is reasonable to assume that the two systems have the same model structure. In the subsequent discussion, the subscript \( K \in \{R, L\} \) is used to denote either the right-hand side (\( R \)) assembly or the left-hand side (\( L \)}
assembly shown in Fig. 2. The source in the right-hand side and left-hand side cylinders are denoted by \( P_{Rs} \) and \( P_{Ls} \), respectively. Since only one hydraulic fluid reservoir is used, the return pressure \( P_{tank} \) is the same for both cylinders. Let the volumetric flowrate at the two ports of the cylinders (see Fig. 2) be \( Q_{K1} \) and \( Q_{K2} \), respectively. Similarly, the pressures at the two ports of the cylinders are denoted by \( P_{K1} \) and \( P_{K2} \), respectively. Assuming laminar flow and ignoring the high bandwidth valve dynamics, the flow in and out of the cylinders can be represented (Jelali & Kroll, 2003; Merritt, 1967) by

\[
Q_{K1} = \bar{u}_{in}^K(u_K)\sqrt{0.5P_{K1} - 0.5P_{tank} - \text{sign}(u_K)(P_{K1} - 0.5P_{K1} - 0.5P_{tank})},
\]

\[
Q_{K2} = \bar{u}_{out}^K(u_K)\sqrt{0.5P_{K2} - 0.5P_{tank} + \text{sign}(u_K)(P_{K2} - 0.5P_{K2} - 0.5P_{tank})},
\]

where \( \bar{u}_{in}^K(u_K) = C_dA_0(2/\rho)^{0.5}u_{in}^K(u_K) \) and \( \bar{u}_{out}^K(u_K) = C_dA_0(2/\rho)^{0.5}u_{in}^K(u_K) \); \( Q_K \) or \( P_K \) associated with subscripts 1 and 2 represent the volumetric flowrate or pressure at the lower and upper ports of the cylinders, respectively. In the previous equations, \( C_d \) and \( A_0 \) denote the orifice flow discharge coefficient and orifice area (m\(^2\)) of the proportional valve, respectively; \( \rho \) is the hydraulic fluid density in kg/m\(^3\); \( u_K \) is the control input voltage to drive the spool of the proportional valve; \( u_{in}^K(u_K) \) and \( u_{out}^K(u_K) \) denote the controlled orifice area ratios at the inlet and the outlet of each proportional directional valve with respect to the corresponding control input voltage \( u_K \). The function \( \text{sign}(u_K) \) equals 1 for \( u_K \geq 0 \) and \(-1 \) for \( u_K < 0 \).

In general, \( \bar{u}_{in}^K(u_K) \) and \( \bar{u}_{out}^K(u_K) \) are related to the characteristics of the proportional directional valve, and their functions are not published by vendors. Instead, these equivalent control input functions can be directly estimated from the steady-state response of the experimental system as a control input voltage is applied. In the case of the right-cylinder hydraulic servo system, if the control input voltage \( u_R \) is kept constant during the linear motion, the piston velocity in the cylinder will finally reach a constant speed and its corresponding fluid flowrates and pressures at the inlet and outlet ports of the cylinder will also remain in...
a steady state. According to Eqs. (1) and (2), the values of $\bar{\bar{u}}_R(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ can be determined by the relationship of $Q/(\Delta P)^{0.5}$, where $\Delta P$ stands for the pressure drop between the driving and the return path of the proportional directional valve, and $Q$ represents the volumetric flowrate of either $Q_{R1}$ or $Q_{R2}$. After repeatedly exploring the steady-state fluid volumetric flowrate response and the pressure response with respect to the different values of control input voltage, the characteristic curves of $\bar{\bar{u}}_R^{\text{in}}(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ for the tested right-cylinder hydraulic servo equipment can be obtained and are plotted in Fig. 3. Figs. 3(a) and (b), respectively, indicate the equivalent control input functions of $\bar{\bar{u}}_R^{\text{in}}(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ at the inlet and outlet ports of the proportional directional valve when the single-rod actuator is moving upward; likewise, Figs. 3(c) and (d) are, respectively, the equivalent control input functions of $\bar{\bar{u}}_R^{\text{in}}(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ at the inlet and outlet ports of the proportional directional valve when the single-rod actuator is moving downward. Any of these characteristic curves appears to be parabolic in the initial and end portions, but follows a straight line in the middle portion. Here, approximated by curve fitting and using the least-square error method, the equivalent control input functions of $\bar{\bar{u}}_R^{\text{in}}(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ can be represented by the following equations:

$$\bar{\bar{u}}_R^{\text{in}} = \begin{cases} 25.2194 \times 10^{-8} & \text{if } 8.06 < u_R \leq 10, \\
[-0.9594(u_R - 8.06)^2] + 25.2194 \times 10^{-8} & \text{if } 4.09 < u_R \leq 8.06, \\
(7.6272u_R - 21.0968) \times 10^{-8} & \text{if } 3.53 < u_R \leq 4.09, \\
2.4893(u_R - 2)^{2} \times 10^{-8} & \text{if } 2.00 < u_R \leq 3.53, \\
0 & \text{if } -1.80 \leq u_R < 2.00, \\
-0.8466(u_R + 1.8)^{2} \times 10^{-8} & \text{if } 3.47 \leq u_R < -1.8, \\
(-2.8320u_R + 7.4659) \times 10^{-8} & \text{if } -7.21 \leq u_R < -3.47, \\
[0.1190(u_R + 10.5798)^{2} - 17.7178] \times 10^{-8} & \text{if } -10.00 \leq u_R < -7.21, \\
\end{cases} \tag{3}$$

$$\bar{\bar{u}}_R^{\text{out}} = \begin{cases} 22.7629 \times 10^{-8} & \text{if } 9.64 < u_R < 10.0, \\
[-1.8529(u_R - 9.64)^2] + 22.7629 \times 10^{-8} & \text{if } 8.64 \leq u_R < 9.64, \\
(3.6982u_R - 11.0424) \times 10^{-8} & \text{if } 3.97 \leq u_R < 8.64, \\
0.9378(u_R - 2)^{2} \times 10^{-8} & \text{if } 2.00 \leq u_R < 3.97, \\
0 & \text{if } -2.30 \leq u_R < 2.00, \\
-1.7116(u_R + 2.3)^{2} \times 10^{-8} & \text{if } 4.01 \leq u_R < -2.30, \\
(-5.8448u_R + 18.4327) \times 10^{-8} & \text{if } 7.35 \leq u_R < 4.01, \\
[10.7042(u_R + 11.4982)^{2} - 36.6441] \times 10^{-8} & \text{if } -10.00 \leq u_R < -7.35. \\
\end{cases} \tag{4}$$

As can be seen in the above equations, $\bar{\bar{u}}_R^{\text{in}}(u_R)$ and $\bar{\bar{u}}_R^{\text{out}}(u_R)$ are obviously different due to the inherent dynamic differences between the inlet and outlet flows in the right
proportional directional valve. Performing the same procedure on the left-cylinder hydraulic servo system yields experimental data related to $\bar{u}_{\text{in}}^L(u_L)$ and $\bar{u}_{\text{out}}^L(u_L)$ that is plotted in Fig. 4; the corresponding estimated functions are similar to Eqs. (3) and (4) but not presented here. Note from Figs. 3 and 4 that the two hydraulic actuator assemblies evidently have different responses. This fact has a significant impact on the motion control of the electro-hydraulic system when one of the requirements for the desired motion is to synchronize the displacement of the two actuators.

Considering the linear motion of two actuators with payloads (see Fig. 2) and applying the Newton’s second law, the dynamic equation governing the motion of the cylinder can be described by

$$M_K \ddot{x}_K = (P_{K1} A_{K1} - P_{K2} A_{K2}) - \text{sign}(u_K) f_K(x_K) - M_K g,$$

(5)

where the subscripts $K \in \{R, L\}$; $x_K$, $\dot{x}_K$ and $\ddot{x}_K$ are, respectively, the position, velocity, and acceleration of the inertial load in the cylinder; $M_K$ is the mass of the payload (including the piston) in the cylinder; $g$ is the earth’s gravity; $A_{K1}$ and $A_{K2}$ are the cross-sectional areas of the bottom side and top side of the piston, respectively; and $f_K$ is the friction force against the moving piston. The inclusion of the gravity term $M_K g$ in Eq. (5) depends on the orientation of the two cylinders. Subtracting $P_{K2} A_{K2}$ from $P_{K1} A_{K1}$ gives the force arising from the differential pressure acting on the piston. The second term on the right-hand side of Eq. (5) represents the combined viscous friction forces acting on the piston and the rod that can be expressed by an empirical function of the velocity,

$$f_K(x_K) = A e^{B x_K} + C x_K^2 + D x_K + E.$$

(6)

Note that the above equation is derived from the combination of the exponential friction model, $(A e^{B x_K} + D x_K + E)$, and the term $C x_K^2$, which is part of the polynomial friction model, since it can be observed from Fig. 5 that there is a nonlinear friction characteristic after the stribeck region. Those friction models can be found in the studies of Armstrong-Helouvry (1991). The parameters $A$, $B$, $C$, $D$, and $E$ must be identified from experimental data. When the piston moves at a constant velocity, the acceleration is equal to zero, and then the characteristic curve of the frictional force can be obtained from the derivation of Eq. (5). The relationships between the estimated friction and the velocity of the right (left) piston moving upward/downward in two cylinders are shown in Fig. 5(a) and (b) (Fig. 5(c) and (d)). It is evident from Fig. 5 that the friction characteristics are different.
between the two cylinders, as well as varying within the same cylinder with motion direction, load, and control inputs. Since the friction model changes with the direction of motion, it can be defined as

\[
f_K(\dot{x}_K) = \begin{cases} 
  f_{K1}(\dot{x}_K) & \dot{x}_K \geq 0 \\
  f_{K2}(\dot{x}_K) & \dot{x}_K < 0 
\end{cases}
\] (7)

for each of the cylinders. Table 2 summarizes coefficients of Eq. (6) obtained using least-square regression on the experimental data. As can be observed from Fig. 5, the frictional force can be clearly divided into static friction, coulomb friction, and viscous friction, no matter when the piston moves upward or downward. Besides, not only are the frictional forces for the piston moving upward and downward different, but also the friction parameters of the left cylinder are different from those of the right cylinder because inconsistent dynamics inherently exist in both cylinders, even though they use the same components.

With the potential of rapid pressure change during closed-loop control, the compressibility of the hydraulic fluid cannot be ignored. Consider the compressibility of the fluid and ignore the valve dynamics (Jelali & Kroll, 2003; Merritt, 1967). The pressure dynamics can be expressed

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Table 2
Hydraulic servo system’s friction parameters

| Parameter | Left cylinder | | Right cylinder | |
|-----------|---------------|-------------------------------------------------|-------------------------------------------------|
|           | Upward motion ($f_{L\text{u}}$) | Downward motion ($f_{L\text{d}}$) | Upward motion ($f_{R\text{u}}$) | Downward motion ($f_{R\text{d}}$) |
| A | 358.11520087 | 211.0138545 | 450.7780046 | 333.3968848 |
| B | -0.0514691 | -0.1050245 | -0.0588830 | -0.0922792 |
| C | 0.0033224 | 0.0007989 | 0.0023003 | 0.0001672 |
| D | -1.2686159 | 1.0632466 | -0.8975068 | 0.9992302 |
| E | 404.6550789 | 243.1870125 | 304.4355496 | 393.8318361 |

---

Fig. 5. The friction characteristic curves: (a) for upward motion at right cylinder, (b) for downward motion at right cylinder, (c) for upward motion at left cylinder, and (d) for downward motion at left cylinder.
as below,

\[
\dot{P}_{K1} = \frac{\beta}{V_K(x_K)} \text{sign}(u_K)(Q_{K1} - A_{K1}\dot{x}_K),
\]

\[
\dot{P}_{K2} = \frac{\beta}{V'_K(x_K)} \text{sign}(u_K)(A_{K2}\dot{x}_K - Q_{K2}),
\]

where \( \beta \) is the bulk modulus of the working fluid in N/m². \( V_{K1}(x_K) \) and \( V_{K2}(x_K) \) are the volumes of the respective chambers (see Fig. 2) in m³. In Eq. (8) and (9), \( V_{K1}(x_K) = A_{K1}\dot{x}_K + V_{PK1} \) and \( V_{K2}(x_K) = A_{K2}(l - x_K) + V_{PK2} \), in which \( l \) is the maximum stroke of the cylinders, and \( V_{PK1} \) and \( V_{PK2} \) are control volumes of the fluid in the inlet pipeline and the outlet pipeline of the proportional directional valve.

2.2. Coupling model of a dual-cylinder hydraulic servo system

For the dual-cylinder electro-hydraulic system of interest, since the working fluid is supplied from a single pump, the flow coupling effect will need to be taken into consideration, see Figs. 1 and 2. Neglecting environmental disturbances, the Bernoulli equation for one-dimensional laminar flows at the flow divider junction shown in Fig. 2 can be written as

\[
P_{Rs} + \frac{1}{2} \rho \left( \frac{Q_{Rs}}{A_{Rs}} \right)^2 + P_{Ls} + \frac{1}{2} \rho \left( \frac{Q_{Ls}}{A_{Ls}} \right)^2 = P_p + \frac{1}{2} \rho \left( \frac{Q_p}{A_p} \right)^2,
\]

where \( P_{Ls} \) and \( P_{Rs} \) are the inlet pressures of the proportional directional valves on the respective sides, in N/m²; \( Q_{Ls} \) and \( Q_{Rs} \) are the inlet volumetric flowrates on the respective sides, in l/min; \( A_{Ls} \) and \( A_{Rs} \) denote the cross-sectional areas (m²) of the inlet fluid pipeline in the left and right proportional directional valves, respectively; and \( P_p, Q_p, \) and \( A_p \) are the fluid pressure (N/m²), total volumetric flowrate (l/min), and pump outlet cross-sectional area in appropriate units.

Note from the conservation of mass that \( Q_p = Q_{Rs} + Q_{Ls} \). Similarly, Fig. 2 demonstrates that \( Q_{R1} = Q_{Rs} \) (for upward motion) or \( Q_{R2} = Q_{Rs} \) (for downward motion) in conjunction with the right proportional directional valve, while \( Q_{L1} = Q_{Ls} \) (for upward motion) or \( Q_{L2} = Q_{Ls} \) (for downward motion) in conjunction with the left proportional directional valve. Therefore, the inlet fluid volume flowrates, \( Q_{Ks} \), either \( Q_{Rs} \) or \( Q_{Ls} \), can be further described as below:

\[
Q_{Ks} = 0.5Q_{K1} + 0.5Q_{K2} + \text{Sign}(u_K)(0.5Q_{K1} - 0.5Q_{K2}).
\]

Consequently, combining Eqs. (10) and (11) and including nonlinear friction and the hydraulic flow coupling effect, the dual-cylinder hydraulic servo system dynamics can be summarized as

\[
M_K\ddot{x}_K = P_{K1}A_{K1} - P_{K2}A_{K2} - \text{sign}(u_K)f_K(\dot{x}_K) - M_{Kg},
\]

\[
\dot{P}_{K1} = \beta h_{K1}(x_K, u_K, p_{Ks}) - \beta h_{K1}(x_K, \dot{x}_K),
\]

\[
\dot{P}_{K2} = \beta h_{K2}(x_K, \dot{x}_K) - \beta h_{K2}(x_K, u_K, p_{Ks}),
\]

where

\[
h_{K1} = \frac{\text{sign}(u_K)}{A_{K1}x_K} \left[ 0.5\dot{x}_K^2 + 0.5\dot{x}_K^2 + \text{sign}(u_K)(0.5\dot{x}_K^2 - 0.5\dot{x}_K^2) \right] \\
\times \sqrt{0.5P_{Ks} - 0.5P_{tank} - \text{sign}(u_K)(P_{K1} - 0.5P_{Ks} - 0.5P_{tank})}
\]

\[
h_{K2} = \frac{\text{sign}(u_K)}{A_{K2}(l - x_K)} \left[ 0.5\dot{x}_K^2 + 0.5\dot{x}_K^2 + \text{sign}(u_K)(0.5\dot{x}_K^2 - 0.5\dot{x}_K^2) \right] \\
\times \sqrt{0.5P_{Ks} - 0.5P_{tank} + \text{sign}(u_K)(P_{K2} - 0.5P_{Ks} - 0.5P_{tank})}
\]

\[
\dot{x}_K = \frac{\dot{x}_K}{x_K}, \quad \dot{x}_K = \frac{\dot{x}_K}{l - x_K}.
\]

2.3. Model validation

In order to verify the modeling performance of the hydraulic servo system, a step input command to one of the cylinder hydraulic servo systems (either the left or the right actuator) is first simply fulfilled and then a comparison between step responses of the experiment and the model is presented. That is, when one cylinder hydraulic servo system is being subjected to verification, another cylinder hydraulic servo system will be inactive.

Fig. 6 presents the position and pressure responses of the left and right cylinders for both the experiment and its corresponding simulation system when a step input voltage of 5 V is applied to the cylinder hydraulic servo system. The figure clearly shows that the experimental responses are slower than the model’s responses because the characteristics of slow-dynamics components, such as the hydraulic pump, the gas-loaded accumulator and so forth, are simply ignored in the model deviation. Moreover, Figs. 7(a) and (b) respectively demonstrate the pressure responses for the step input voltages of 3 and 7 V applied to the right-cylinder hydraulic servo system; Figs. 7(c) and (d) are, respectively, the corresponding results for the step input voltages of 3 and 4 V applied to the left-cylinder hydraulic servo system. Obviously, these results indicate that the experimental system has a sluggish dynamic response in comparison with the plant model. It can be observed that the greater the step control input voltage, the greater the difference between the pressure responses of the hydraulic model and those of the experimental system. All this further confirms that the model did not capture the slow transient response of the system, but it does match the steady-state response of the system.
3. Motion synchronization control

Taking into consideration the effect of unbalanced loading as well as the system uncertainties verified in Section 2.3, here a better controller design will be used to reduce the requirement for a more accurate model of the electro-hydraulic system with unbalanced loading and system uncertainties. Due to the inconsistent dynamics between the two independent hydraulic servo systems, it is difficult to design a controller based on the one and then expect it to fit both systems. Therefore, this paper proposes a nested fuzzy controller design approach, shown in the control block diagram of Fig. 8(a), which comprises two cylinder controllers and a FCC to achieve the positioning associated with motion synchronization of dual-cylinder hydraulic systems. The individual cylinder controller consists of a FFC and a FTC, as shown in Fig. 8(b). The cylinder controller operates independently and is designed to track a desired position reference for each individual cylinder. The FCC is designed to improve motion synchronization between the two cylinders by augmenting the control signal in the individual valves.

3.1. Feedforward controller design

Based on a single-cylinder hydraulic actuator model, the FFC algorithm implements the equivalent inverse dynamics by assuming that the actuator output is the desired reference trajectory. For upward movement, assuming zero tank pressure $P_{tank}$, Eqs. (1) and (2) can be modified to

$$Q_{K1} = A_{K1} \ddot{x}_d = \bar{u}_{in}^m(u_K) \sqrt{P_{KS} - P_K1},$$  

$$Q_{K2} = A_{K2} \ddot{x}_d = \bar{u}_{out}^m(u_K) \sqrt{P_K2}.  \tag{16}$$

In order to simplify the FFC design, here the coupling effect given in Eqs. (10) is temporally ignored—i.e., $P_{R1} = P_{L1} = P_{in}$—while this coupling effect will be depressed and compensated for by the FCC in the latter design. Therefore, the above equations can be expressed by

$$P_{K1} = P_{KS} - \frac{A_{K1}^2 \ddot{x}_d^2}{[\bar{u}_{in}^m(u_K)]^2}, \tag{17}$$

$$P_{K2} = \frac{A_{K2}^2 \ddot{x}_d^2}{[\bar{u}_{out}^m(u_K)]^2}. \tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (5) yields

$$\frac{A_{K1}^3 \ddot{x}_d^3}{[\bar{u}_{in}^m(u_K)]^3} + \frac{A_{K2}^3 \ddot{x}_d^3}{[\bar{u}_{out}^m(u_K)]^3} = A_{K1} P_{KS} - f_K(\ddot{x}_d) - M_K(\ddot{x}_d + g), \tag{19}$$

where $\ddot{x}_d$ and $\ddot{x}_d$ are the velocity and the acceleration of the desired trajectory profile, respectively. In Eq. (19),
\[ u_K = \bar{u}_{K1}(u_K) = \sqrt{\frac{(m_{KJ})^2 A_{K1} x_d^2 + A_{K2} x_d^2}{(m_{KJ})^2 [A_{K1} P_{Kr} - f_K(\dot{x}_d) - M_K(\dot{x}_d + g)]}} \]

The FFC input \( \bar{u}_{K1} \) can be determined if the payload \( M_K \) and proportional constant \( m_{KJ} \) are known.

### 3.2. Fuzzy tracking controller design

Although the FFC approach provides a simpler way to design a tracking controller, it easily leads to a large position-tracking error because it ignores the model uncertainties. To compensate for the inaccuracies caused by those nonlinear properties and disturbances, a FTC was inserted to enhance the position-tracking performance of the individual actuators along the desired trajectory. Basically, a typical fuzzy controller can be constructed by the following four components: (1) a fuzzification module, (2) a fuzzy knowledge-based rule base, (3) a fuzzy inference engine, and (4) the defuzzification module. In this paper, a PD-like fuzzy algorithm (Chao & Teng, 1997; Driankov,
Hellendoorn & Reinfrank, 1996) was chosen to yield the desired response. To interpret the fuzzy rule base, the position and velocity response—the pair of \( [e, \dot{e}] \) of a standard second-order system excited by a step input—are considered in the design of fuzzy logic control. By adopting an adequate response of a second-order controlled system, the fuzzy rule base can be built from the phase portrait of \([e, \dot{e}]\). In this study, seven membership functions (MF), \{NB, NM, NS, ZE, PS, PM, and PB\}, that represent \{negative big, negative medium, negative small, zero, positive small, positive medium, and positive big\}, respectively, will be used for each FTC. Triangular MFs \( \mu(x) \) are used for fuzzy sets NM, NS, ZE, PS, and PM,

\[
\mu(x) = \begin{cases} 
0 & x \leq a_i \\
\frac{(x-a_i)}{(b-a_i)} & a_i < x \leq b_i \\
\frac{(c_i-x)}{(e-b_i)} & b_i < x < c_i \\
1 & c_i \leq x \end{cases}, \quad i \in \{NM, NS, ZE, PS, PM\}.
\]

(21)

Trapezoidal MFs are used for the fuzzy sets NB and PB, i.e.,

\[
\mu_{NB}(x) = \begin{cases} 
1 & x \leq b_i \\
\frac{(c_i-x)}{(e-b_i)} & b_i < x \leq c_i \\
0 & c_i < x \end{cases}, \quad \text{and}
\]

\[
\mu_{PB}(x) = \begin{cases} 
0 & x \leq a_i \\
\frac{(c_i-x)}{(e-b_i)} & a_i < x \leq b_i \\
1 & b_i \leq x \end{cases}.
\]

(22)

respectively. In Eqs. (21) and (22), parameters \( a_i, b_i, \) and \( c_i \) are the breakpoints of the \( i \)th triangular or trapezoidal membership function of the input or output variable, \( x \). When the inputs of position-tracking error \( e \) and velocity-tracking error \( \dot{e} \) are fed into the fuzzy controller, the control input \( u \) to the cylinder plant can be determined from fuzzy rules presented in Fig. 9, according to the fuzzy inference engine of Mamdani’s MAX–MIN method and the defuzzification module of the center-of-gravity method. With two antecedents \( [e, \dot{e}] \) and 7 × 7 fuzzy rules in Fig. 9, the control input produced by the fuzzy rule base behaves as the PD-like controller that is often used for those applications whose responses can be depicted using a two-dimensional phase plane (Driankov et al., 1996). In practice, the fuzzy rules of the control input can be divided into four regions, as marked in Fig. 9. The fuzzy rules in regions 1, 2, 3, and 4 dominate the controlled system’s function at the rise time, the maximum overshoot time, the convergence state, and the steady state, respectively. Based on the information of \([e, \dot{e}]\) and its related control input, the fuzzy controller can intuitively make a good response within a very short time without any prior knowledge about the system modeling.

The FTC here is used to enhance the trajectory-tracking task of the individual cylinder together with the FFC. To design a PD-like fuzzy controller for each cylinder, both of the position-tracking errors \((x_L-x_d)\) and \((x_R-x_d)\) and the velocity-tracking errors \((v_L-v_d)\) and \((v_R-v_d)\) are introduced as the input signals \([e, \dot{e}]\) for the left and the right cylinders, respectively. Based on the maximum velocity along the trajectory in the controlled system being 120 mm/s, the adjustable margin of velocity being 30 mm/s, and with the FFC controller for the dual-cylinder electro-hydraulic servo system, the fuzzy membership functions for the left and right FTCs are given in Tables 3 and 4. Notice that the maximum position-tracking error is restricted to \( \pm 1.5 \) mm and the maximum velocity of each piston of the cylinders is limited to 120 mm/s for the sake of eliminating the big position-tracking error. In order to synchronize the positions of the left and right actuators, the spans of the fuzzy membership functions of position errors \( e \) of both FTCs are planned to be the same, as are the control commands \( u \) of both FTCs. However, the spans of the membership functions of \( \dot{e} \) of both FTCs are different in supporting different responses, due to the distinct mechanical characteristics of the proportional directional valve and the actuator, as shown in Figs. 6 and 7.

3.3. Fuzzy coordination controller design to enhance motion synchronization

According to the design mentioned above, the individual cylinder controller is designed to improve the position-tracking performance of the respective actuator under model uncertainty and disturbances. It does not consider the inherent coupling effect in the dual-cylinder hydraulic dynamics. In addition, the individual cylinder controller is not designed to achieve identical closed-loop dynamics between the two actuators. Without explicit consideration to achieve identical closed-loop dynamics or synchronization errors (Chiu & Tomizuka, 2001; Sun & Chiu, 2002), synchronization between the two cylinders cannot be achieved. For a dual-cylinder electro-hydraulic left system, the motion synchronization and the individual tracking performance are equally important. To address motion synchronization, a FCC is proposed to connect the two individual cylinders, and to actively coordinate and

Fig. 9. The fuzzy rules of control input function for PD-like fuzzy controller.
synchronize the motions between the two closed-loop controlled hydraulic actuators.

As shown in Fig. 8, the inputs to the FCC are the position synchronization error \((x_R - x_L)\) and the velocity synchronization error \((v_R - v_L)\) between the two cylinders. From Tables 3 and 4 it can be found that the interval [minimum, maximum] of each fuzzy membership function of the control input of the right FTC is different from that of the left FTC. This phenomenon is also true for the fuzzy membership functions of the velocity-tracking error for both cylinder controllers. In order to achieve motion synchronization and positioning, the fuzzy membership functions of the FCC will be designed according to these fuzzy membership functions and create a compromise between them. Finally, the FCC membership functions are defined in Table 5. The control output of the FCC will slow down the right actuator and speed up the left actuator simultaneously when the position and velocity synchronization errors are both positive. Similarly, the control output of the FCC will speed up the right actuator and slow down the left actuator simultaneously when the position and velocity synchronization error are both negative.

4. Simulation and experiment results

4.1. Simulation results

In order to evaluate the performance of the controller designs mentioned above, some different cases of loadings...
upon two pistons are considered in the simulation studies for each of the FFC, FTC, and FCC controller designs, based on the system model derivation in Section 2. When the maximum velocity and positioning distance are respectively defined as 120 mm/s and 900 mm for the test experimental system, the desired trajectory can be designed to arrive at the destination of 900 mm in 7.5 s (Cheng & Chen, 1998). According to the analysis of the friction force associated with the experimental system, as shown in Fig. 5, the static friction force of the hydraulic servo system can be estimated as 762 N. Using the system parameters summarized in Table 1 and the experimental data shown in Figs. 6 and 7, the proportional constant of the FFC input functions (see Eq. (19)) for the right (left)-cylinder hydraulic servo systems is estimated to be $m_{RU} = 0.42$ and $m_{RD} = 1.18$ ($m_{LU} = 0.46$ and $m_{LD} = 2.3$), respectively. In addition, since the loading on each cylinder is an unknown parameter in the dual-hydraulic system, the mean value of maximum and minimum loadings, 105 kg, is chosen as the parameter of the FFC design so that the maximum loading uncertainty can be constrained within a known region.

To evaluate the position-tracking performance of the FFC design, two cases of loading, 210/0 and 0/0 kg on the left/right pistons, are considered in the simulation studies. Fig. 10 illustrates the position responses, position-tracking errors, and synchronization errors for the case of the unbalanced loadings of 210/0 kg on left/right cylinders. The results clearly indicate that the right piston position response is always faster than the desired trajectory, but the left piston position is constantly behind the desired trajectory. That is, when the loading on the piston is lighter than the designated loading of 105 kg, the control input produced by the FFC will enforce the position response more quickly than will the demand of the desired trajectory, and the maximum position-tracking error will be around 35 mm. Conversely, when the loading on the piston is greater than the designated loading of 105 kg, the FFC input will result in a slower response and the maximum position-tracking error will be around –140 mm. It can also be observed that the piston position response controlled by the FFC design linearly follows the desired trajectory profile, fast or slow, because of its open-loop control methodology. For the worst unbalanced loadings, the synchronization error between the two cylinders can go up to around 175 mm, as shown in Fig. 10(c). For the balanced case of 0/0 kg on left/right cylinder, Fig. 11 presents the position-tracking error and synchronization error. This figure clearly shows that each piston position response is always faster than the desired trajectory because the loading is less than 105 kg. However, the response of the position-tracking error of the right piston and that of the left piston shown in Fig. 11(a) are not identical because of the inconsistent dynamics of the two electro-hydraulic actuators. This phenomenon can be verified from the FFC design presented in Eqs. (15)–(20), because some nonlinear dynamics of the system have been
either linearized or ignored. In comparison with the case of unbalanced loading shown in Fig. 10, the synchronization error between two cylinders with balanced loadings shown in Fig. 11(b) is also depressed to 33 mm. Nevertheless, based on simulation studies it is difficult to successfully exclude the effects of system uncertainties and disturbances using the derived dynamic models.

Next, to verify the performance of the proposed FTC design, the simulation for the unbalanced case of 210/0 kg on left/right cylinder was performed. Fig. 12 shows that the position-tracking errors of the left and right cylinder motions have been largely reduced to within the error bounds of 0.5 and 1.5 mm, respectively. In comparison with Fig. 10, Fig. 12 also demonstrates that a cylinder controller can effectively reduce the position-tracking error because the FTC design runs as a closed-loop control, whereas the FFC design runs as an open-loop control. Therefore, it can be concluded that the FTC design attains better tracking performance upon the desired trajectory than does the FFC design. Moreover, the synchronization error of the FTC design is within ±0.5 mm, except in the launch region, as shown in Fig. 12(b). Undoubtedly, the FTC design can passively improve the synchronization error of the dual-cylinder more than the FFC design.

Finally, to evaluate the performance of the FCC design, the case of unbalanced loadings (210/0 kg on the left/right cylinder) as discussed in the FFC and FTC experiments is also verified. The simulation results shown in Fig. 13 clearly demonstrate that the proposed add-on FCC design can further effectively reduce the synchronization error between the two actuators to within almost 0.1 mm during the overall motion process (excluding the launch and stop regions), regardless of the unbalanced loadings. As well, the proposed integrated fuzzy controller design also outperforms the cylinder controller design, as is evident from comparing the simulation results in Fig. 13 with those in Fig. 12, since the position tracking and synchronization errors are extremely decreased.

4.2. Experiment results

The experimental setup for the positioning and motion synchronization of a dual-cylinder electro-hydraulic system, using the sampling rate of 200 Hz, is illustrated in Fig. 1. The plug-in PCI-1602 multi-function DAQ Board and PIO-DA8 D/A board, from ICP-DAS, were used to fetch the position signals from potentiometers and to feed the control commands into the linear amplifiers of the two proportional directional valves, respectively. Filtered position signals are used to estimate cylinder velocity. The working pressure of the experimental system is set at 4.903 MPa and the corresponding volumetric flowrate leaving the hydraulic vane pump is estimated to be 27.68571/min. Since the head end area of the piston is
1.25664 \times 10^{-3} \text{m}^2$, the maximum motion speed of the single-rod cylinder will never exceed 367.19 mm/s. With two cylinders operating together, the maximum speed of the cylinders must be less than 183.6 mm/s. The maximum speed is set at 150 mm/s with a 180 kg load on each cylinder.

To evaluate the performance of the integrated fuzzy controller design, the maximum unbalanced loadings of 0/210 or 210/0 kg on left/right cylinder were considered here. In the case of 210/0 kg being loaded on left/right actuators, Fig. 14 shows the experiment results for the initial positions of the left and right actuators located at 20 and 0 mm, respectively. The proposed add-on FCC approach can effectively reduce the synchronization error between the two actuators to almost within \pm 4 mm, except at the start portion, regardless of the unbalanced loadings. By comparison with Fig. 13, Fig. 14 shows that the position-tracking error and synchronization error are a little larger than those mentioned in the previous simulation case. However, the experimental study clearly confirms that the FCC approach applied on an actual system has just as agreeable a position tracking and synchronization performance as does its simulation study, except for the influence from actual measurement noise. In addition, regarding the performance of the integrated fuzzy controller for the payloads of 0/210 kg on the left/right single-rod actuator without initial position error, Fig. 15 presents experimental results that are opposite to those in Fig. 14. Figs. 14 and 15 show that both the position-tracking error and the motion synchronization can be improved at the same time and quickly, even during the launching period. Furthermore, in consideration of the problem of the accumulated position error, Fig. 16 demonstrates the experimental results of a multi-stage positioning process compensated by the proposed method for unbalanced loadings. Both the synchronization error and the position tracking error are depressed to within \pm 5 mm. In addition, it is worth noting that the measurement deviation of the position sensor is $5 \text{V} \times 0.0005 = 0.0025 \text{V}$, i.e., 2.5 mm, which is based on the potentiometer specifications of a stroke of 1 m, a supply voltage of 5 V, and an independent linearity of 0.05%, as shown in Table 1. For the experimental studies of the proposed integrated fuzzy controller, the position-tracking error and synchronization error of the dual-cylinder are within the acceptable error range, which is twice the position measurement, i.e., \pm 5 mm, as shown in Fig. 8. If a precise position sensor is used, better control performance by the proposed control approach can be obtained.

5. Conclusions

In this paper, an integrated fuzzy controller design, which is realized by individual cylinder controllers combined with

![Fig. 14. Experimental results of the FCC design for the loadings of 210/0kgs on left/right cylinder: (a) position-tracking error at the left cylinder, (b) position-tracking error at the right cylinder, and (c) synchronization error.](image1)

![Fig. 15. Experimental results of the FCC design for the loadings of 0/210kgs on left/right cylinder: (a) position-tracking error at the left cylinder, (b) position-tracking error at the right cylinder, and (c) synchronization error.](image2)
a FCC, has been proposed to improve the position tracking and motion synchronization performances of a dual single-rod hydraulic actuator system. Simulation studies and experimental verifications demonstrated the effectiveness of the proposed controller in reducing the position-tracking error as well as maintaining the motion synchronization of the hydraulic actuators with different loading scenarios. Without modification to system components and design, the proposed control approach can maintain the position-tracking error and position-synchronization error to within 2 times of the measurement resolution. In the future, more accurate noncontact position transducers will be adopted to reduce sensor noise. Expanding the experimental setup to a high-pressure hydraulic system will also be investigated.

References


